Formation of large disturbances in separated fluid-fluid flows

M. J. McCready, D. D. Uphold
Department of Chemical Engineering
University of Notre Dame
Notre Dame, IN 46556 USA

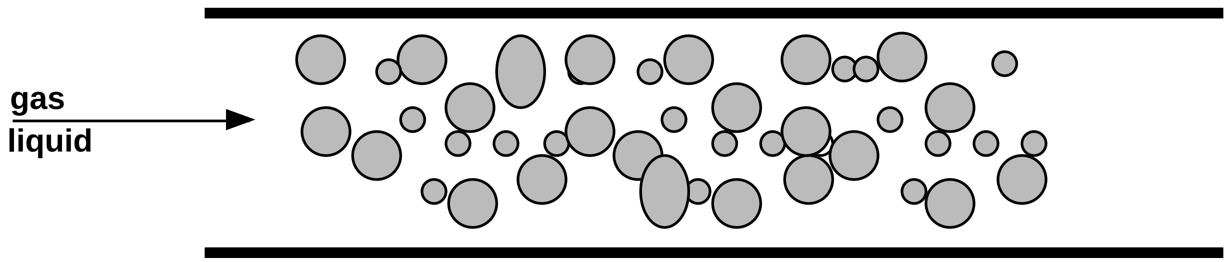
Acknowledgments

NASA, μ-gravity;
DOE, Basic Energy Sciences

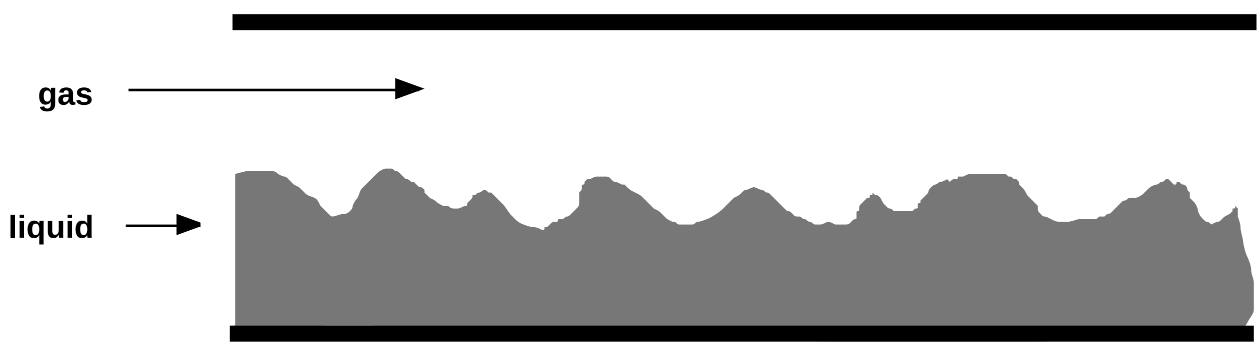
A way to organize flow regimes to make progress with current theoretical tools is:

Limiting Flow Configurations

Dispersed



Separated



This talk will be confined to separated flows

Conceptual limits also exist for prediction of the presence of slugs or roll waves.

We could do:

1. Stability of a traveling slug
("Necessary conditions for the existence of stable slugs" Ruder, Hanratty and Hanratty, IJMF 1989)

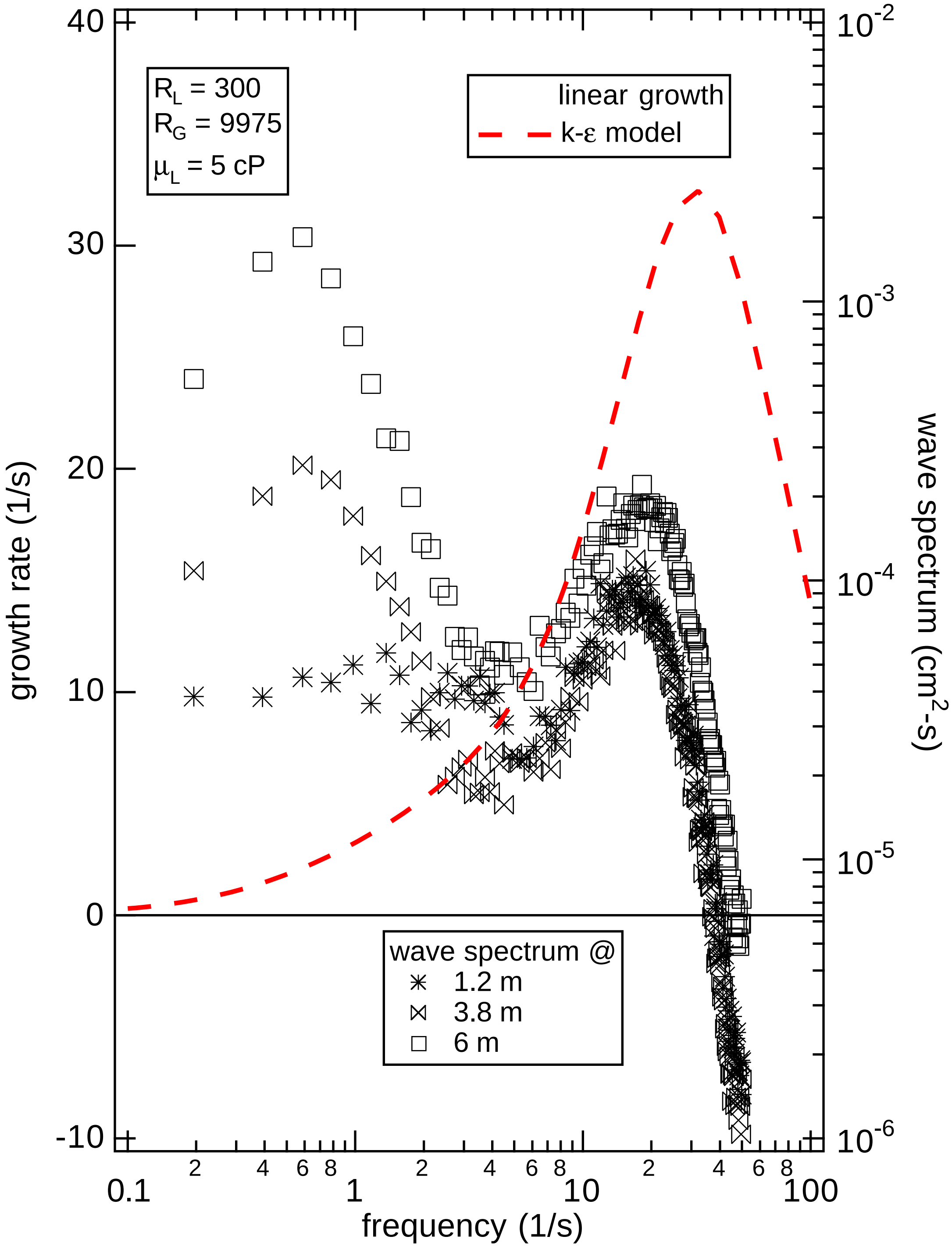
2. Stability of the two-layer base state

(We will examine this case.)

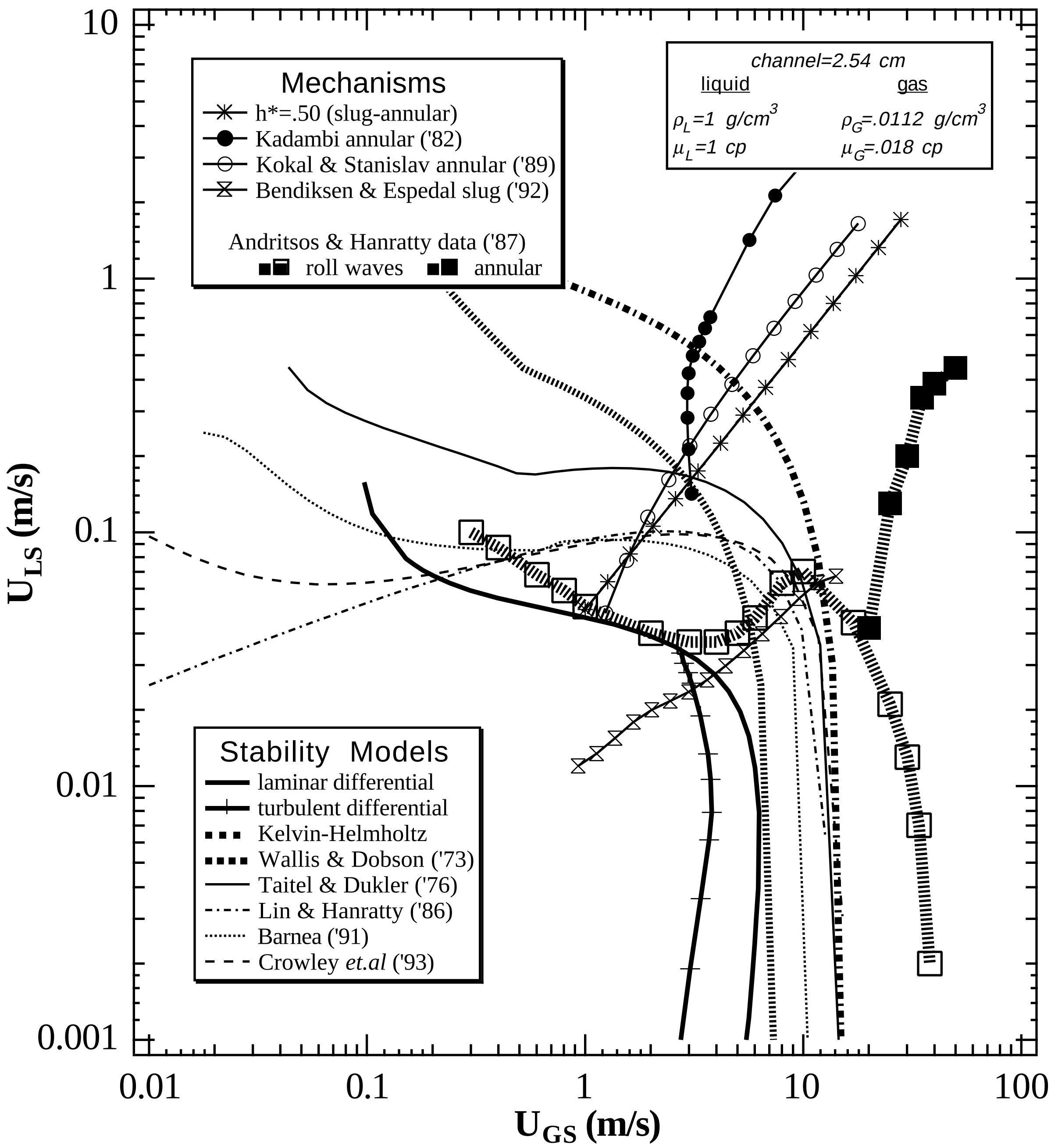
Measured transition to roll waves

(Bruno and McCready, 1988)

Long wave peak grows with distance.
Can linear theory predict this??



