Interfacial waves
in
multifluid flows:
Nonlinear spatial evolution

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Acknowledgments

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If you wanted to just work on the most important problem, everyone could work on flow regimes!

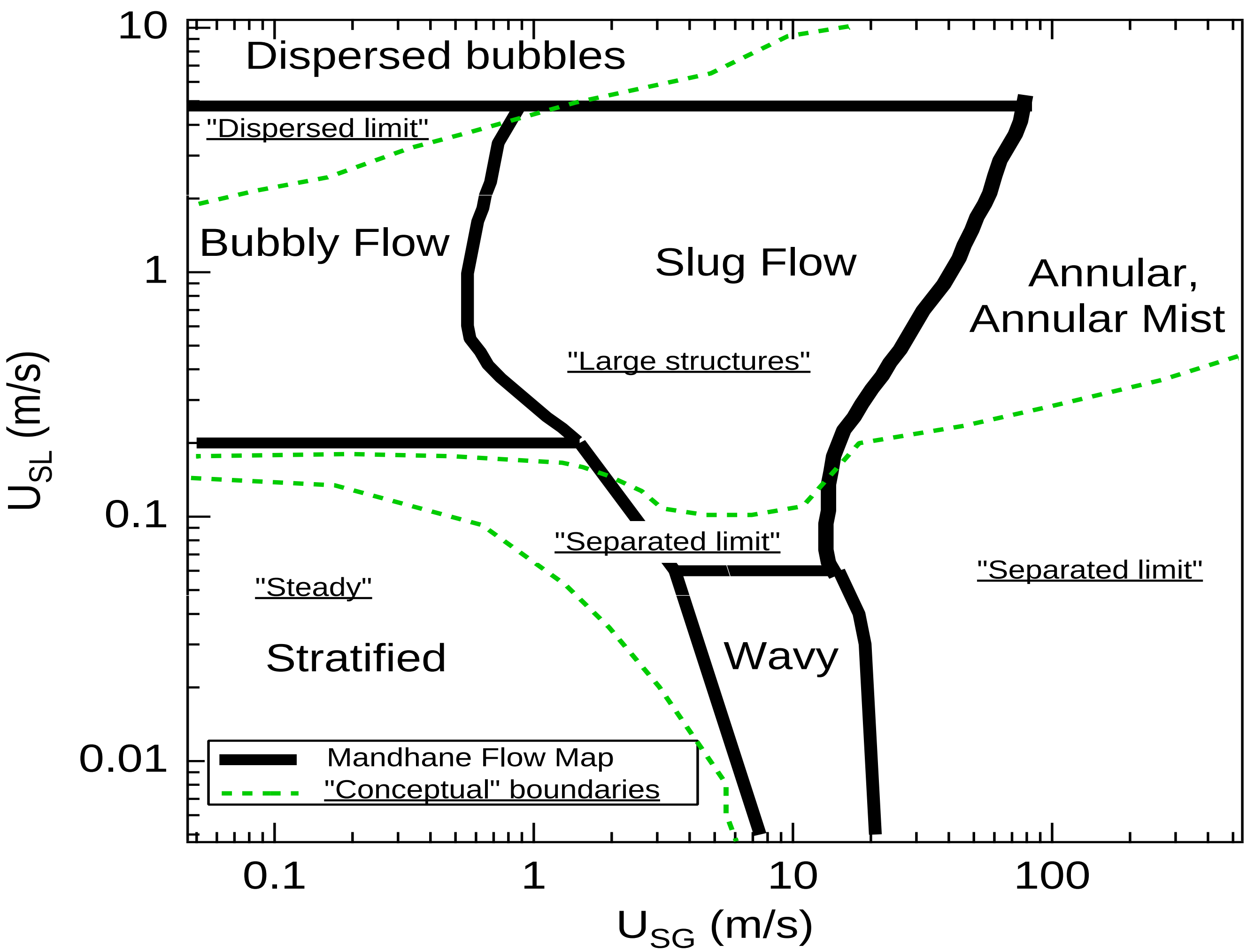

Figure 1a. Flow regime map for horizontal flow (Mandhane et al., 1974) with rough boundaries for different limiting analyses added.

Flow geometry of interest

Two-layer, horizontal two-fluid flow

Problem gets interesting when waves begin to form!!

Linear stability of long waves in two-layer channel flow

This notebook has been written in *Mathematica* by

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Version: 6/13/98

This notebook solves the longwave stability problem for two-layer, pressure driven channel flow

References:

C. -S Yih (1967) "Instability due to viscosity stratification", J. Fluid Mech., **27** pp337-352**.**
S. G. Yiantsios and B. G. Higgins (1988) "Linear stability of plane Poiseullie flow of two-superposed fluids", Phys. Fluids**, 31** pp3225-3238**.**

The 0 order and first order terms for the wave velocity for a pressure driven channel flow are obtained from a long wave expansion. All of the necessary manipulations are shown for this direct perturbation solution method. I plan to add the solution method using the adjoint system (see P. J. Blennerhassett, Trans. Roy. Soc. 298, pp451-494 (1980)) sometime in the next few months.

Why study waves?

1. Application to *flow regime transition* and overall flowrate – pressure drop properties of separated multifluid flows, atomization, heat and mass transfer

2. Interesting academic problems that can be used to advance understanding of analytical and numerical solution techniques, develop new experiments and to educate students.

Problems of interest today

A. We will examine initial wave formation experimentally and theoretically with a particular interest in the weakly nonlinear problem.

   We will see that nonlinear effects can stabilize linearly unstable waves at small amplitude.

- Two possibilities exist for the wave field at this point:

   1. The waves exist as a family of steady states
   2. The waves are continually oscillating

B. We will show a wave experiment that prevents the complication of wave evolution with distance

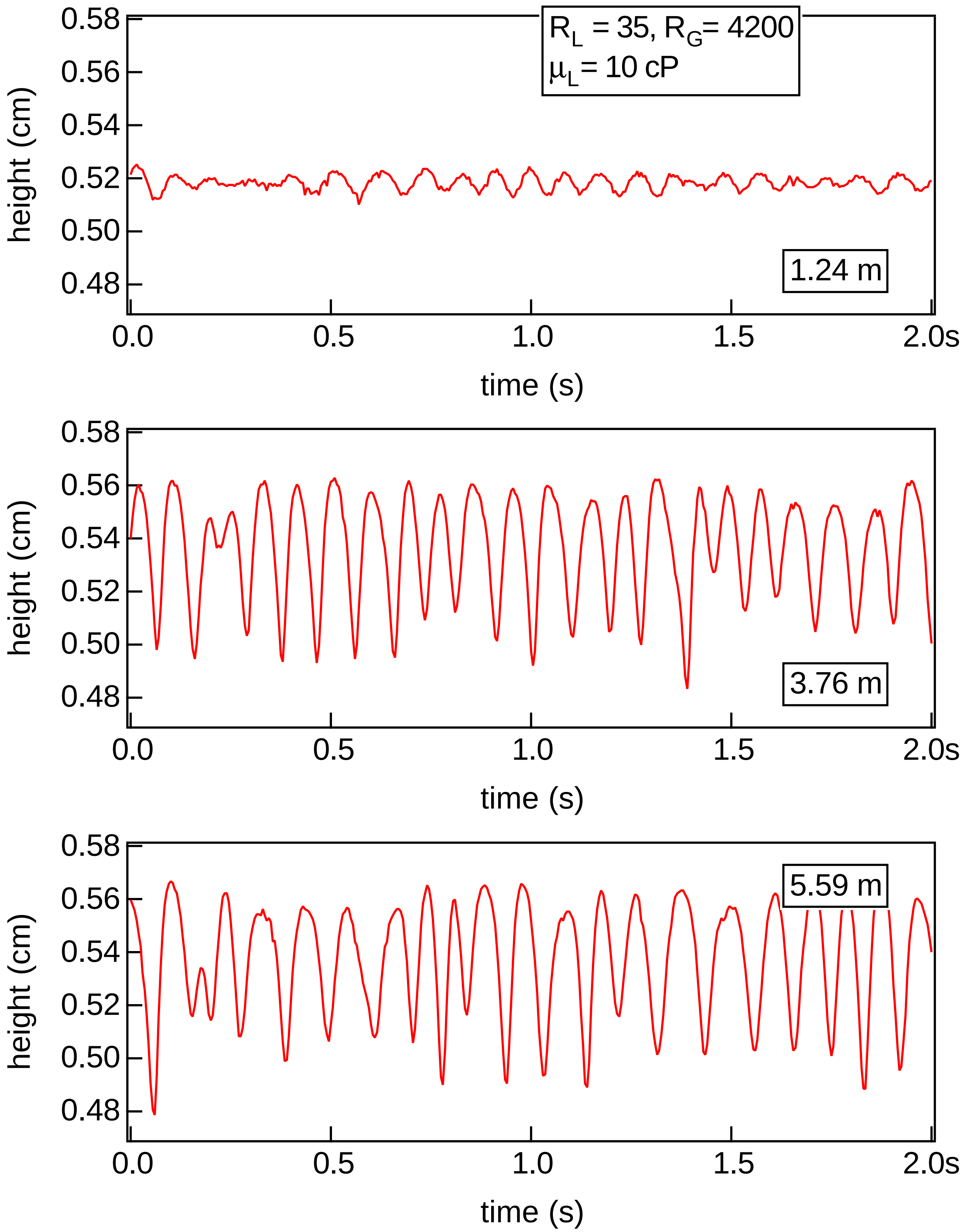
- Two-liquid, matched density, rotating Couette device

C. Continuing work on

   1. Two-layer oscillatory flows
   2. Finding the limits of linear stability
   3. Region of subcritical and supercritical bifurcation

Measurements of interfacial waves for a gas-liquid flow in a horizontal channel at increasing distance (fetch).

