## 14A.1 Average heat transfer coefficients.

The total heat transfer rate is:

$$\begin{aligned} Q &= w\hat{C}_p(T_{b2} - T_{b1}) \\ &= (10,000\ \mathrm{lb}_m/\mathrm{hr})(0.6\ \mathrm{Btu/lb}_m{\cdot}\mathrm{F})(200 - 100\ \mathrm{F}) \\ &= 600,000\ \mathrm{Btu/hr} \end{aligned}$$

The total inside surface area of the tubes is:

$$A = \pi D L_{\mathrm{tot}} = (\pi)(1.00 - 2 \times 0.065\ \mathrm{in})(1/12\ \mathrm{ft/in})(300\ \mathrm{ft}) = 68.3\ \mathrm{ft}^2$$

The various temperature differences between the inner tube surfaces and the oil are:

$$\begin{aligned} (T_0 - T_b)_1 &= 213 - 100 = 113\ \mathrm{F} \\ (T_0 - T_b)_a &= (113 + 13)/2 = 63\ \mathrm{F} \\ (T_0 - T_b)_{\ln} &= (113 - 13)/\ln(113/13) = 46.2\ \mathrm{F} \end{aligned}$$

Insertion of these values into Eqs. 14.1-2,3,4 then gives the heat transfer coefficients:

$$\begin{aligned} h_1 &= (600,000)/(68.3 \times 113) = 78\ \mathrm{Btu/hr{\cdot}ft^2{\cdot}F} \\ h_a &= (600,000)/(68.3 \times 63) = 139\ \mathrm{Btu/hr{\cdot}ft^2{\cdot}F} \\ h_{\ln} &= (600,000)/(68.3 \times 46.2) = 190\ \mathrm{Btu/hr{\cdot}ft^2{\cdot}F} \end{aligned}$$

## 14A.7 Free-convection heat transfer from an isolated sphere.

For the conditions of this problem, the thermal expansion coefficient $\beta = 1/T_f$ is (1/338.7 K), and the other physical properties are the same as in part **(a)** of Problem 14A.6. (Note that, for the correlations in §14.6, $\beta$ and $\rho$ are evaluated at $T_f$ rather than $\overline{T}$ for calculation of Gr.) Then

$$\begin{aligned}
\text{GrPr} &= \left(\frac{D^3\rho^2 g\beta\Delta T}{\mu^2}\right)\left(\frac{\hat{C}_p\mu}{k}\right) \\
&= \frac{(2.54\text{ cm})^3(0.001042\text{ g/cm·s})^2(980.7\text{ cm/s}^2)(100/[1.8\times 338.7])}{(2.023\times 10^{-4}\text{ g/cm·s})^2}(0.703) \\
&= 4.92\times 10^4
\end{aligned}$$

Eq. 14.6-4 gives

$$\begin{aligned}
\text{Nu}_m^{\text{lam}} &= \frac{0.878\times 0.671}{[1+(0.492/\text{ Pr})^{9/16}]^{4/9}}(\text{ GrPr})^{1/4} \\
&= \frac{(0.878)(0.671)}{[1+(0.492/0.703)^{9/16}]^{4/9}}\left(4.92\times 10^4\right)^{1/4} \\
&= 6.73
\end{aligned}$$

Hence,

$$\begin{aligned}
h_m &= 6.73k/D = (6.73)(26.9\times 10^{-5}\text{ W/cm·K})/(2.54\text{ cm}) \\
&= 0.000712\text{ W/cm}^2\text{·K}
\end{aligned}$$

and the convective heat loss rate is

$$\begin{aligned}
Q &= \pi D^2 h_m(T_0 - T_\infty) \\
&= \pi(2.54\text{ cm})^2(0.000712\text{ W/cm}^2\text{·K})([100/1.8]\text{ K}) \\
&= 0.80\text{ W} = 0.20\text{ cal/s}
\end{aligned}$$

By the methods of §16.5, one can calculate that the rate of heat loss by radiation is of comparable magnitude: 1.0 W for a perfectly black sphere in a large enclosure with walls at 100°F.