Transition predictions from many models for the "easiest" case



Slug prediction from basestate wave growth

There are four issues.

1. Need to have a good prediction for the properties of the base state
(i.e., two-layer flow with no large waves present).

   Not trivial since both phases are turbulent

2. First waves that usually occur are short wavelength and probably do not directly lead to slugs.

   Thus short waves may be present when you want to calculate stability

3. Simplified theories (Inviscid or averaged-equations) usually do not work very well.

4. Loss of linear stability is probably necessary but not sufficient for formation of long waves.

Gas-liquid flow interfacial stability problem turbulence model: k-ε

Basic equations:

Liquid-phase: $0 \le y^\bullet \le d_1$

$$\rho_1\left[\frac{\partial u_i^\bullet}{\partial t^\bullet} + u_j^\bullet\frac{\partial u_i^\bullet}{\partial x_j^\bullet}\right] = -\frac{\partial p^\bullet}{\partial x_i^\bullet} + \rho_1 g^\bullet \sin(\theta) + \frac{\partial}{\partial x_j^\bullet}\left[\left(\mu_1 + \mu_t^\bullet\right)\left(2s_{ij}^\bullet\right)\right] \qquad \text{(3-1a)}$$

$$\rho_1\left[\frac{\partial k_i^\bullet}{\partial t^\bullet} + u_i^\bullet\frac{\partial k^\bullet}{\partial x_i^\bullet}\right] = \frac{\partial}{\partial x_i^\bullet}\left[\left(\mu_1 + \frac{\mu_t^\bullet}{\sigma_{ke}}\right)\left(\frac{\partial k^\bullet}{\partial x_i^\bullet}\right)\right] + \mu_t^\bullet\left(2s_{ij}^\bullet\right)\frac{\partial u_i^\bullet}{\partial x_j^\bullet} - \rho_1\varepsilon^\bullet - 2\mu_1\left(\frac{\partial\sqrt{k^\bullet}}{\partial x_i^\bullet}\right)^2 \qquad \text{(3-1b)}$$

$$\rho_1\left[\frac{\partial \varepsilon^\bullet}{\partial t^\bullet} + u_i^\bullet\frac{\partial \varepsilon^\bullet}{\partial x_i^\bullet}\right] = \frac{\partial}{\partial x_i^\bullet}\left[\left(\mu_1 + \frac{\mu_t^\bullet}{\sigma_{\varepsilon}}\right)\left(\frac{\partial \varepsilon^\bullet}{\partial x_i^\bullet}\right)\right] + c_1 f_1\mu_t^\bullet\frac{\varepsilon^\bullet}{k^\bullet}\left(2s_{ij}^\bullet\right)\frac{\partial u_i^\bullet}{\partial x_j^\bullet} \qquad \text{(3-1c)}$$

$$+ 2\mu_1\mu_t^\bullet\left(\frac{\partial^2 u_i^\bullet}{\partial x_j^\bullet \partial x_l^\bullet}\right)^2 - \rho_1 c_1 f_1\frac{\varepsilon^{\bullet 2}}{k^\bullet}$$

Gas-phase: $d_1 \le y^\bullet \le d_1 + d_2$

$$\rho_2\left[\frac{\partial u_i^\bullet}{\partial t^\bullet} + u_j^\bullet\frac{\partial u_i^\bullet}{\partial x_j^\bullet}\right] = -\frac{\partial p^\bullet}{\partial x_i^\bullet} + \rho_2 g^\bullet \sin(\theta) + \frac{\partial}{\partial x_j^\bullet}\left[\left(\mu_2 + \mu_t^\bullet\right)\left(2s_{ij}^\bullet\right)\right] \qquad \text{(3-2a)}$$

$$\rho_2\left[\frac{\partial k^\bullet}{\partial t^\bullet} + u_i^\bullet\frac{\partial k^\bullet}{\partial x_i^\bullet}\right] = \frac{\partial}{\partial x_i^\bullet}\left[\left(\mu_2 + \frac{\mu_t^\bullet}{\sigma_{ke}}\right)\left(\frac{\partial k^\bullet}{\partial x_i^\bullet}\right)\right] + \mu_t^\bullet\left(2s_{ij}^\bullet\right)\frac{\partial u_i^\bullet}{\partial x_j^\bullet} - \rho_2\varepsilon^\bullet - 2\mu_2\left(\frac{\partial\sqrt{k^\bullet}}{\partial x_i^\bullet}\right)^2 \qquad \text{(3-2b)}$$

$$\rho_2\left[\frac{\partial \varepsilon^\bullet}{\partial t^\bullet} + u_i^\bullet\frac{\partial \varepsilon^\bullet}{\partial x_i^\bullet}\right] = \frac{\partial}{\partial x_i^\bullet}\left[\left(\mu_2 + \frac{\mu_t^\bullet}{\sigma_{\varepsilon}}\right)\left(\frac{\partial \varepsilon^\bullet}{\partial x_i^\bullet}\right)\right] + c_1 f_1\mu_t^\bullet\frac{\varepsilon^\bullet}{k^\bullet}\left(2s_{ij}^\bullet\right)\frac{\partial u_i^\bullet}{\partial x_j^\bullet} \qquad \text{(3-2c)}$$

$$+ 2\mu_2\mu_t^\bullet\left(\frac{\partial^2 u_i^\bullet}{\partial x_j^\bullet \partial x_l^\bullet}\right)^2 - \rho_2 c_1 f_1\frac{\varepsilon^{\bullet 2}}{k^\bullet}$$

Stability problem becomes:

k=1 (liquid-phase) $0 \le y \le 1$
k=2 (gas-phase) $1 \le y \le n_2 + 1$ (3-17)

$$\frac{\left(\hat{\mu}_k u'_{b,k}\right)''}{m_k} + \left(\Gamma_{b,k}\hat{\phi}''_k\right)'' - 2\alpha^2\left(\Gamma_{b,k}\hat{\phi}'_k\right)' + \alpha^4\Gamma_{b,k}\phi_k = i\alpha R\frac{r_k}{m_k}\left\{\left(u_{b,k}-c\right)\left(\hat{\phi}''_k - \alpha^2\hat{\phi}_k\right) - u''_{b,k}\hat{\phi}_k\right\}$$

$$\frac{\left(\hat{\mu}_k k'_{b,k}\right)' + \hat{\mu}_k u''_{b,k}}{m_k} + \Gamma_{b,k}\left(\hat{k}''_k - \alpha^2\hat{k}_k\right) + \Gamma'_{b,k}\hat{k}_k + 2\frac{\hat{\mu}_{b,k}u'_{b,k}}{m_k}\left(\hat{\phi}''_k + \alpha^2\hat{\phi}_k\right) + \frac{k'_{b,k}}{k_{b,k}}\left(\frac{k'_{b,k}}{2k_{b,k}}\hat{k}_k - \hat{k}'_k\right)$$
$$= i\alpha R\frac{r_k}{m_k}\left\{\left(u_{b,k}-c\right)\hat{k}_k - k'_{b,k}\hat{\phi}_k\right\}$$

$$\frac{\left(\hat{\mu}_k \varepsilon'_{b,k}\right)'}{m_k} + \Gamma_{b,k}\Gamma_{b,k}\left(\hat{\varepsilon}''_k - \alpha^2\hat{\varepsilon}_k\right) + \Gamma'_{b,k}\hat{\varepsilon}_k + 2c_1 f_1\frac{\hat{\mu}_{b,k}u'_{b,k}}{m_k}\left(\hat{\phi}''_k + \alpha^2\hat{\phi}_k\right) + r_k R c_2 f_2\frac{\varepsilon_{b,k}}{k_{b,k}}\left(\frac{\varepsilon_{b,k}}{k_{b,k}}\hat{k}_k - 2\hat{\varepsilon}_k\right)$$
$$+\frac{\left(u''_{b,k}\right)^2}{m_k}\left[c_1 f_1\frac{\varepsilon_{b,k}}{k_{b,k}}\left(\hat{\mu}_k + \hat{\varepsilon}_k - \frac{m_k\mu_{b,k}}{k_{b,k}}\hat{k}_k\right) + \frac{2m_k}{r_k R}\left(\hat{\mu}_k + 2\mu_{b,k}\hat{\phi}'''_k\right)\right]$$
$$= i\alpha R\frac{r_k}{m_k}\left\{\left(u_{b,k}-c\right)\hat{\varepsilon}_k - \varepsilon'_{b,k}\hat{\phi}_k\right\}$$

$$\hat{\mu}_k = c_\mu f_\mu r_k R\frac{k_{b,k}}{\varepsilon_{b,k}}\left(2\hat{k}_k - \frac{k_{b,k}}{\varepsilon_{b,k}}\hat{\varepsilon}_k\right)$$

The interfacial boundary conditions at @ $y = 1$ are

$$\hat{\phi}_1 = \hat{\phi}_2 \qquad \text{(3-18c)}$$

$$\hat{\phi}_1 + u_{b,1}\hat{h} = c\hat{h} \qquad \text{(3-18d)}$$

$$\hat{\phi}'_1 - \hat{\phi}'_2 = \hat{h}\left(u'_{b,1} - u'_{b,2}\right) \qquad \text{(3-18e)}$$

$$\hat{\phi}''_1 + \alpha^2\hat{\phi}_1 + \hat{h}u''_{b,1} = m_2\left(\hat{\phi}''_2 + \alpha^2\hat{\phi}_2 + \hat{h}u''_{b,2}\right) \qquad \text{(3-18f)}$$

$$\left(\hat{\phi}'''_1 + \Gamma'_{b,1}\hat{\phi}''_1 + u'_{b,1}\hat{\Gamma}'_1 - 3\alpha^2\hat{\phi}_1\right) + i\alpha R\left(u'_{b,1}\hat{\phi}_1 - u_{b,1}\hat{\phi}'_1\right) - m_2\left(\hat{\phi}'''_2 + \Gamma'_{b,2}\hat{\phi}''_2 + u'_{b,2}\hat{\Gamma}'_2 - 3\alpha^2\hat{\phi}_2\right)$$
$$- i\alpha r_2 R\left(u'_{b,2}\phi_2 - u_{b,2}\phi'_2\right) - i\alpha R\left[\left(1-r_2\right)F + \alpha^2 S\right]\hat{h} = i\alpha Rc\left(r_2\hat{\phi}'_2 - \hat{\phi}'_1\right) \qquad \text{(3-18g)}$$

$$\hat{k}_1 = \hat{\varepsilon}_1 = \hat{\mu}_1 = \hat{k}_2 = \hat{\varepsilon}_2 = \hat{\mu}_2 = 0 \qquad \text{(3-18h)}$$