Under these conditions, waves grow up and saturate.



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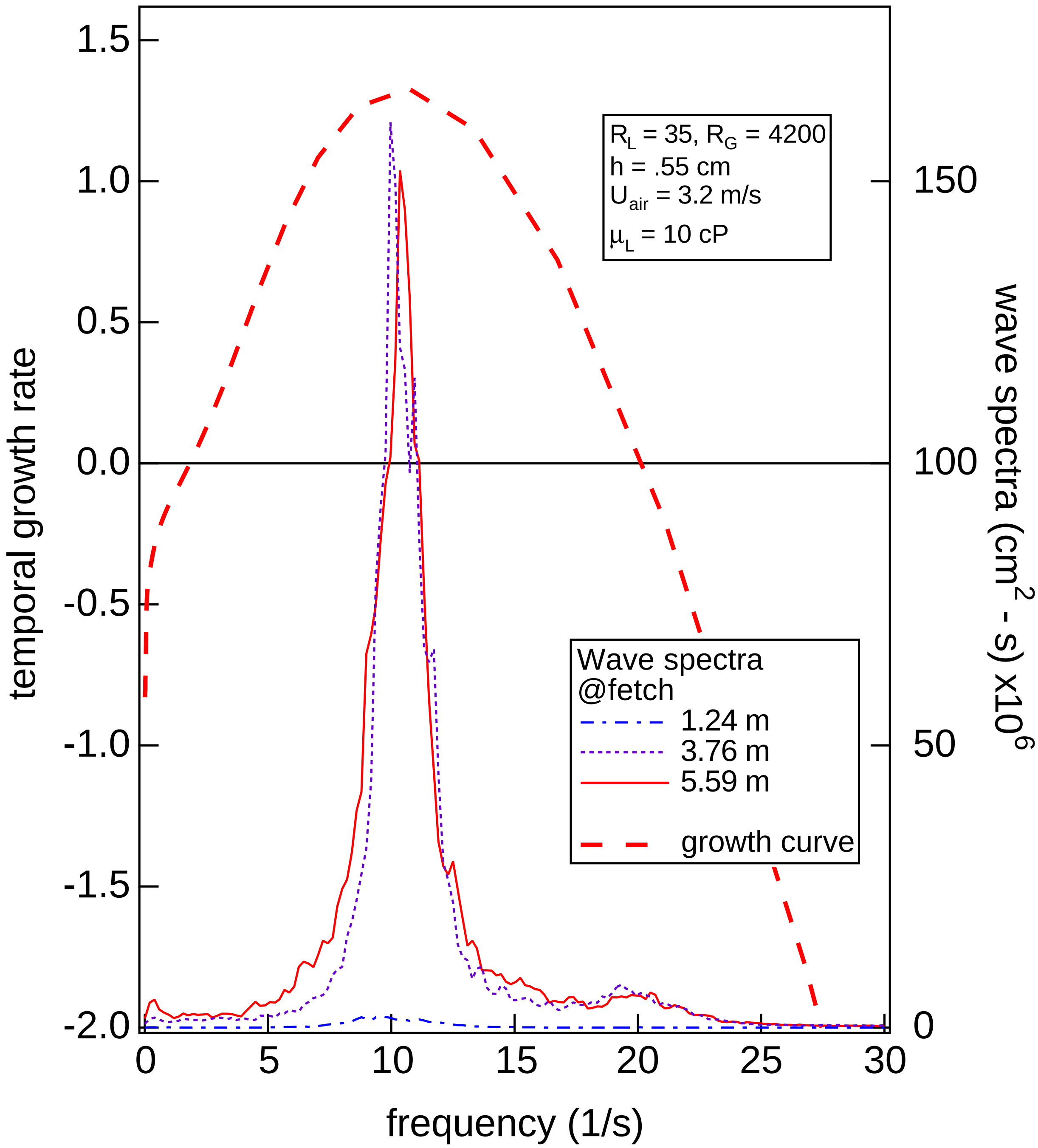
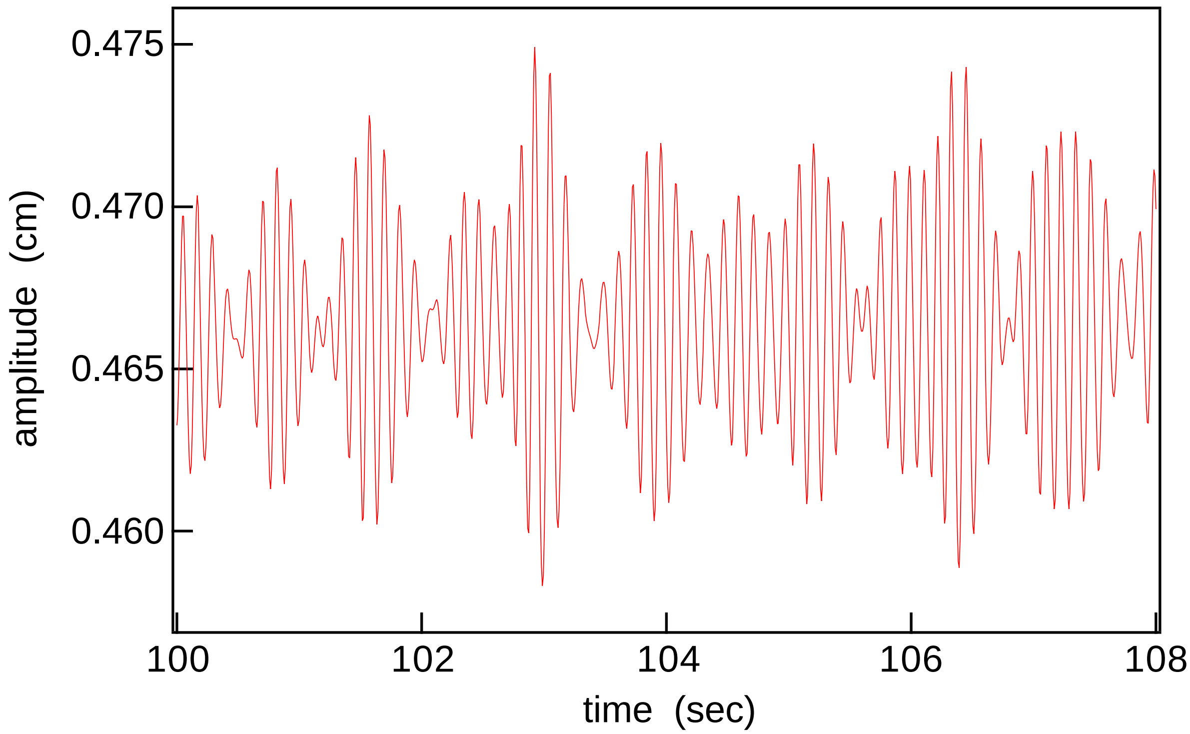
Comparison of linear stability prediction and wave spectrum. Wavelength remains in the region of the fastest growing waves.



Even if the waves have saturated (on average), individual waves can continue to oscillate



Wave tracing at R_L = 30, R_G = 3930, μ_L = 16 cP, h = .47 cm. Very severe beating is present. Local dynamics prevent any chance of steady waves.

