ChEg 542
Fall 1998
HW #8
Due 12/14/98

1. Diffusion operators

The diffusion operator can be written as

$$D(y) = \frac{1}{x^n}\frac{d}{dx}\left(x^n \frac{d\,y(x)}{dx}\right)$$

for Cartesian, cylindrical and spherical geometries for n = 0,1,2 respectively.

a. Find the first several eigenvalues for each of the three cases and discuss the differences and similarities of the three geometries.

b. Sketch the first two eigenfunctions for each of the cases and again discuss the differences and similarities between the three geometries.

c. Now consider the boundary value problem, on the domain, (0,1) for this operator where there is an inhomogeneous term:

$$\frac{1}{x^n}\frac{d}{dx}\left(x^n \frac{d\,y(x)}{dx}\right) = x^\gamma$$

with homogeneous boundary conditions

$$\alpha_1\, y(0) + \beta_1\, y'(0) = 0$$

$$\alpha_2\, y(1) + \beta_2\, y'(1) = 0$$

Do unique solutions exist for all values of α_1, α_2, β_1, β_2, and γ for n = 0,1,2? If not explain why not and define the values for which unique solutions exist.

2. V&M, 3.25
3. V&M, 3.26
4. V&M, 7.1
5. V&M, 7.3

6. Spectral numerical solutions

Use the Mathematica notebook, http://www.nd.edu/~mjm/spectral.numerical.nb, to solve the following problem using a Chebyshev polynomial approach. Compare your answer with an anayltical solution and show how it converges with increasing number of terms.

$$\frac{d^2y(x)}{dx^2} + xy(x), \text{ where } y(1)=y(-1) = 1$$