Comparison of base state predictions

"Separated phase" friction factor model does not work well

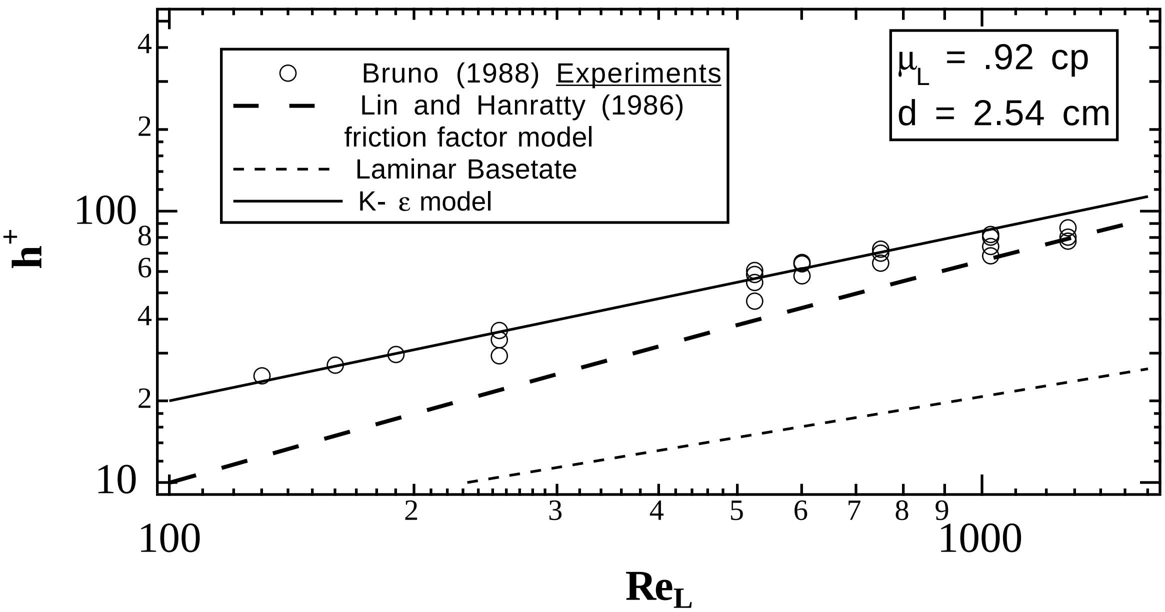


Figure 4.7 h^+ - Re_L correlation for horizontal gas-liquid channel flow. Re_G = 4,000 - 15,000

Note:

From averaged momentum equations, the liquid holdup and pressure drop are predicted by using a friction factor for the gas, a friction factor for the liquid and an interfacial friction factor to account for the non smooth gas-liquid interface.

These correlations are usually the same as for pipe flow, e.g., $f = 0.079\, R^{-.25}$ with, (sometimes) a hydraulic radius correction.

Comparison of k-ε base state with a correlation intended to predict the effects of waves (Andritsos and Hanratty, 1987)

Some of the effect of "waves" could be inaccuracies in the friction factor model formulation

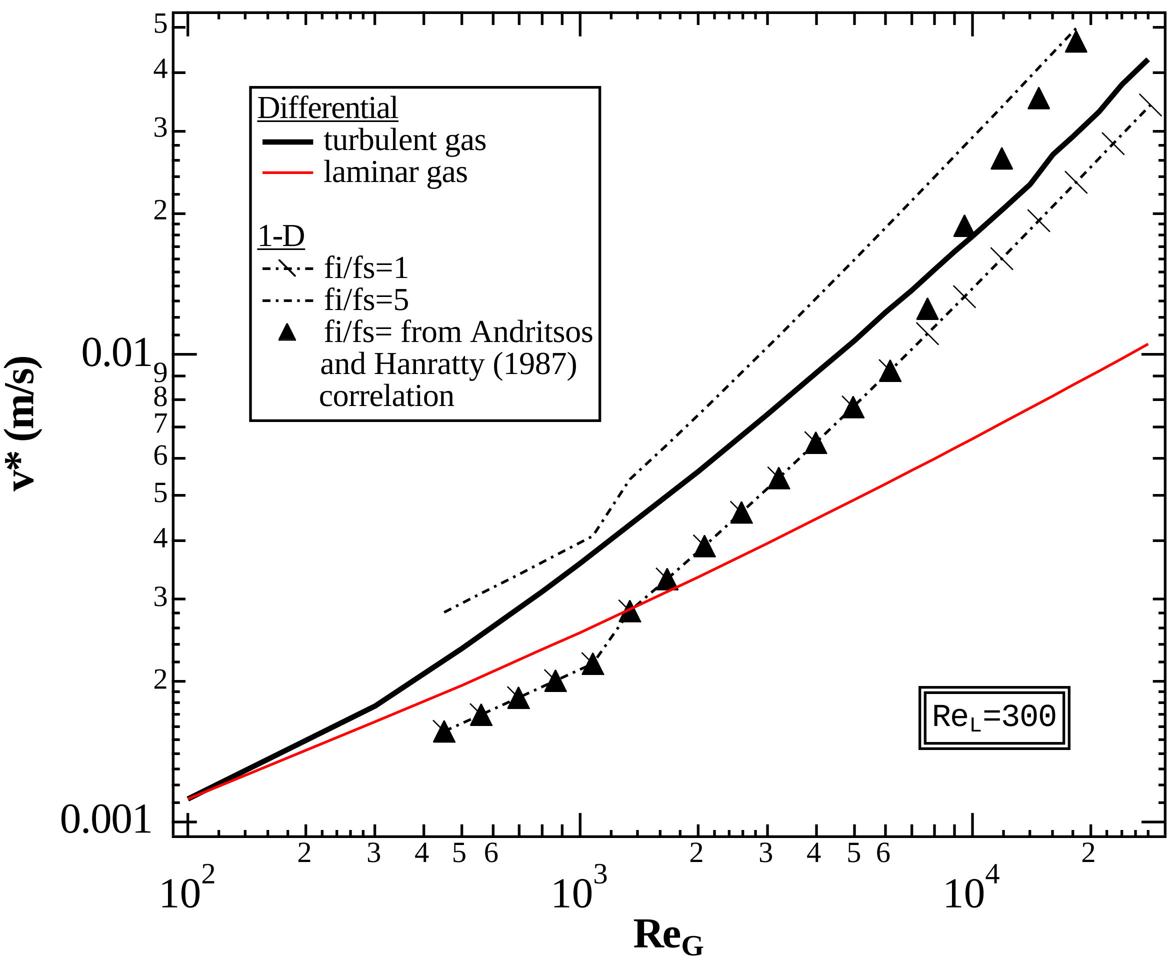
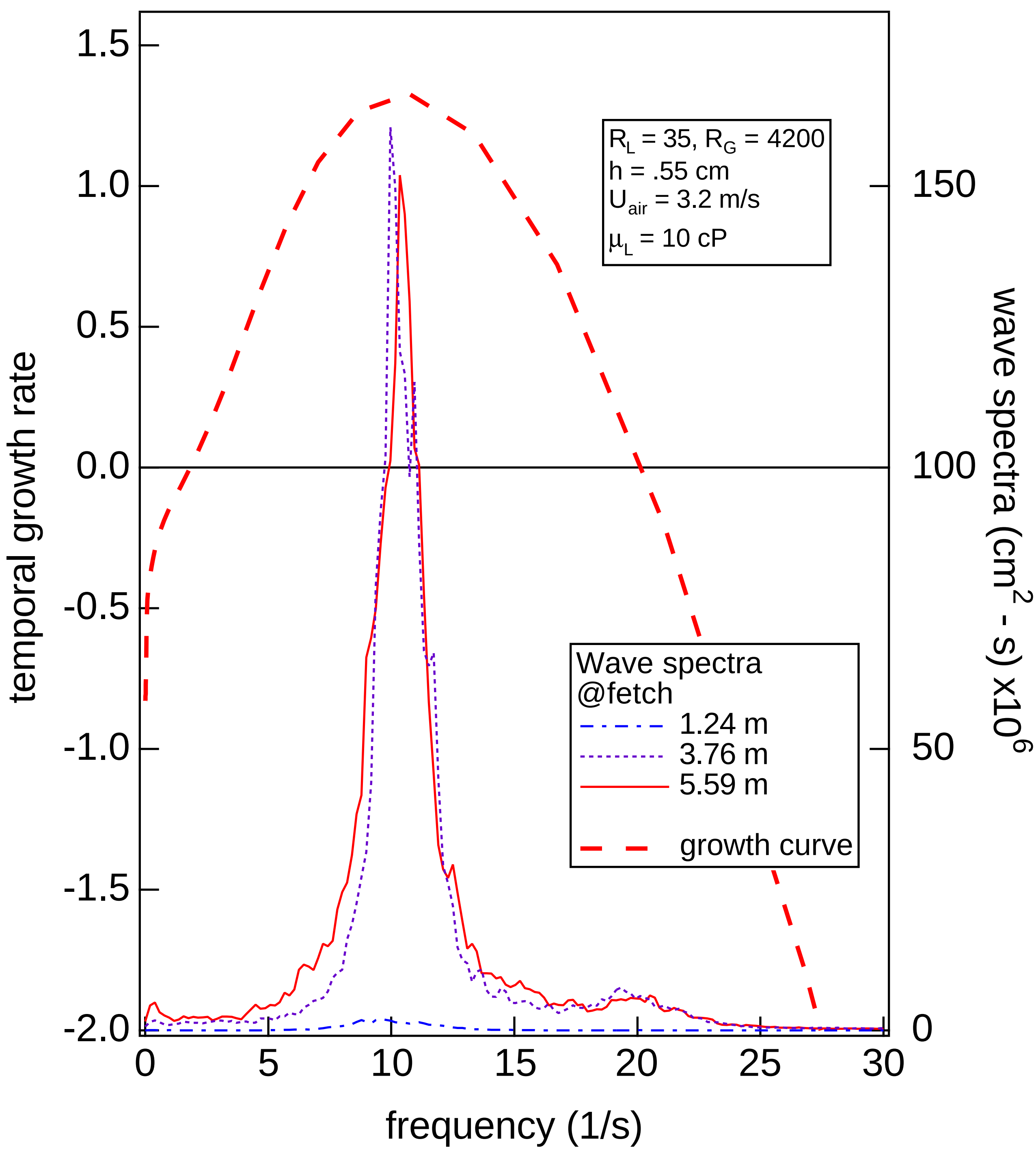