The plot of amplitude versus wavelength shows no relation

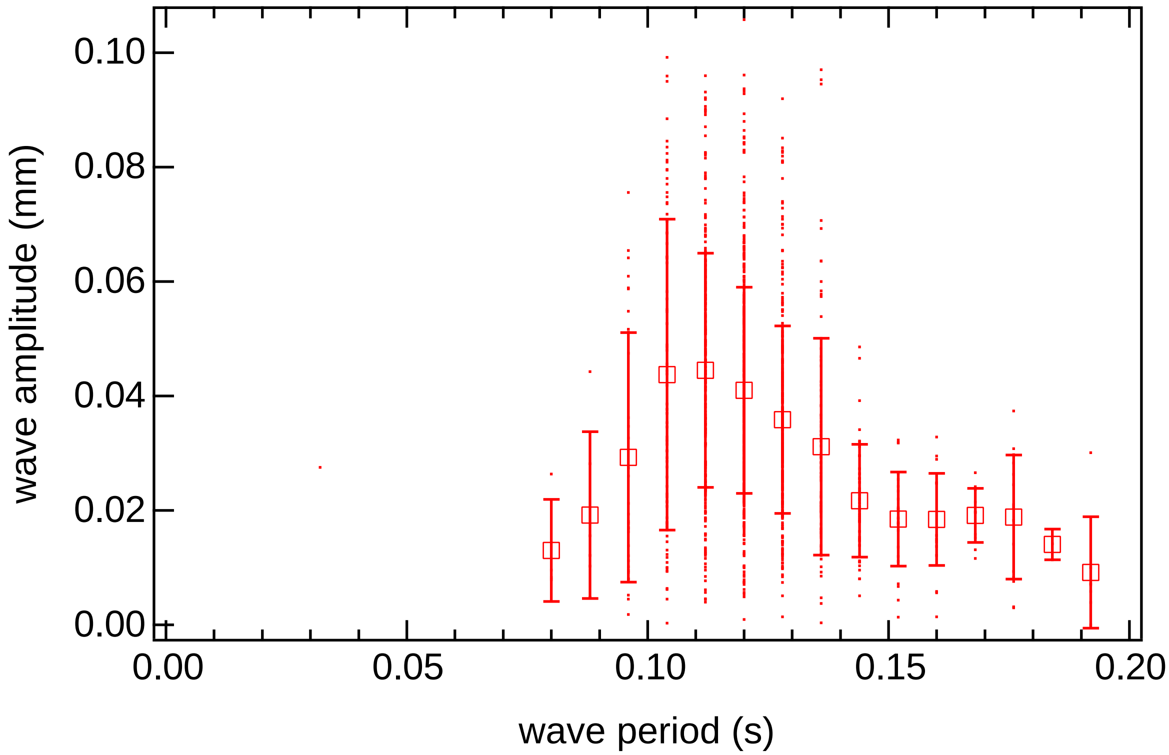
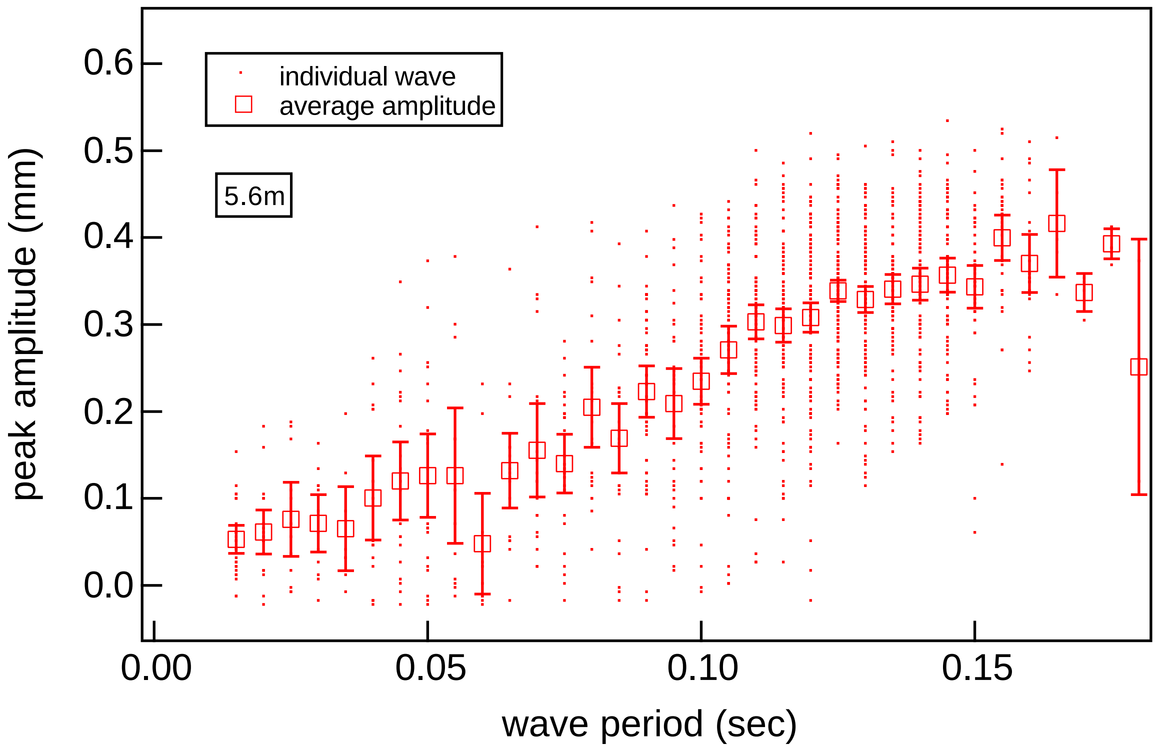


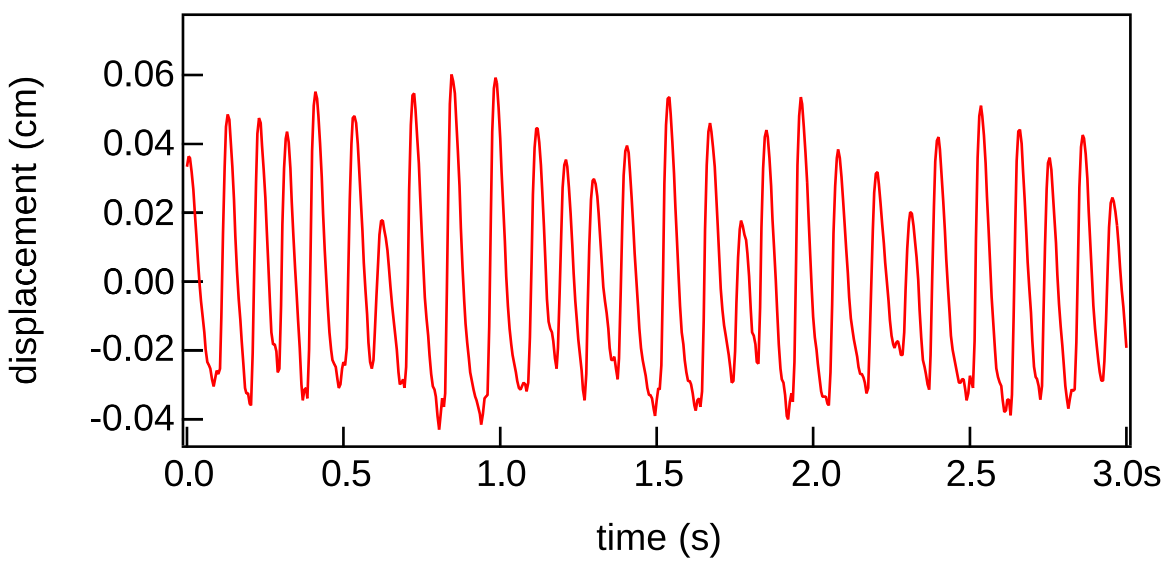
Figure 7. Amplitude-period plot for R_L = 30, R_G = 3930, μ_L = 16 cP, h = .47 cm. No relation between amplitude and time period exists.

Or they can all choose a steady configuration

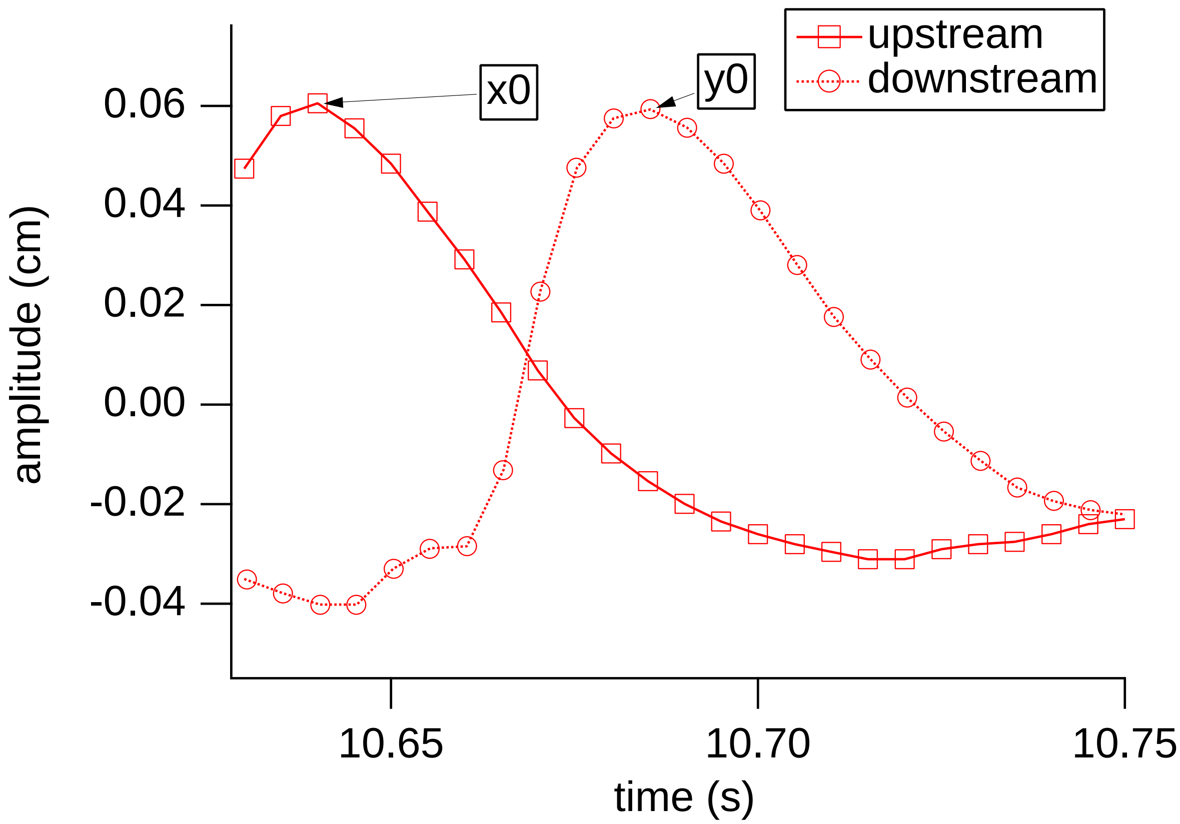
In this plot there is a definite relation between time period and wavelength indicating that the wave field is comprised of a family of steady waves



Amplitude-time period relation for $R_L = 78$, $R_G = 3640$, $\mu_L = 10$ cP, h = .7 cm.