**14B.2 Local overall heat transfer coefficient.**

Let 0 and 1 denote the inner and outer surfaces of the tube, and $h_0$ and $h_1$ denote the local heat transfer coefficients on those surfaces at the cross-section where the oil bulk temperature is 150°F. According to the development in §9.6, the temperature drops within a cross-section have the same ratio as the corresponding resistance terms that sum to $1/(r_0U_0)$:

$$\frac{T_1 - 150}{213 - T_1} = \left[\frac{1}{r_0 h_0} + \frac{\ln(r_1/r_0)}{k_{01}}\right] / \left[\frac{1}{r_1 h_1}\right]$$

The numerator on the right is

$$\frac{1}{(0.435/12)(190)} + \frac{\ln(0.5/0.435)}{220}$$
$$= 0.1452 + 0.0006 = 0.1458 \text{ hr·ft·F/Btu}$$

in which a thernal conductivity of $k_{01} = 220$ Btu/hr·ft·F has been used for copper at $T \approx 190$°F, based on Tables 9.1-5 and F.3-5. To calculate the denominator, we use Eq. 14.7-3 for the heat transfer coefficient for filmwise condensation on horizontal tubes. Iteration is required, since the temperature difference across the condensate film is unknown. As a first approximation, we choose $T_1 = 190$°F, and use the physical properties at 200°F from Example 14.7-1:

$$\begin{aligned} \Delta\hat{H}_{\text{vap}} &= 978 \text{ Btu/lb}_m \\ k &= 0.393 \text{ Btu/hr·ft·F} \\ \rho &= 60.1 \text{ lb}_m/\text{ft}^3 \\ \mu &= 0.738 \text{ lb}_m/\text{hr·ft} \end{aligned}$$

Then Eq. 14.7-3 gives

$$\begin{aligned} h_1 = h_m &= 0.725\left[\frac{k^3\rho^2 g\Delta\hat{H}_{\text{vap}}}{\mu D(T_d - T_1)}\right]^{1/4} \\ &= 0.725\left[\frac{0.393^3\rho^2(4.17\times10^8)(978.8)}{(0.738)(0.5/12)(T_d - T_1)}\right]^{1/4} \\ &= 12,500(213 - T_1)^{-1/4} \end{aligned}$$

Equating the heat flow through the numerator and denominator resistances gives

$$(T_1 - 150)/0.1458 = (213 - T1)r_1h_1 = (213 - T_1)^{3/4}(0.5/12)(12,500)$$

or

$$213 - T_1 = 0.0000183(T_1 - 150)^{4/3}$$

Successive substitutions of $T_1$ in the right-hand term give a rapidly converging sequence of left-hand values: to the solution: $T_1 = 212.9975, 212.9954, 212.9954, \ldots$. Thus, the outer-surface temperature of the tubes at this cross-section is 212.9954° F. The temperature drop through the tube wall is $0.0006/0.1458(212.995 - 150) = 0.25$F. Thus, the thermal resitances of the tube wall and condensate film are unimportant here, as assumed in Problem 14A.1.

**14B.3 The hot-wire anemometer**

*a*. The physical properties of interest at $p$ = 1 atm and a film temperature of 335°F are:

$$\rho = 0.0499 \ \text{lb}_m/\text{ft}^3$$

$$\hat{C}_p = 0.242 \ \text{Btu/lb}_m\cdot°\text{F}$$

$$\mu = 0.0594 \ \text{lb}_m/\text{ft}\cdot\text{hr} = 1.64\times 10^{-5} \ \text{lb}_m/\text{ft}\cdot\text{s} \ \text{(from Eq. 1.4.14)}$$

$$k = \left(0.242 + \frac{5}{4}\left(\frac{1.986}{29}\right)\right)\left(1.64\times 10^{-5}\right) = 5.373\times 10^{-6}\ \text{Btu/ft}\cdot\text{s}°\text{F}$$

(from Eq. 9.3-15)

Pr = 0.74 (from Eq. 9.3-16)

Also the Reynolds number is $\text{Re} = \dfrac{(0.01/12)(100)(0.0499)}{\left(1.64\times 10^{-5}\right)} = 254$