Figure 4.5 Friction velocity versus Re_G at constant Re_L.
d = 2.54 cm, μ_L = 1 cp, P=14.7 psia, T=298 K

Growth and saturation of waves when only short waves are unstable



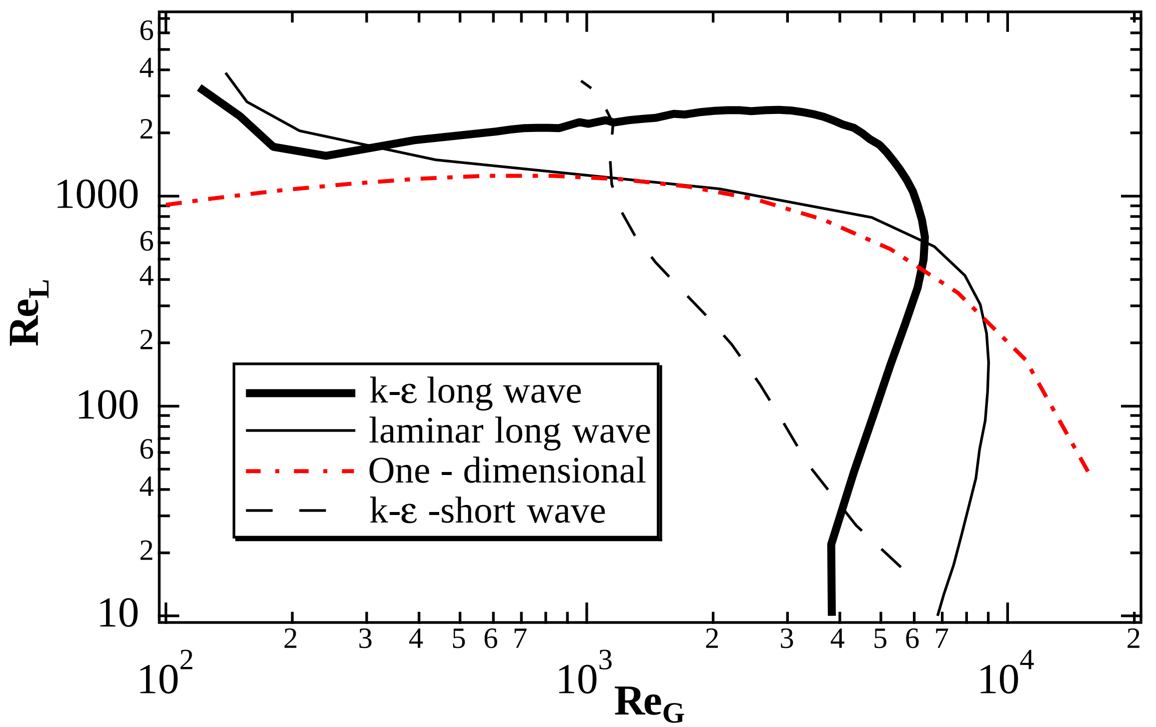
Thus, it makes sense to consider the transition as occurring when long waves are unstable.

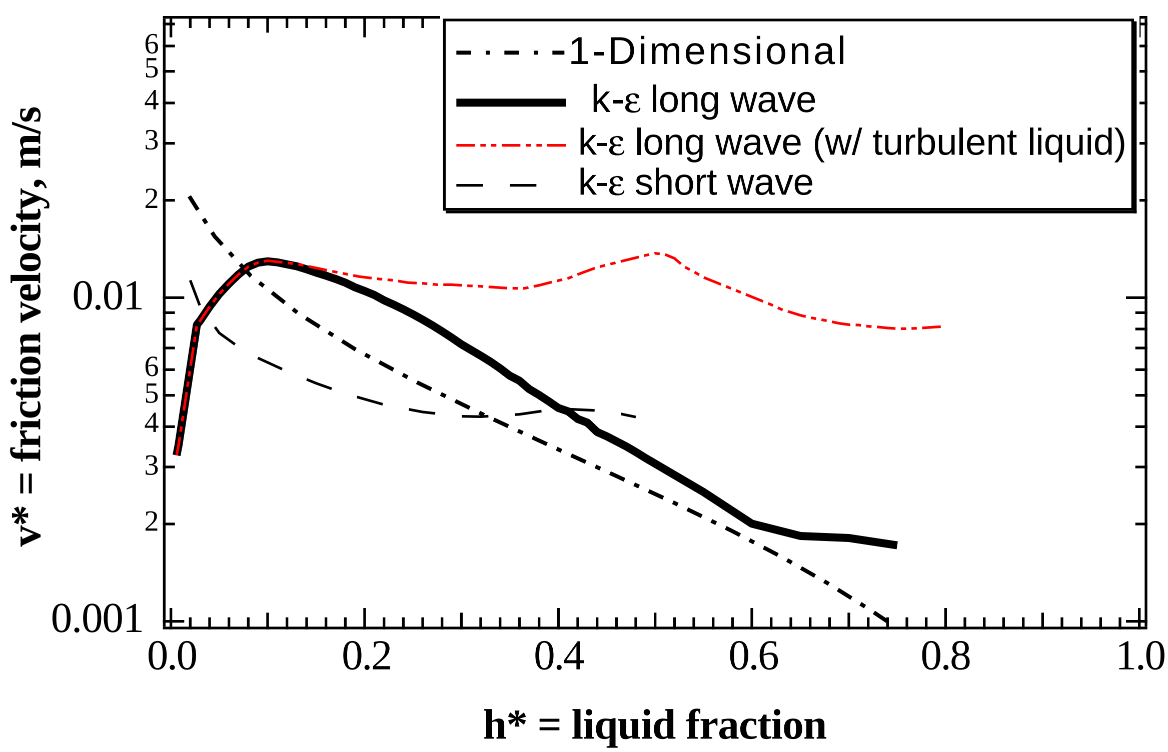
Simplified theories are appealing because the calculations are much easier.

Obvious simplifications are averaging over the liquid or gas depth. Result is similar to, but not quite the same, as boundary-layer approximation.

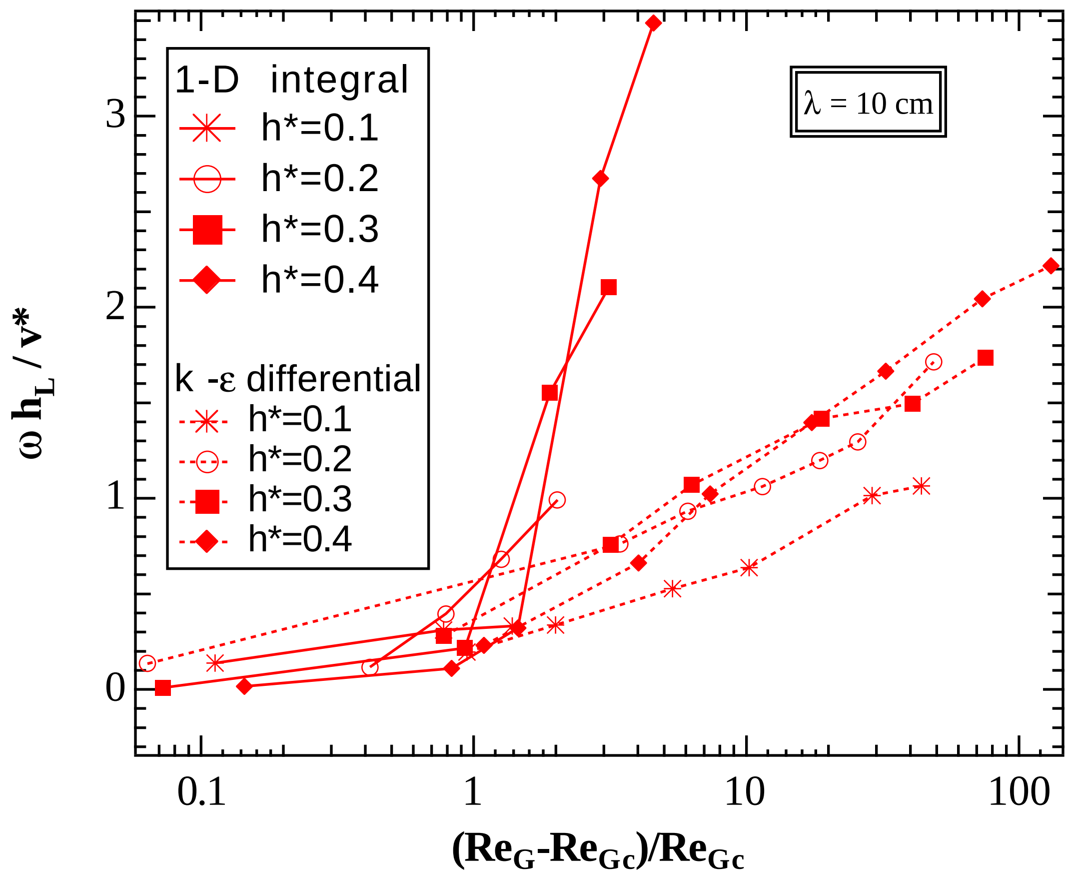
Unfortunately, these simplifications are probably too simple.