Calculation of wave evolution rate



Surface tracings are represented as vectors:

$\mathbf{X} = \{ x0,x1,x2, ..., xn\}^T$

$\mathbf{Y} = \{ y0,y1,y2, ..., yn\}^T$

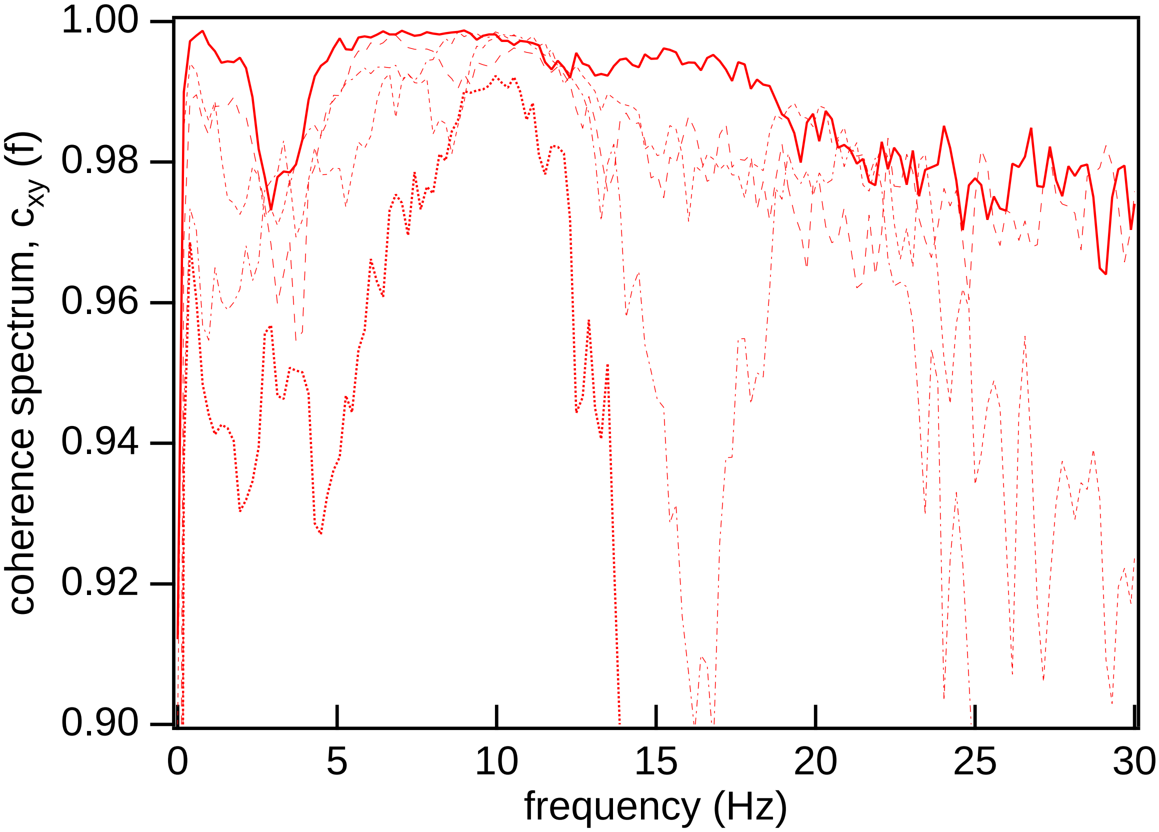
Surface evolution is determined by calculating the Euclidian norm:

E = norm[X,Y]

Measured wave coherence distances compared to *cross - coherence* spectra

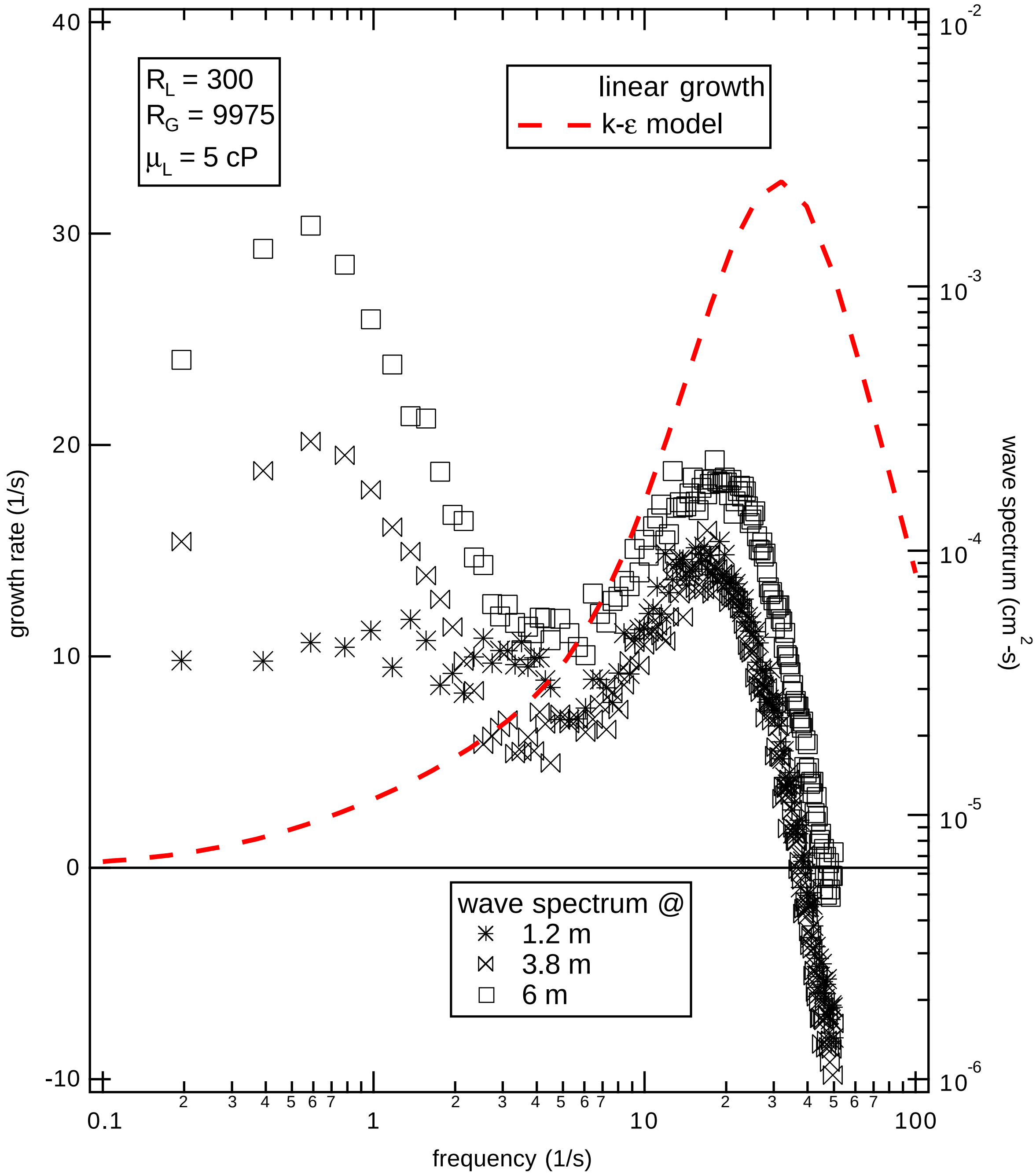
Table 1

R_L	Estimated Coherent distance (cm)
5	3
14	12
35	15
55	22
78	26



For many conditions, waves do not saturate at all.

Prediction of the growth of the low frequency peak is one of the most important questions of wave theory.



Wave spectra as a function of distance showing continued growth of the low frequency peak. The linear stability prediction uses a k-ε turbulence model that is described in a thesis by Uphold.

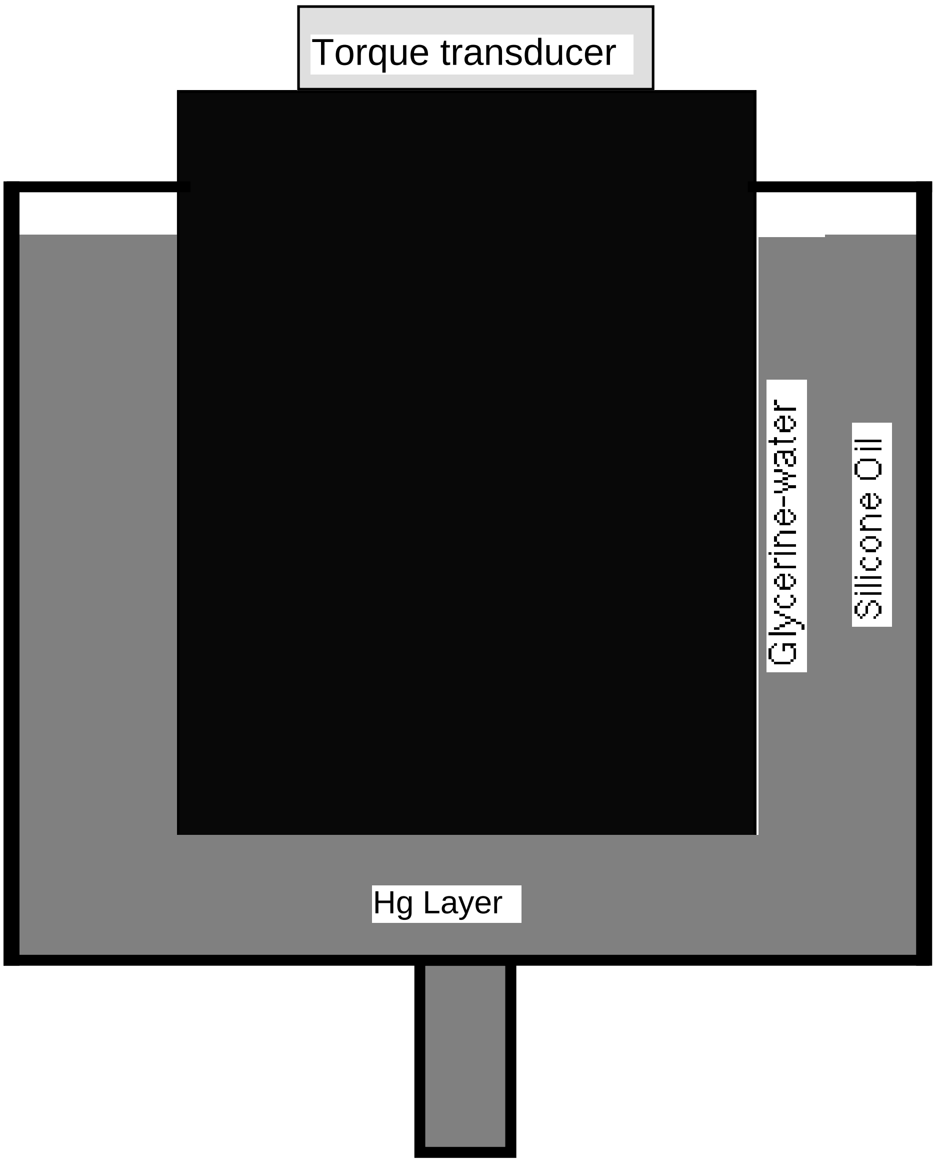
These complications are interesting but make study of waves difficult and confuse generic behavior and dynamics that are caused by the particular system.