Then Eq. 14.4-8 gives

$$\text{Nu}_m = (5.99 + 2.29)(0.905) + 0.92(-3.54 + 1062)^{-1/3}(6.33)(0.905) = 8.010$$

Then we get the heat transfer coefficient from

$$h_m = \frac{\text{Nu}_m k}{D} = \frac{(8.010)\left(5.373\times 10^{-6}\right)}{(0.01/12)} = 0.0516 \ \text{Btu/ft}^2\cdot\text{s}°\text{F} = 186 \ \text{Btu/ft}^2\cdot\text{hr}°\text{F}$$

Finally, the heat loss from the wire is

$$Q = h_m A\Delta T = h_m\cdot \pi DL\cdot(T_0 - T_\infty) = (186)\left(\pi\frac{0.01}{12}\frac{0.5}{12}\right)(600-70)\,\text{Btu/hr} = 10.75 \ \text{Btu/hr} = 3.15 \ \text{W} = 10.75 \ \text{W}$$

*b*. For an approach velocity of 300 ft/s, Re = 762. Equation 14.4-8 gives $\text{Nu}_m = 14.20$, and $Q(300)/Q(100) = 14.20/8.010 = 1.77$. This is very close to $\sqrt{3} = 1.73$ from King's relation.

**15A.1 Heat transfer in double-pipe heat exchangers.**

**(a)** In the absence of heat loss to the surroundings, Eqs. 15.4-7,8 give

$$w_c\hat{C}_{pc}(T_{c2} - T_{c1}) = -w_h\hat{C}_{ph}(T_{h2} - T_{h1})$$

with each flow rate $w$ expressed from plane 1 toward plane 2. Insertion of the data then gives

$$Q_c = (5000)(1.00)(T_{c2} - 60) = -(-10{,}000)(0.60)(200 - 100) = 600{,}000 \text{ Btu/hr}$$

whence

$$T_{c2} = 60 + 120 = 180°\text{F}$$

The log-mean temperature difference is

$$(\Delta T)_{\ln} = (20 - 40)/\ln(20/40) = 28.85\text{F}$$

and the required heat exchange area, from Eq. 15.4-15, is

$$A_0 = \frac{Q_c}{U_0(\Delta T)_{\ln}} = \frac{(600{,}000 \text{ Btu/hr})}{(200 \text{ Btu/hr·ft}^2\text{·F})(28.85\text{F})} = 104 \text{ ft}^2$$

**(b)** Eq. 15B.1-2 gives

$$\begin{aligned}\frac{Q}{A} &= \frac{U_1\Delta T_2 - U_2\Delta T_1}{\ln(U_1\Delta T_2/U_2\Delta T_1)} \\ &= \frac{(50 \times 20) - (350 \times 40)}{\ln((50 \times 20)/350 \times 40))} \\ &= 4926 \text{ Btu/hr·ft}^2\end{aligned}$$

The required heat exchange area is then

$$A = \frac{Q}{Q/A} = \frac{600{,}000}{4926} = 122 \text{ ft}^2$$

**(c)** The minimum usable flow rate of water to cool the oil to 100°F in counterflow is

$$w_c = \frac{(10{,}000)(0.60)(200 - 100)}{(1.00)(200 - 60)} = 4286 \text{ lb}_m\text{/hr}$$

whereas the minimum usable flow rate of water in parallel flow is

$$w_c = \frac{(10{,}000)(0.60)(200 - 100)}{(1.00)(100 - 60)} = 15{,}000 \text{ lb}_m\text{/hr}$$

**(d)** If parallel flow is used, with $w_c$ =15,500 $\text{lb}_m$/hr of water, the outlet water temperature will be

$$T_{c2} = 60 + (10{,}000)(0.60)(200 - 100)/(15{,}500)(1.00) = 98.71°\text{F}$$

Then $(\Delta T)_{\ln} = (140 - 1.29)/\ln(140/1.29) = 29.6°\text{F}$ and the required heat exchange area is

$$A = \frac{Q}{U(\Delta T)_{\ln}} = \frac{(10{,}000)(0.60)(200 - 100)}{(200)(29.6)} = 101 \text{ ft}^2$$