Stability boundaries for laminar, one-dimensional and k-ε models





Growth rate comparison

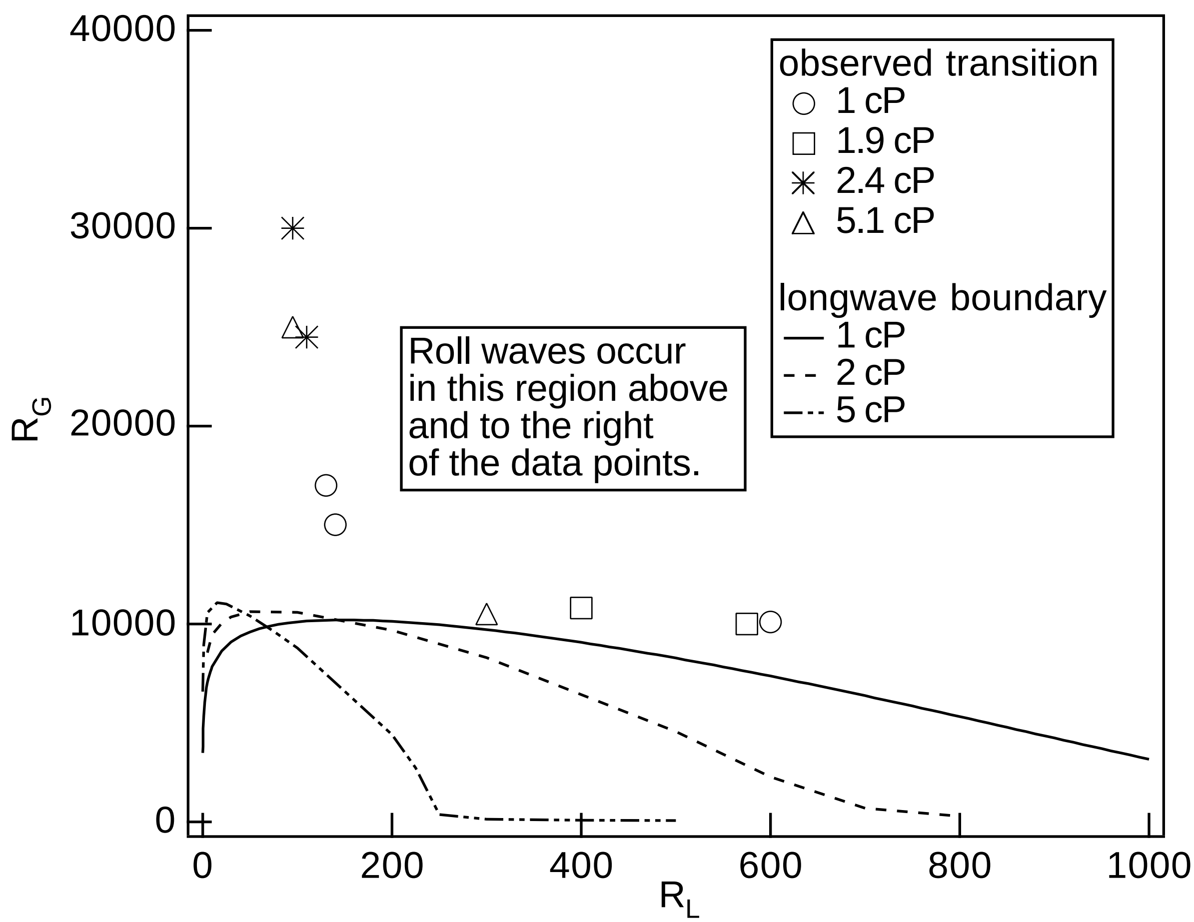


Dimensionless growth rate of long waves (λ =10 cm) versus reduced gas Reynolds number, where Re_{Gc} is the value of Re_G at criticality.

- Thus we have concern that the approximate models give incorrect predictions for stability boundary and growth.

Roll wave transition experiments and long wave stability boundary

Further, simply predicting the stability boundary may not be enough to tell if large disturbances form



Can we find something that might work that could be (at least) used for further study?

<u>"Engineering models"</u>

Two categories:

A. "Extra" linear

i. growth curve always positive
ii. modified wave growth rate
iii. shear stress phase angle

- These three are "interpretations" of the rigorous linear stability calculations

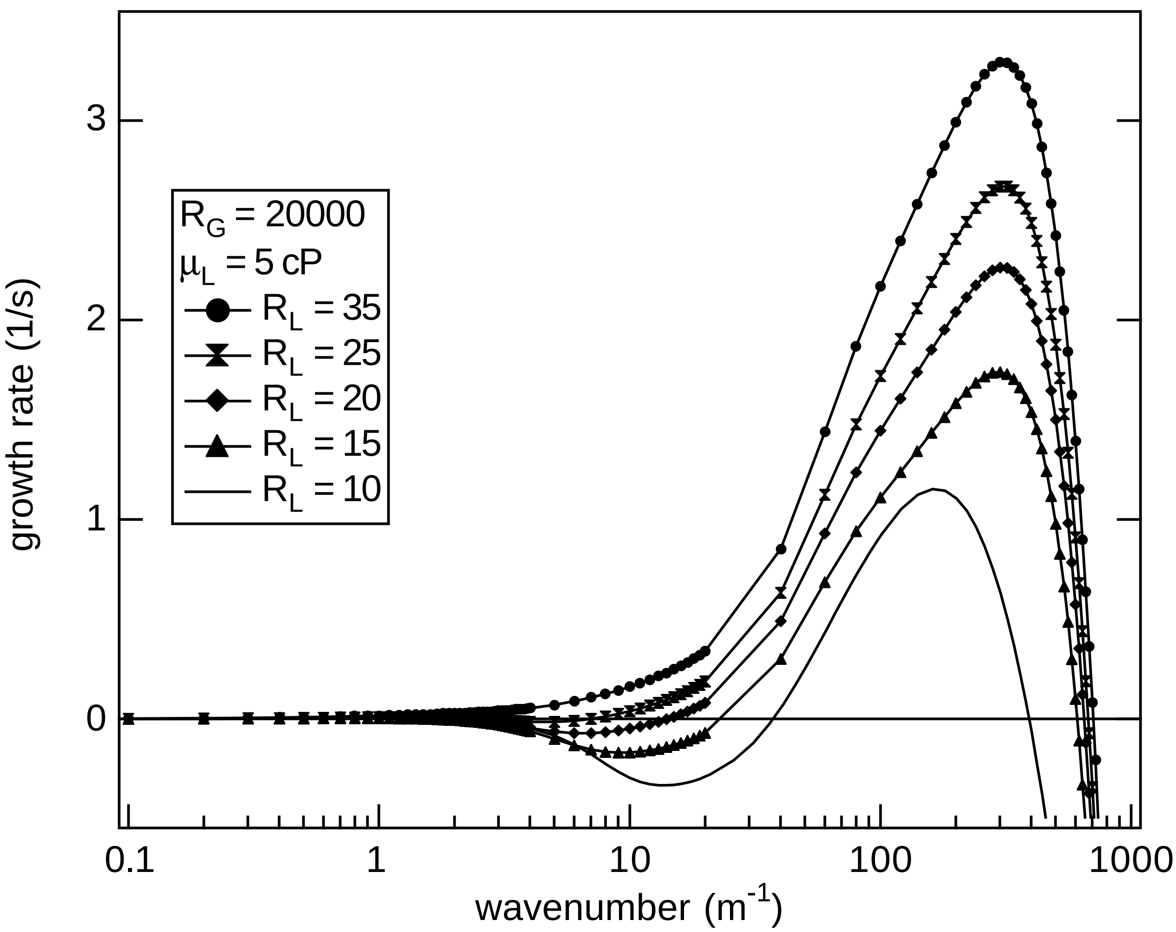
B. Weakly Nonlinear

i. Value of Landau Coefficient at wavenumbers below peak in growth rate

- This would be a logical, but not rigorous extension of weakly-nonlinear theory

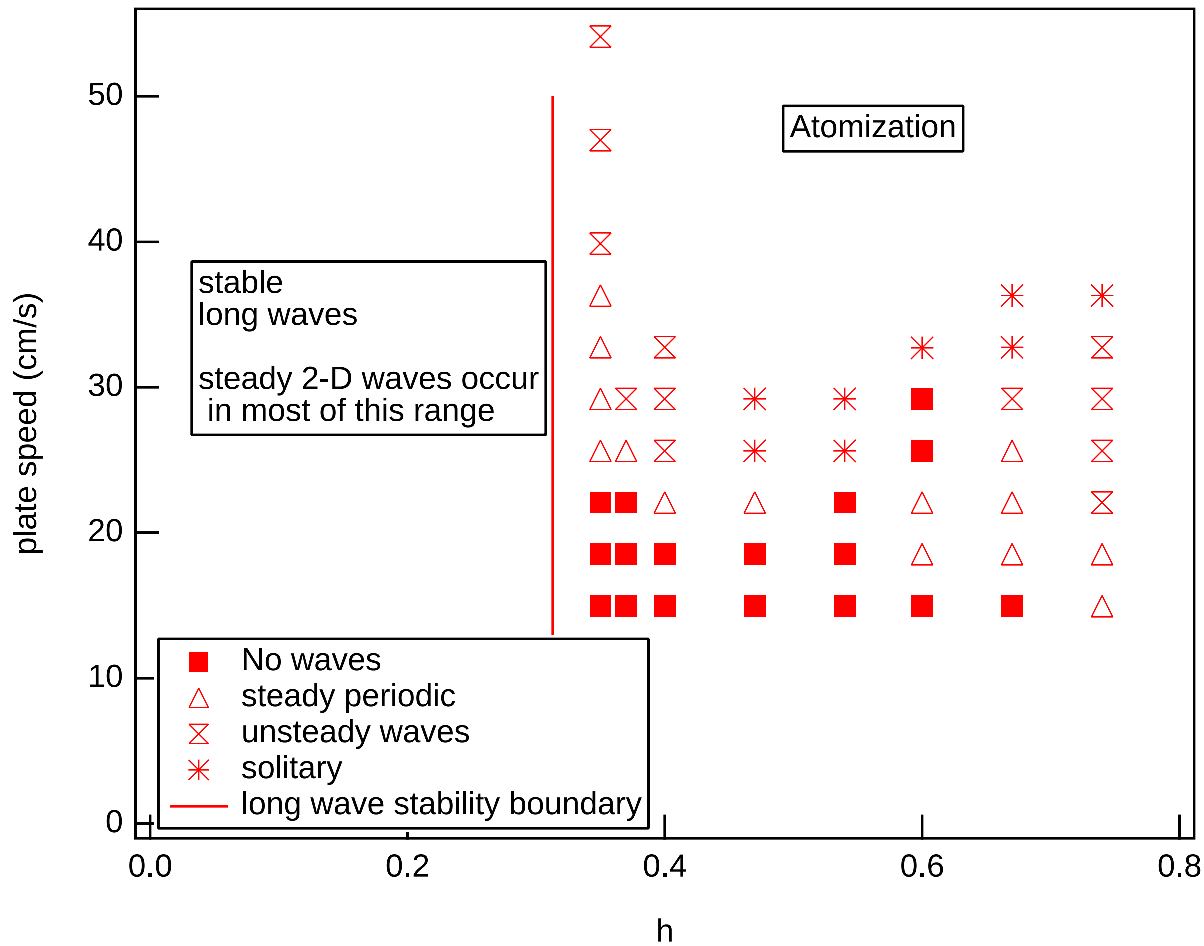
Growth curves (from laminar stability code) across the roll wave transition at low R_L

Long waves are unstable always but the entire range is unstable only for $R_L > 20$



Thus, we might surmise that a necessary condition for long waves is that the growth curve us unstable all the way down to 0 wavenumber.