We wanted to perform a more precise experiment that eliminated the effects of initial conditions and finite length

The resulting experiment closely reproduces the situation described in C. -S. Yih's 1967 paper since there is no density difference between our fluids and the viscosity difference is responsible for the formation of waves.

Rotating, Two-(matched density) liquid, Couette Flow, Wave Experiment

Outside cylinder is Plexiglas®, Inside cylinder is Aluminum painted black



Outside cylinder is rotated.

Theoretical analysis (linear theory)

The complete differential linear problem can be formulated as

$U = \Phi'(y)\,\mathrm{Exp}[i\,k\,(x-c\,t)]$, $u = \phi'(y)\,\mathrm{Exp}[i\,k\,(x-c\,t)]$,

$V = -\,i\,k\,\Phi\,(y)\,\mathrm{Exp}[i\,k\,(x-c\,t)]$, $v = -\,i\,k\,\phi(y)\,\mathrm{Exp}[i\,k\,(x-c\,t)]$

where $\Phi(y)$ and $\phi(y)$ are the *disturbance* stream functions

$\Phi = \Phi' = 0$ @y=1, [1a]

$\phi = \Phi$, @y=0, [1b]

$\phi' - \frac{u_b'\,\phi}{u_b(0)-c} = \Phi' - \frac{U_b'\,\Phi}{u_b(0)-c}$ @y=0, [1c]

$\phi'' + k^2\,\phi = \mu\,(\Phi'' + k^2\,\Phi)$, @y=0, [1d]

$$\frac{1}{\nu R}(\phi''' - 3\,k^2\,\phi') + i\,k\,(\phi\,u_b' - \phi'\,(u_b(0) - c)) + \frac{i\,k\,\phi}{(u_b(0) - c)}\,\frac{(F + k^2\,T)}{R^2} =$$

$$\frac{\rho}{R}(\Phi''' - 3\,k^2\,\Phi') + \rho\,i\,k\,(\Phi\,U_b' - \Phi'\,\sigma) + \frac{i\,\rho\,k\,\phi}{(u_b(0) - c)}\,\frac{F}{R^2}\ ,$$

@y=0, [1e]

$$i\,k\,(U_b - c)\,(\Phi'' - k^2\,\Phi) - i\,k\,U_b''\,\Phi = R^{-1}(\Phi^{iv} - 2\,k^2\,\Phi'' + k^4\Phi),$$

for $0 \le y \le 1$ [1f]

$$i\,k\,(u_b - c)\,(\phi'' - k^2\,\phi) - i\,k\,u_b''\,\phi = (\nu\,R)^{-1}(\phi^{iv} - 2\,k^2\,\phi'' + k^4\phi),$$

for $-1/d \le y \le 0$ [1g]

$\phi = \phi' = 0$, @ $y = -d^{-1}$. [1h]

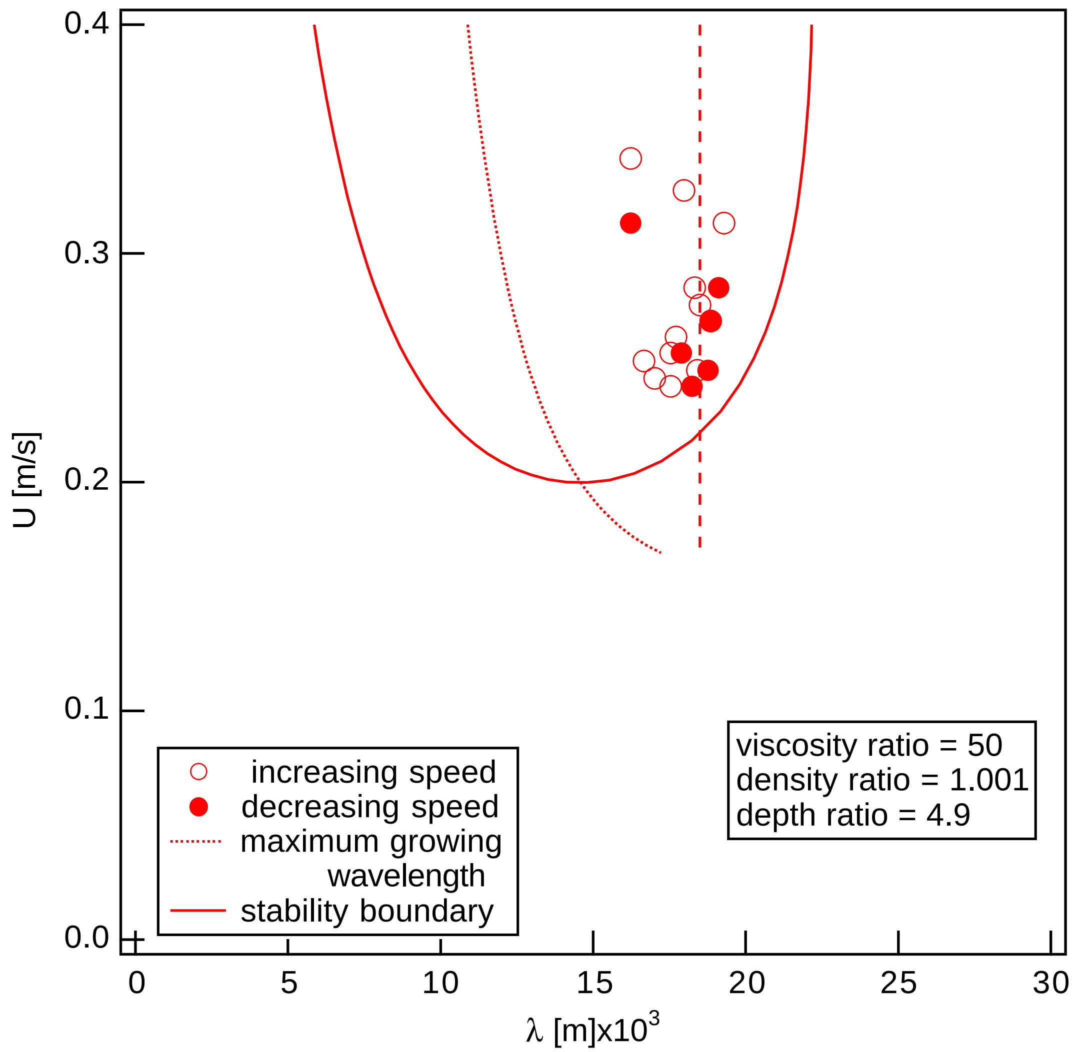
viscosity ratio ==> $\mu = \mu_2/\mu_1$, density ratio ==> $\rho = \rho_2/\rho_1$,

ratio of kinematic viscosities ==> ν, $\sigma = u_b(0) - c$

depth ratio ==> $d = D_2^\dagger/D_1^\dagger$. wavenumber ==> k

liquid average velocity profiles ==> u_b gas velocity ==> U_b

Measured and predicted wavelengths as a function of rotation rate for Couette flow device



M. Sangalli, C. T. Gallagher, D. T. Leighton, H. -C. Chang and M. J. McCready
Phys. Rev. Lett. 75, pp 77-80 (1995).

Weakly- nonlinear theory

$$\frac{D\mathbf{u}}{Dt} = -\nabla p + \frac{1}{R}\nabla^2\mathbf{u}$$

Spectral reduction of Navier-Stokes equations and boundary conditions

$\psi \equiv (\mathbf{u}, p, h)$

If system is such that a single dominant mode exists then:

$\psi = A\,\zeta + \overline{A\,\zeta} + \xi$

$A \equiv$ Complex Amplitude Function
$\zeta \equiv$ dominant eigenfunction
$\xi \equiv$ Linear combination of eigen functions of stable modes

Center manifold projection to produce a Stuart - Landau equation.
(Blennerhassett, 1980; Renardy & Renardy, 1993).

$$\frac{\partial A}{\partial t} = L(\lambda)\,A + \beta\,|A^2|A$$

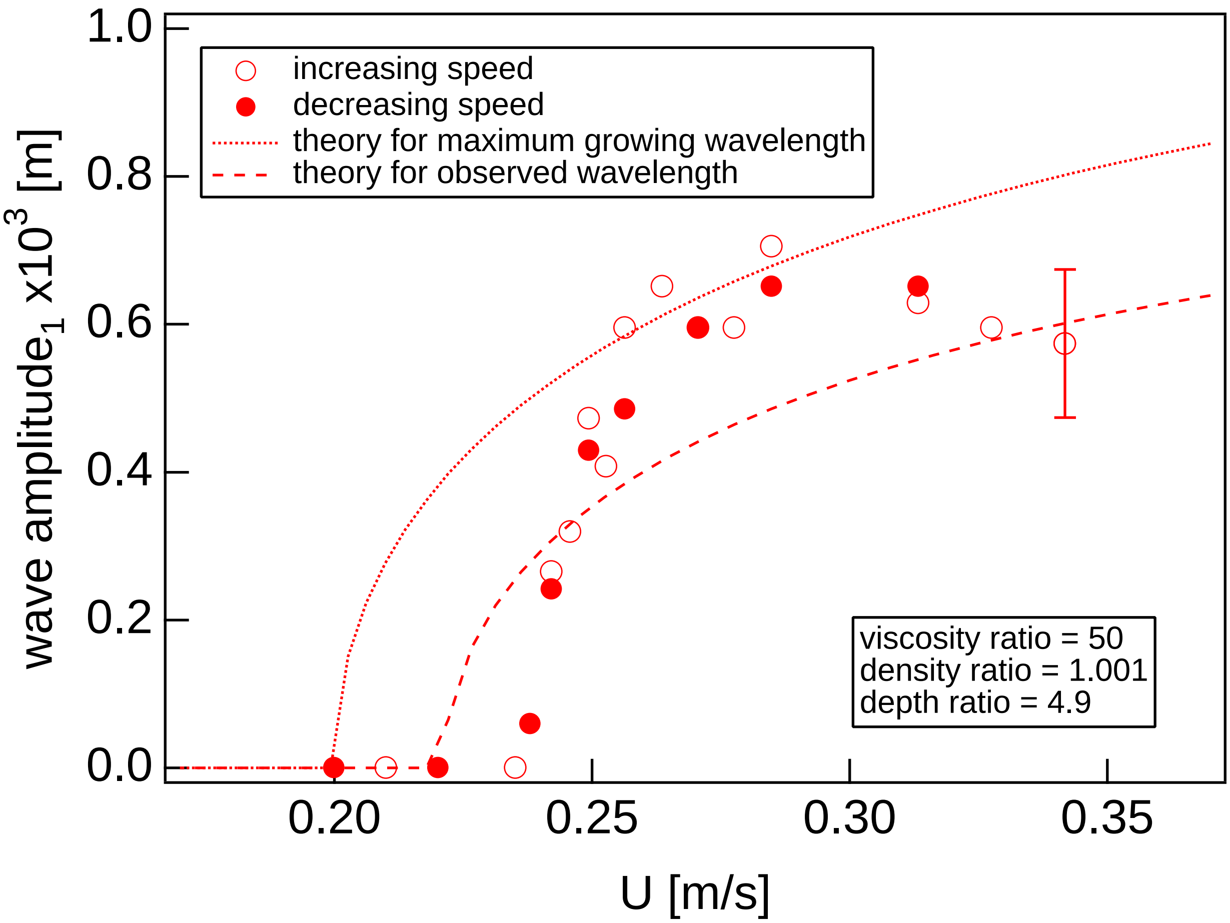
$\beta \equiv$ Landau constant
$L(\lambda) \equiv$ Linear mode behavior

Experimental verification

Comparison of experiment with weakly-nonlinear theory:

Measured and predicted wave amplitude for interfacial waves in rotating Couette device

The bifurcation is supercritical



M. Sangalli, C. T. Gallagher, D. T. Leighton, H. -C. Chang and M. J. McCready
Phys. Rev. Lett. 75, pp 77-80 (1995).

Results from nonlinear theory for gas-liquid flows
(that are verified experimentally)

M. Sangalli, M. J. McCready and H. –C. Chang,
Phys. Fluids **9**, pp 919-939. (1997)

a. Typical amplitude ratio is $A_2 \sim O(A_1^2)$
b. Phase relation between fundamental and overtone will change with liquid depth because of the different relative phase speeds
c. Amplitude ratio is expected to change to $A_2 \sim O(A_1)$ close to the resonance condition
d. Fundamental-overtone phase angle is unstable close to resonance

3. Dominant stabilization is usually through the cubic self interaction[#] term and the system tends to be supercritical, but not always.

[#] We can't show this experimentally

Interesting unresolved issues

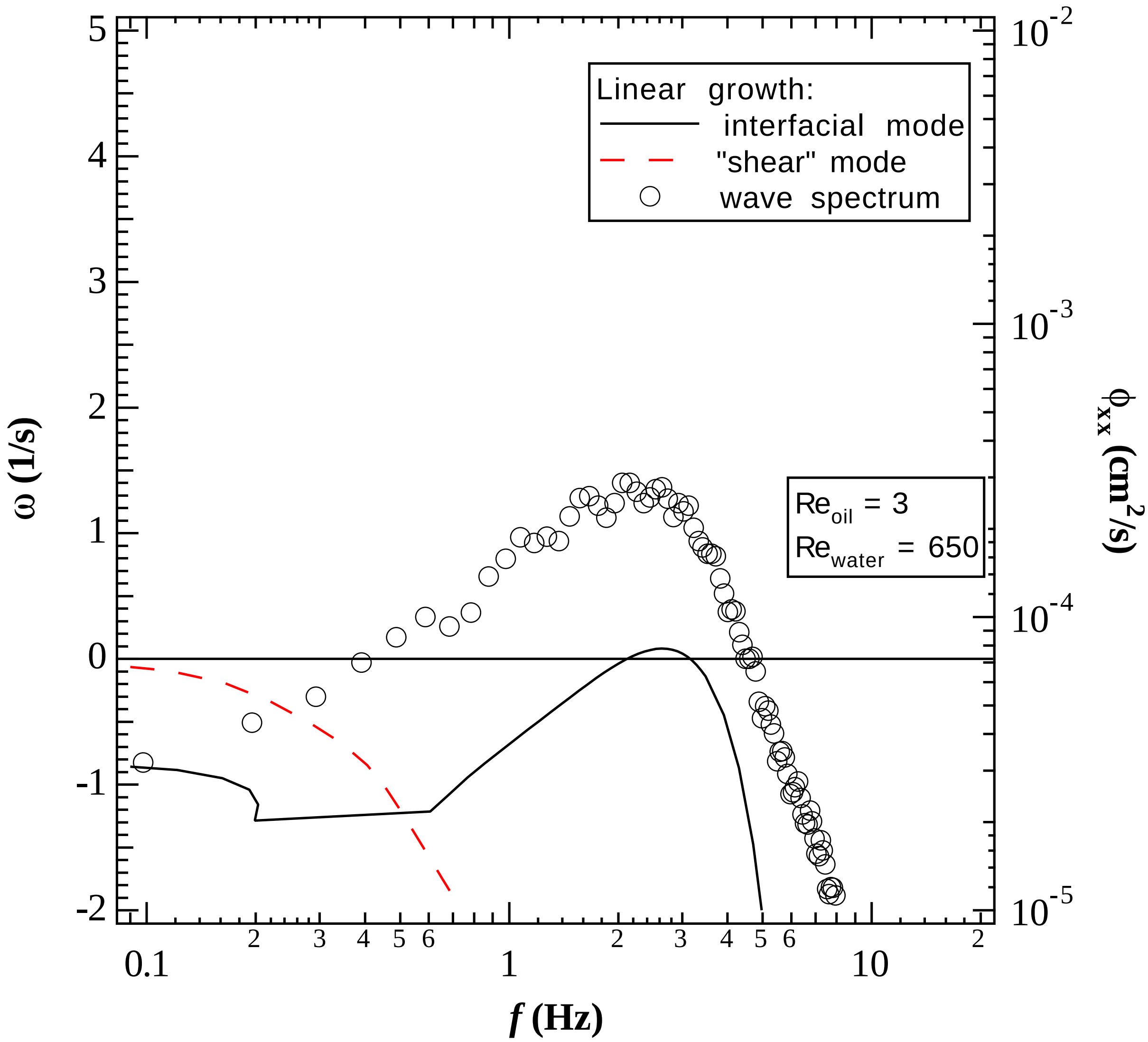
1. Oscillatory flow in Couette device.

   Weak and strong oscillations in the flow often occur in gas-liquid flow systems. Oscillatory flow is also a nice model for transient flows.

   We are examining the purely oscillatory problem and have found a <u>new mechanism for wave formation</u> that requires oscillation or transient flow!!