However, at least for one system (our density matched, rotating two-layer Couette flow) this is not the case.



Plot from Gallagher et al. (1996), Phys. Fluids.

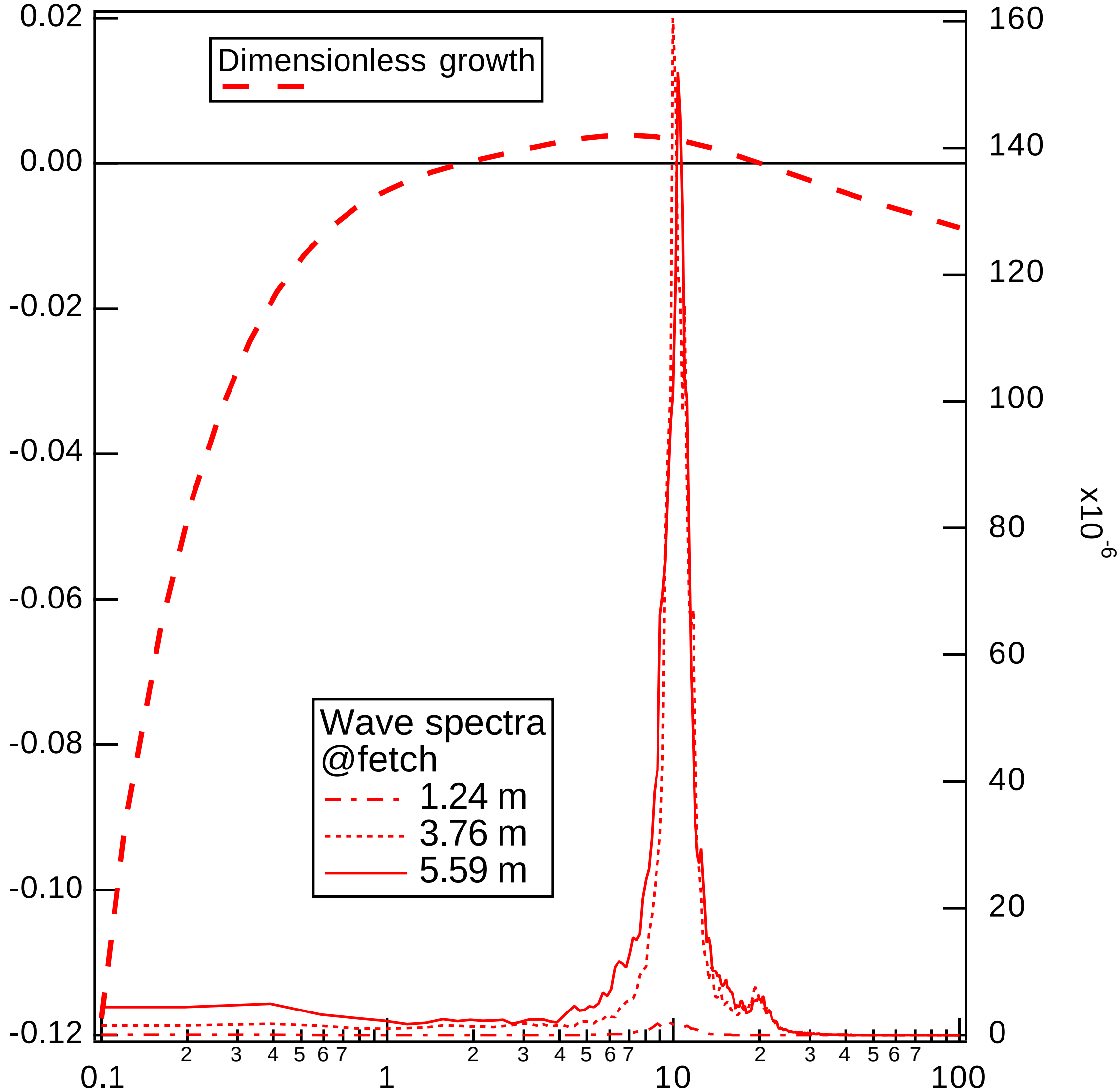
ii. Modified (nondimensional) linear growth rate

$$\xi = \frac{\text{temporal growth}}{\text{wavenumber * wave speed}}$$

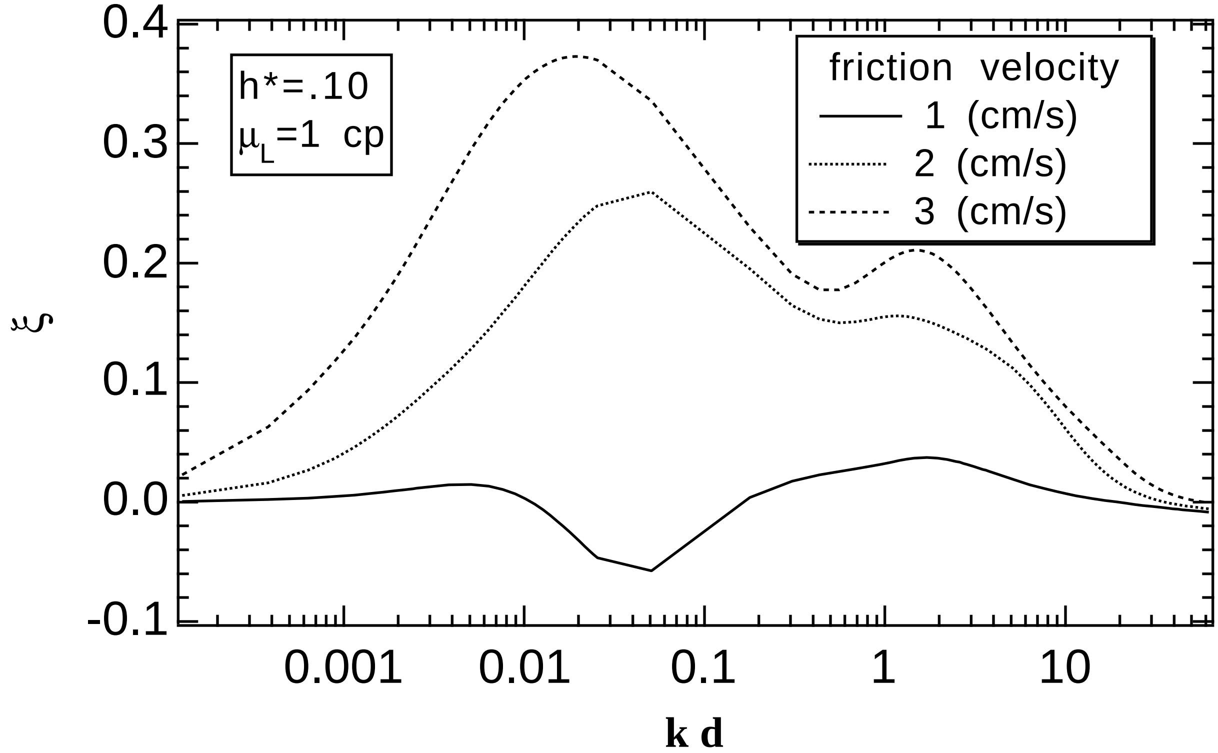
This is a rough measure of dispersion.

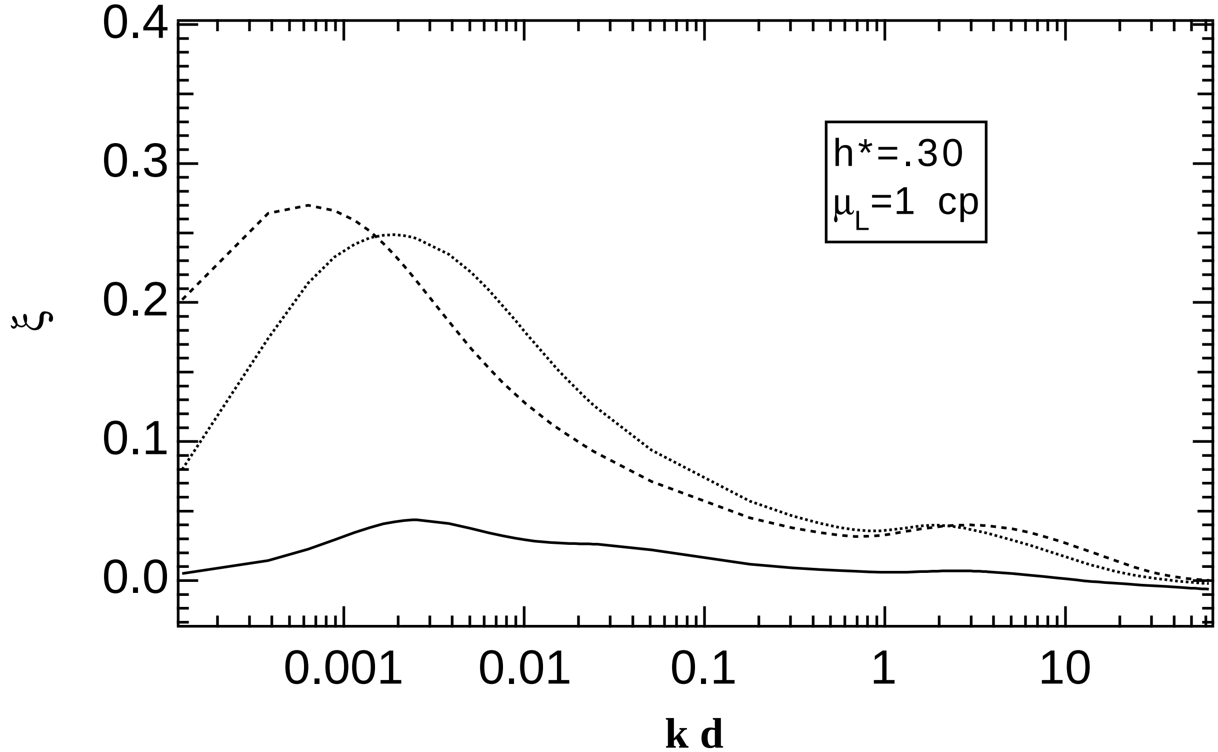
- Rational is that higher dispersion causes large waves, which are necessarily many modes, to break up.