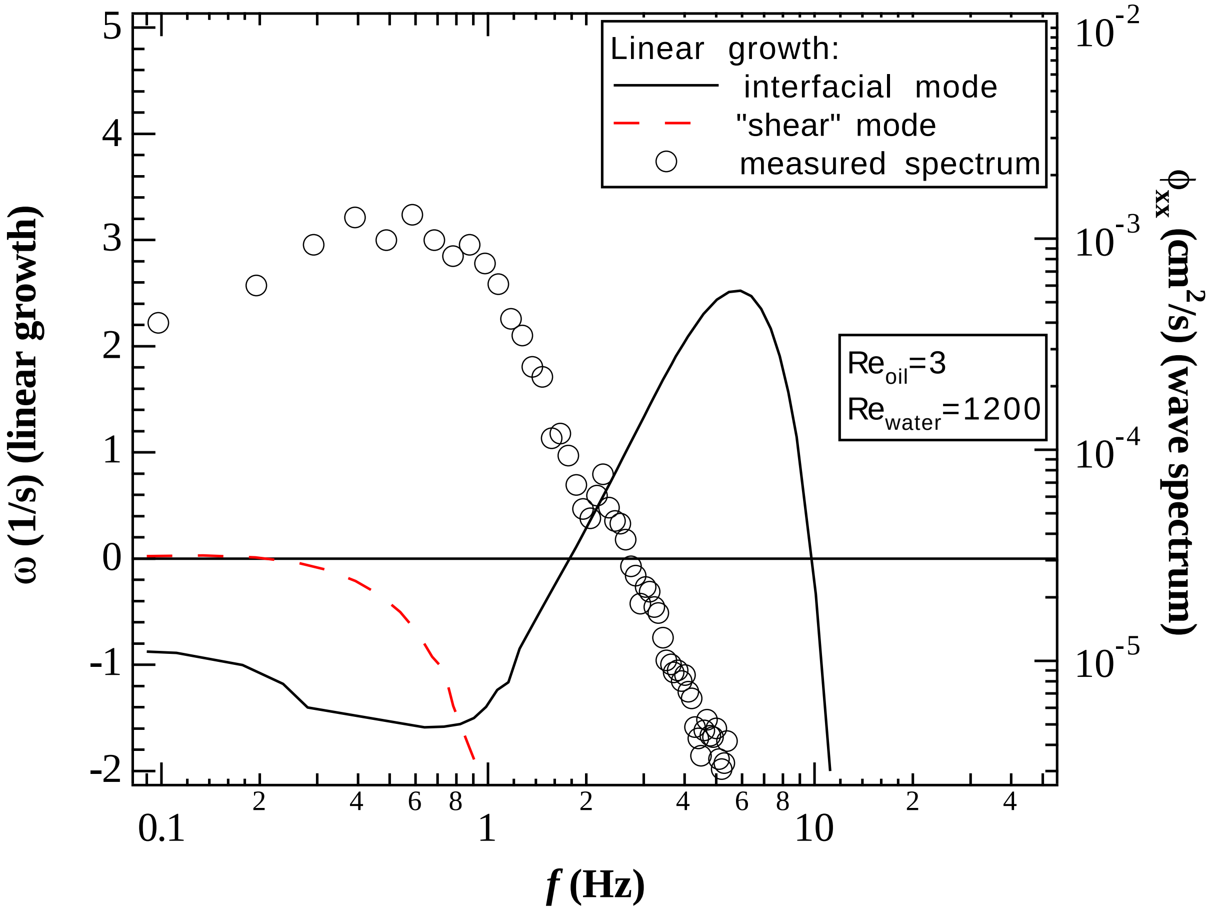
   (send me email to request a preprint)

2. Finding the limits of linear stability theory



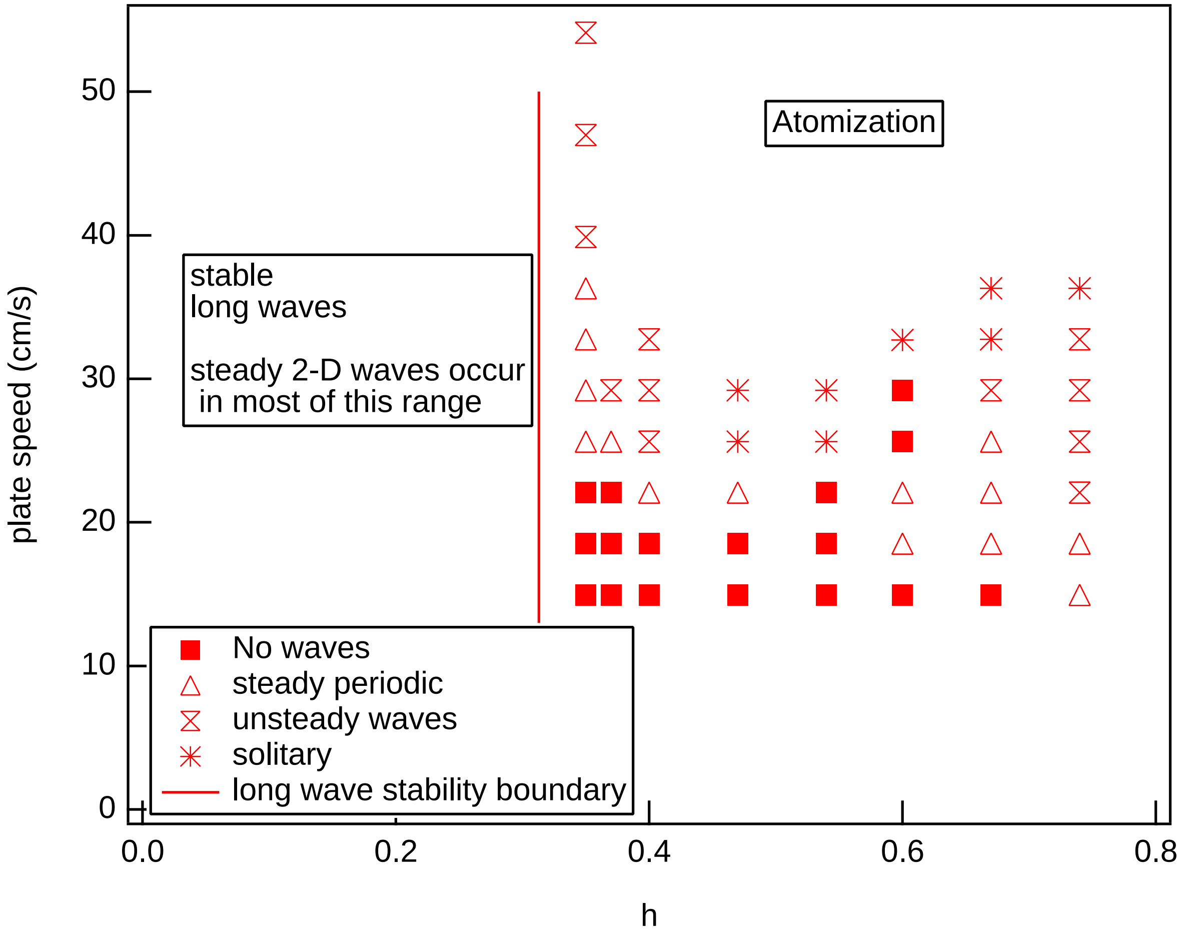
Close to neutral stability we can usually predict the region of unstable waves

At some point, we loose this ability and we don't know the bounds on when this occurs.



We would say that it was just because we crossed the <u>long wave stability boundary</u>, but we have data in our Couette device that are unstable to long waves where no large amplitude or low frequency waves form.

Long waves are unstable to the right of the line.



Gallagher, Leighton & McCready, *Phys. Fluids* **8**, pp2385-2392 (1996)

3. Regions of subcritical bifurcation.

The interfacial wave instability seems to generally be supercritical. We have three different experiments that we use to study waves which are all usually supercritical

1. gas-liquid channel (μ = 50 -1000, ρ = 1000)

    • stabilization is usually cubic self-interaction,

$$\beta_1 A_1 A_1^* A_1$$

2. oil-water channel (μ = 1/15, ρ = 1.2)

    • stabilization by mean flow interaction at low oil flow

$$\beta_0 A_0 A_0^* A_1$$

    • stabilization by cubic self interaction at higher oil flow

3. liquid-liquid rotating Couette (μ = 55, ρ = 1.001)

    • stabilization by overtone interaction, $P_1 A_1^* A_2$

The natural question that arises is: are there subcritical regions of wave transition somewhere?

We cannot check for all cases because there are 7 parameters in the two-layer system and no general theory.

•• We can think about this a bit and get some ideas.

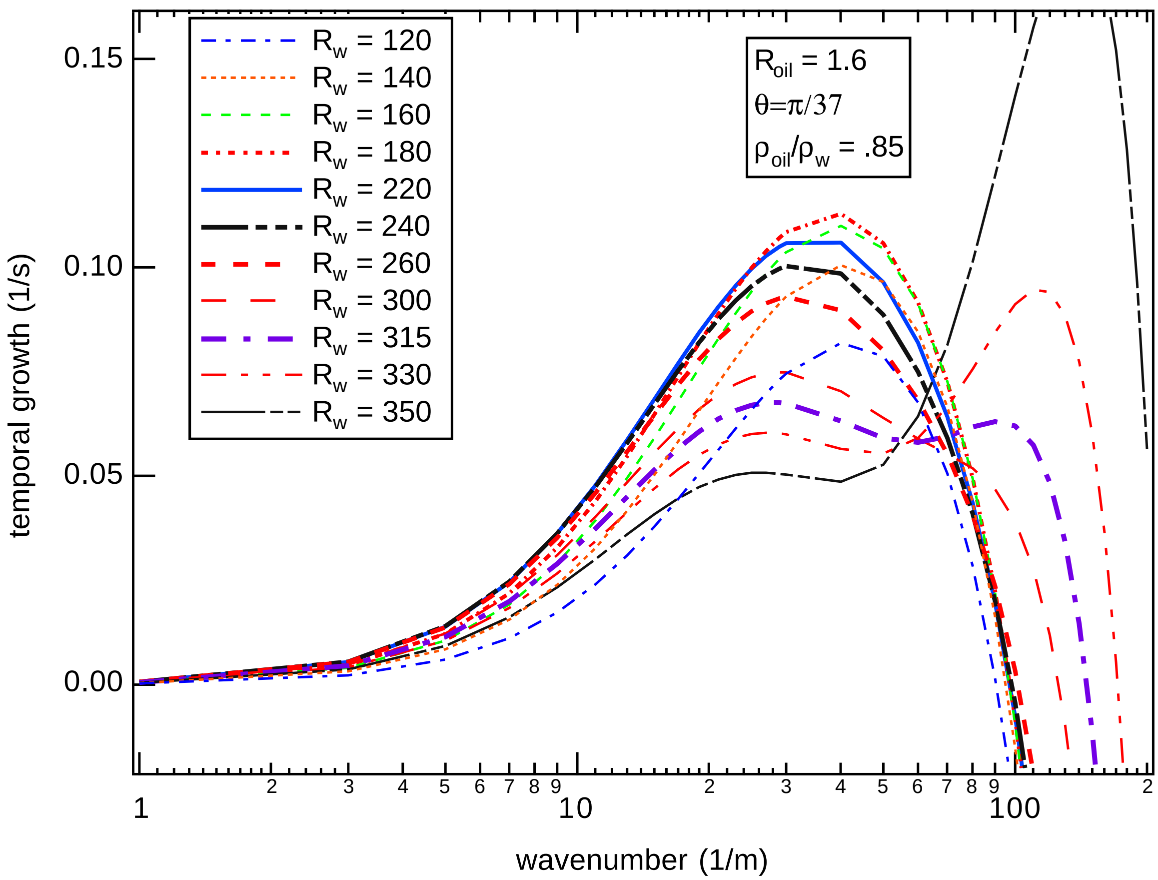
Subcritical 1. (obvious and not interesting)

If a gas-phase internal mode becomes unstable first for, say, air-liquid, $R_L = 2$, $R_G = 8000$, $\mu_L = 10$ cP, this mode is subcritical as expected from single-phase theory.
Destabilization is from *mean flow interaction* as expected.