For low shear, this is similar to the growth rate

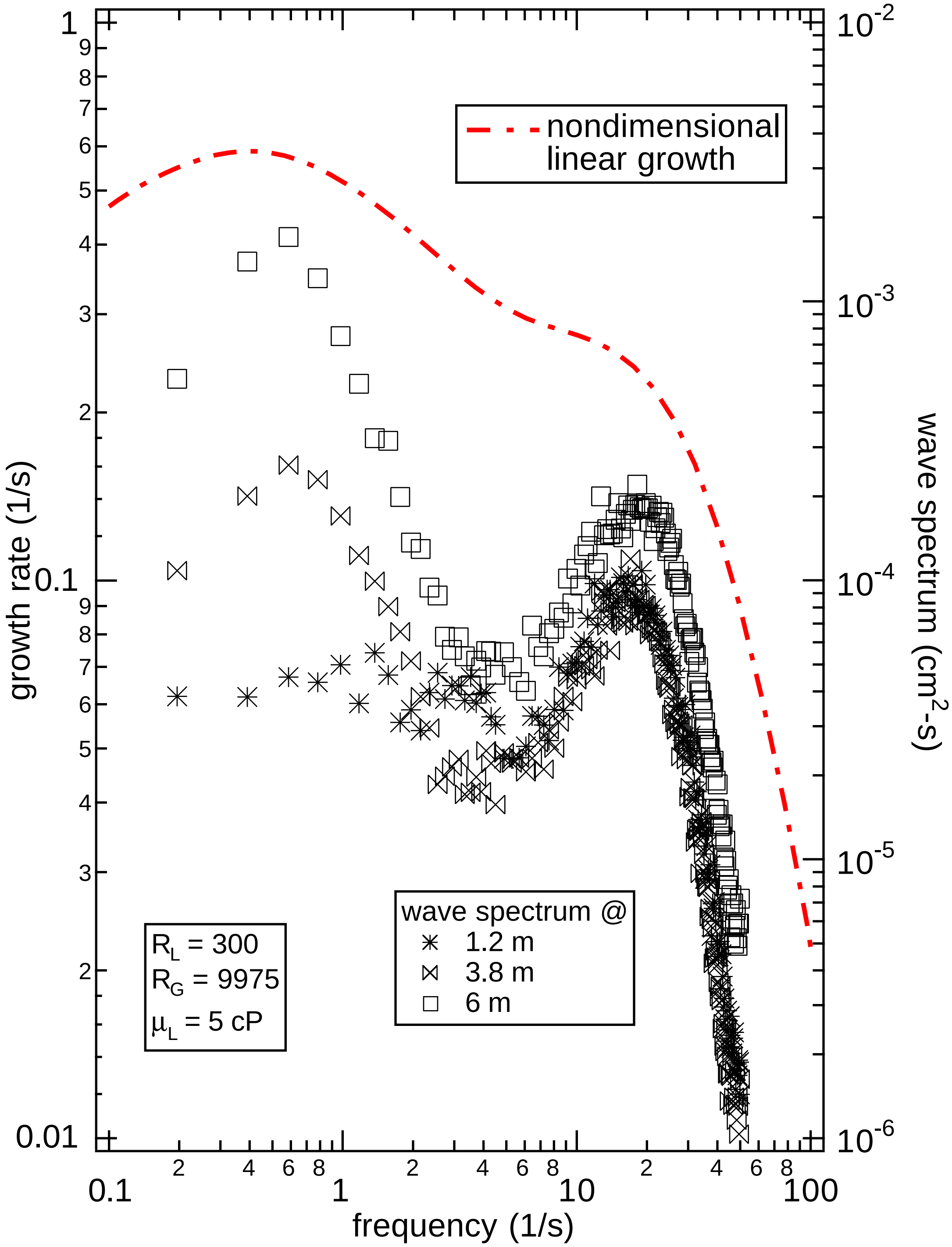


Modified growth rate as a function of friction velocity and depth





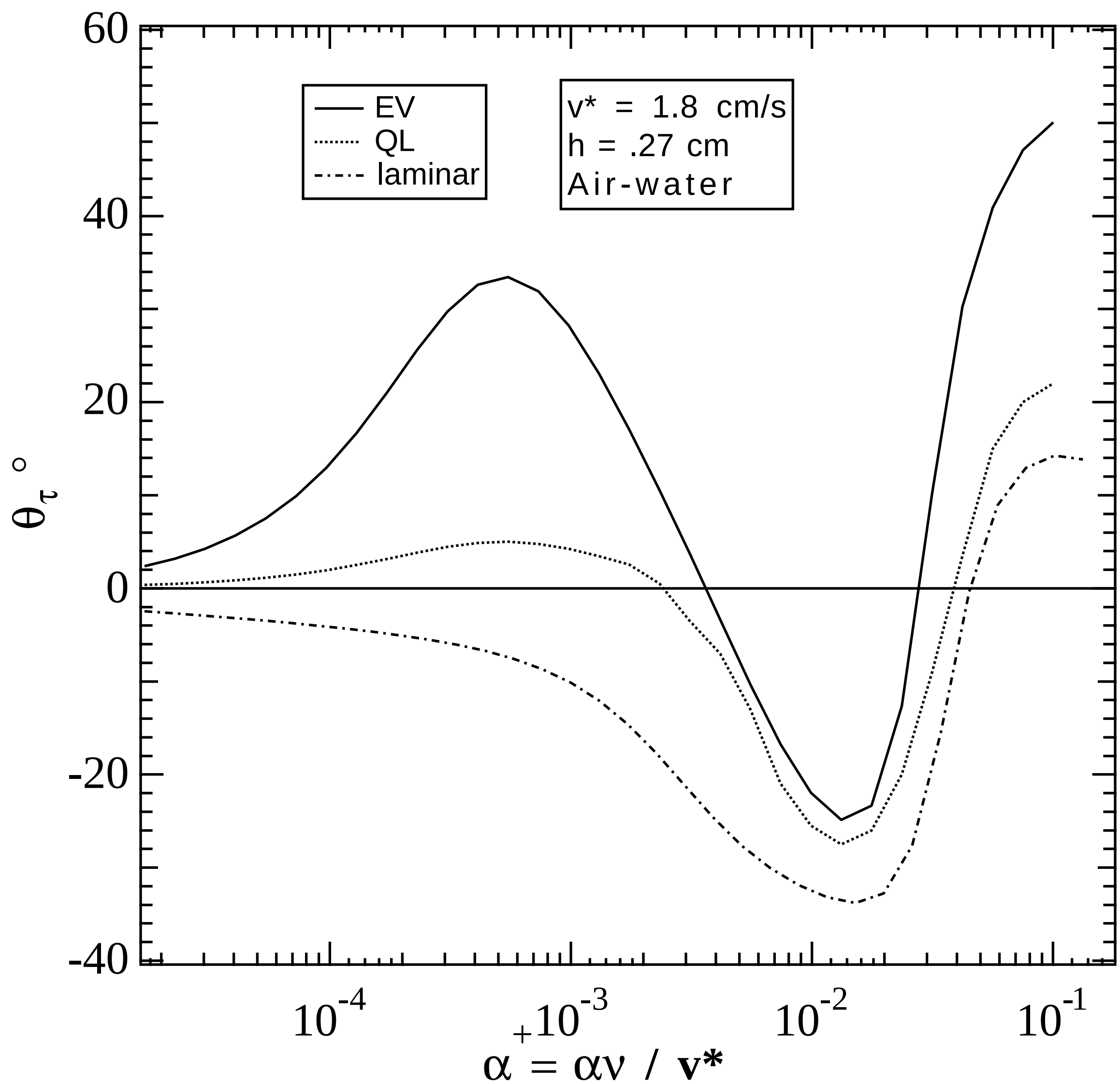
Growth of roll wave precursor peak compared to dimensionless growth rate, ξ



Phase angle of shear stress often has an pronounced peak in the wavenumber range where roll waves show up.

- Rational is that as wave reach finite amplitude, this mechanism would be more efficient either because the phase angle for shear stress changes or because the finite amplite of the wave makes growth more efficient

(we have seen real good agreement for vertical annular flows)



Weakly- nonlinear theory

$$\frac{D\mathbf{u}}{Dt} = -\nabla p + \frac{1}{R}\nabla^2 \mathbf{u}$$

Spectral reduction of Navier-Stokes equations and boundary conditions

$$\psi \equiv (\mathbf{u}, p, h)$$

If system is such that a single dominant mode exists then:

$$\psi = A\,\zeta + \overline{A\,\zeta} + \xi$$

$A \equiv$ Complex Amplitude Function

$\zeta \equiv$ dominant eigenfunction

$\xi \equiv$ Linear combination of eigen functions of stable modes

Center manifold projection to produce a Stuart - Landau equation.

(Blennerhassett, 1980; Renardy & Renardy, 1993).