<u>Subcritical 2.</u> (much more interesting)

Oil-water channel flow inclined downward at π/37 in the flow direction.

Linear stability shows region where increasing the water flow rate *decreases* the wave growth!!

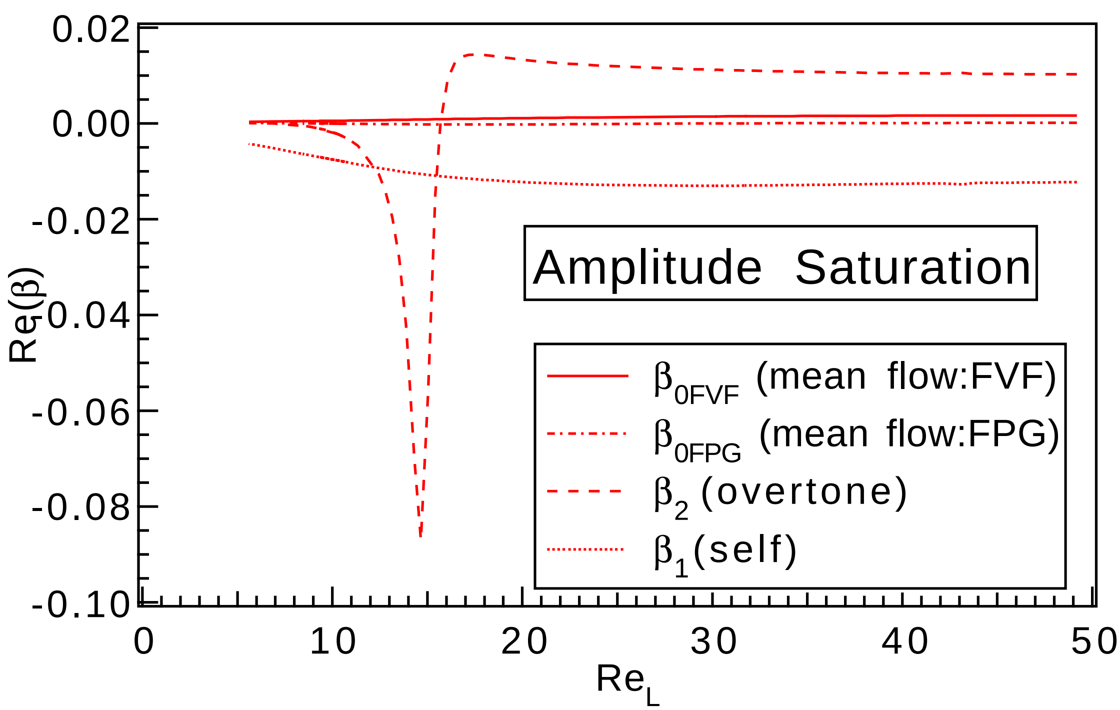


Calculation of Landau coefficient shows that the *overtone interaction* is source of the <u>destabilization</u>. If this is large enough, it dominates the stabilization from the cubic self-interaction and the mean flow interaction.

Subcritical 3.

Consistent with inviscid theory, close to resonance, that is where a fundamental wave travels at the same speed at its overtone, the overtone interaction dominates.

Generally for depths slightly larger than resonance conditions, the overtone interaction is destabilizing and large enough to cause a positive Landau coefficient.



We have tried to do experiments in these regions and not yet seen any significant effects.

Conclusions

1. Linearly unstable short waves can be stabilized by nonlinear effects to saturate at small amplitude

   This can lead to either a family of steady waves or continually oscillating waves

2. The complication of wave evolution with distance can be overcome with a rotating Couette device that employs matched-density liquids

- This device has confirmed weakly nonlinear theory for the initial onset of waves
- It has shown that *long wave instability* does not necessarily imply growth of large amplitude long waves
- New experiments agree with linear theory for oscillatory flows which has identified a new mechanism for formation of waves.

3. All of the interesting problems regarding interfacial waves are not solved. We are still working on:

- Oscillatory flows
- Limits of linear theory
- Regions of sub and supercritical bifurcations.

Why study waves?

1. Prediction of flow regime, pressure drop, holdup, interphase transport and atomization

    - pipeline and process piping flows,
    - energy exchange devices,
    - oil and gas well risers,

2. Prediction of transport rates and atomization
    - reactors, absorbers,
      (chemical process equipment)

3. Environmental flows
    - underground, fractured rock flows to
      clean up ground water??

    - Ocean, lake, river surfaces
      Gas exchange,
      remote sensing (weather conditions)
      environmental change(??)

Academic interest of waves

1. Linear stability *theory* is well established but there may still be some *interesting results*.

- Still very few rigorous comparisons of predicted and measured growth rates

- Practical limits of linear theory are not well established
  (Can we always predict the wavelength?)

2. Weakly nonlinear theory is developing and there are probably many new results to be uncovered and there is a need for experiments to assure correctness.

3. Vigorous efforts are occurring on the fully nonlinear, interface simulation problem. (This one is far from being solved.)

Major open questions in separated flows:

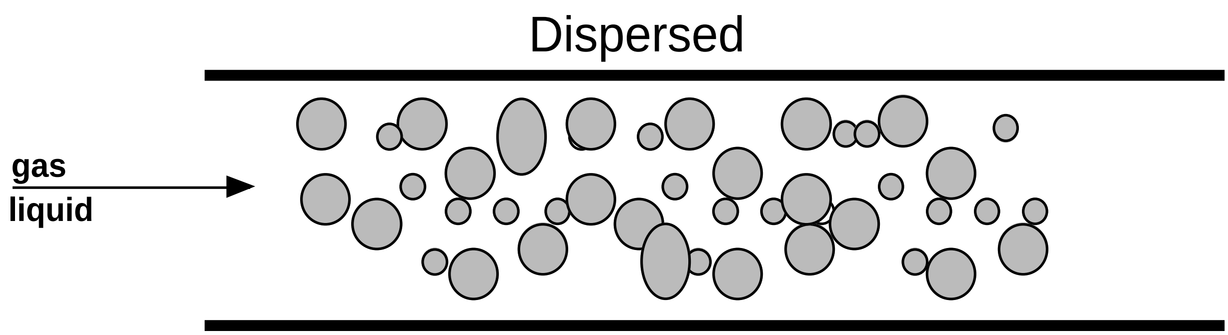
1. Prediction of the interfacial wave field for arbitrary conditions

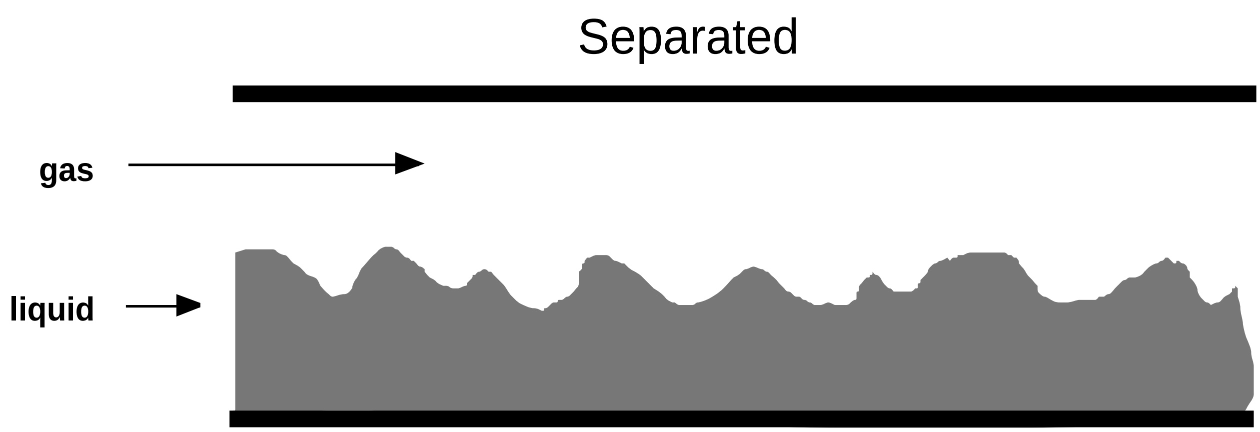
2. Prediction of the formation of large disturbances

3. Prediction of atomization rates

A sensible approach is to think of the flow regime problem in terms of:

Limiting Flow Configurations





This talk with be confined to separated flows