$$\frac{\partial A}{\partial t} = L(\lambda)\, A + \beta\, |A^2| A$$

β $\equiv$ Landau constant

$L(\lambda)$ $\equiv$ Linear mode behavior

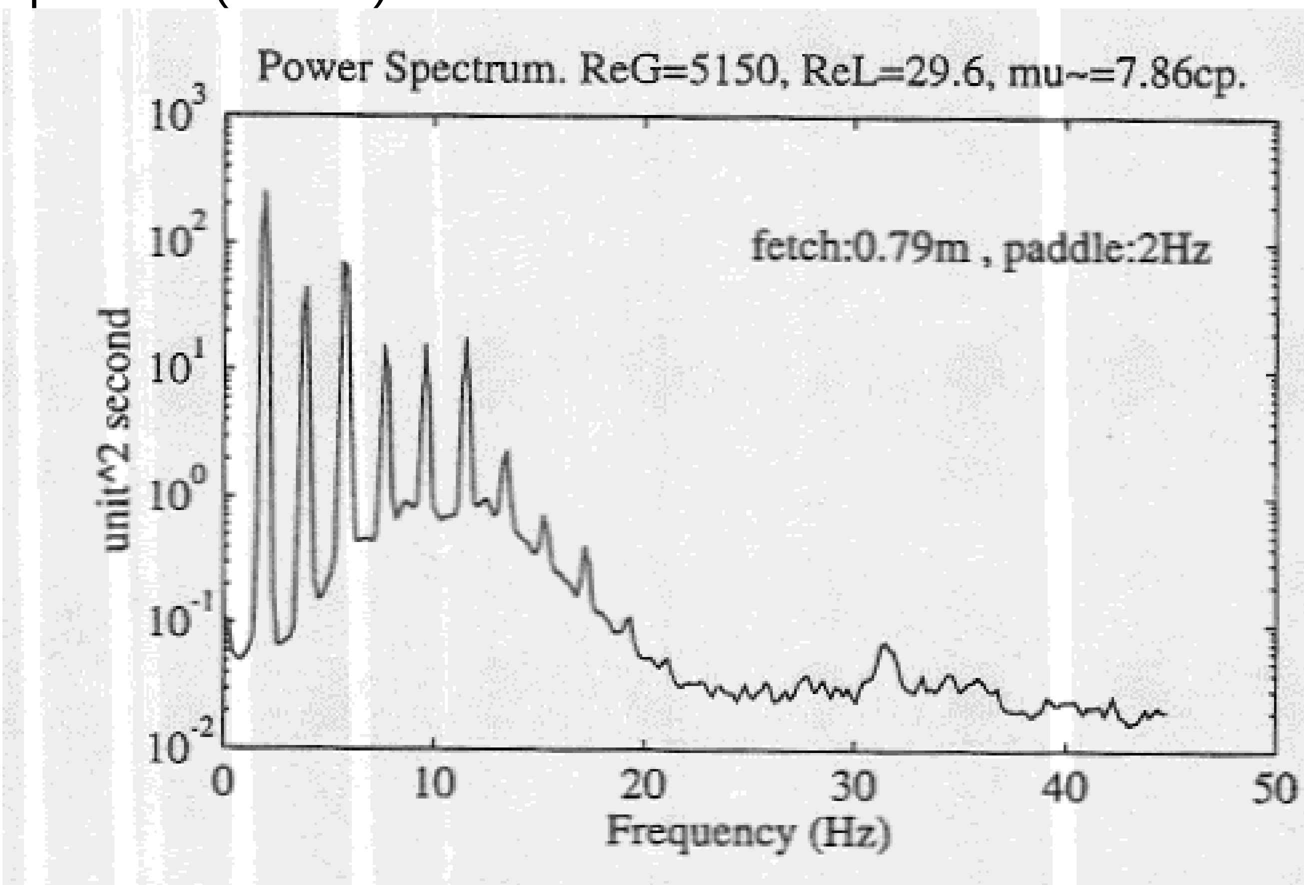
If β is positive conditions are subcritical and nonlinearly enhanced growth will occur.

- •• For most conditions, region below the peak has a positive β.

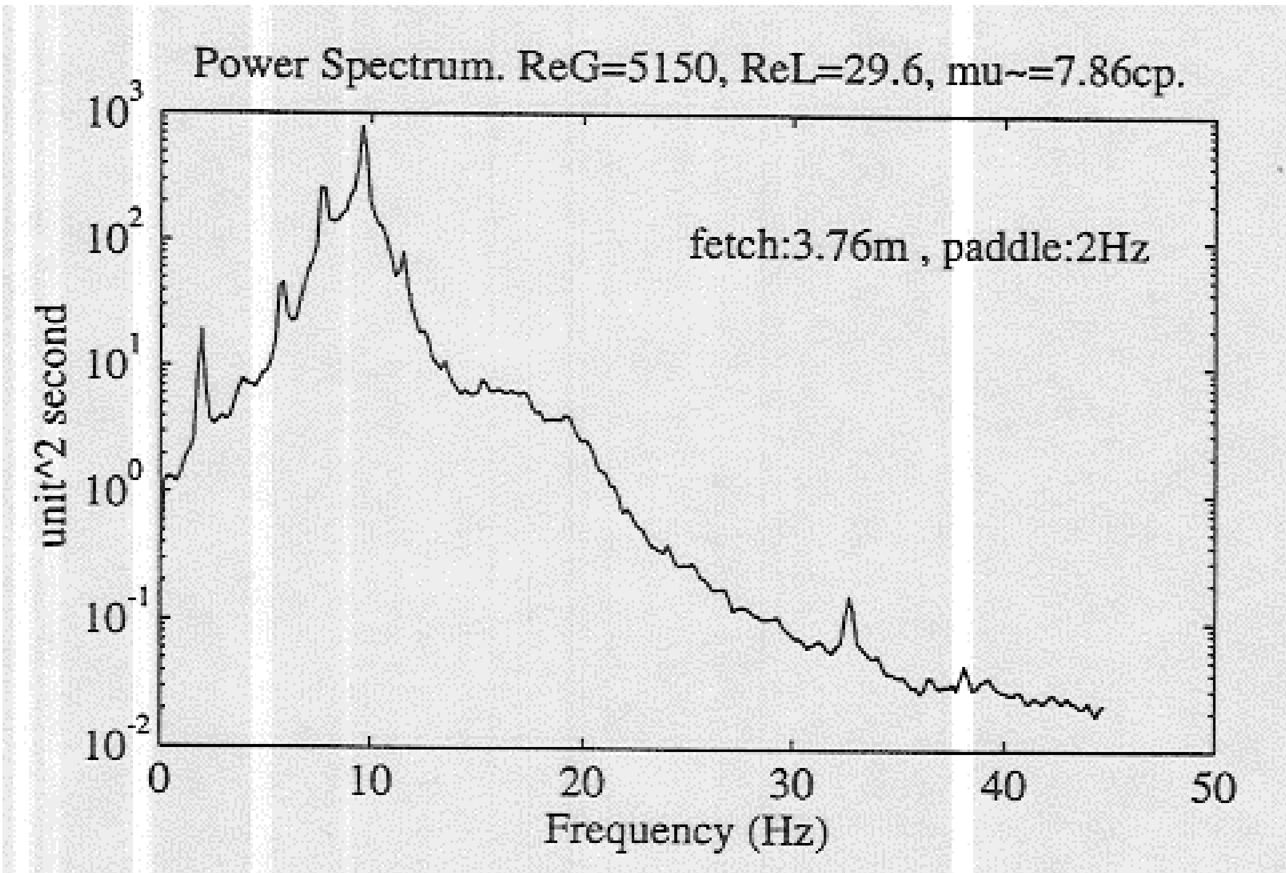
Can we see nonlinear destabilization ??

Mechanical paddle was used to induce waves at frequencies below peak. Frequency of 1 to 10 Hz was tried.

upstream (0.79 m)

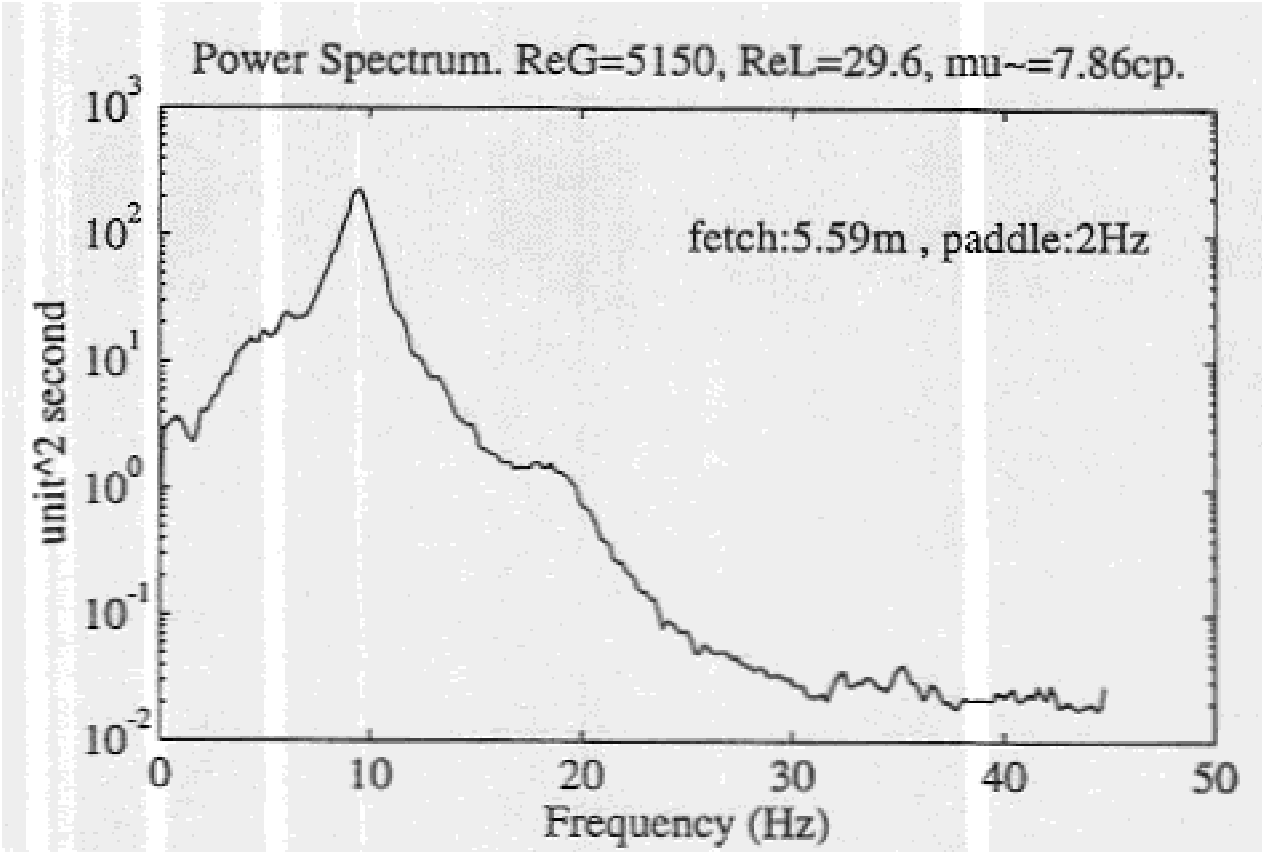


Middle (3.76 m)



Power Spectrum. ReG=5150, ReL=29.6, mu~=7.86cp.
fetch:3.76m , paddle:2Hz
unit^2 second
10^3
10^2
10^1
10^0
10^-1
10^-2
0
10
20
30
40
50
Frequency (Hz)

End (5.59 m)


Power Spectrum. ReG=5150, ReL=29.6, mu~=7.86cp.
fetch:5.59m , paddle:2Hz
unit^2 second
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
0
10
20
30
40
50
Frequency (Hz)

Conclusions

1. Friction factor representation of base state is often not accurate

   two-layer turbulence model does better but the issue of how to handle the interfacial waves is not completely resolved.

(Celik and Rodi, PCH 5 p217 (1984); have used finite values of k and ε at the interface to account for waves)

2. Simplified theories (Inviscid or averaged-equations) usually do not work very well.

3. Loss of linear stability is probably necessary but not sufficient for formation of long waves.

4. "Extra linear" conjectures show some promise

   i. Growth rate in entire low range
   ii. Modified growth rate
   iii. Shear stress phase angle

5. Weakly nonlinear theory is not yet telling us much. "Positive" Landau coefficient does not predict nonlinear growth

   Need to look for specific mode interactions between long waves and shorter waves.