12A.1 Unsteady-state heat conduction in an iron sphere

a. The thermal diffusivity of the sphere is given by Eq. 9.1-8:

$$\alpha = \frac{k}{\rho \hat{C}_p} = \frac{30}{(436)(0.12)} = 0.573 \text{ ft}^2 / \text{hr}$$

b. The center temperature is to be 128°F; hence

$$\frac{T_{\text{ctr}} - T_0}{T_1 - T_0} = \frac{128 - 70}{270 - 70} = 0.29$$

Then, from Fig. 12.1-3, $\alpha t / R^2 = 0.1$, and

$$t = 0.1\left(\frac{R^2}{\alpha}\right) = 0.1\left(\frac{(1/24)^2}{0.573}\right) = 3.03 \times 10^{-4} \text{hrs} = 1.1 \text{ s}$$

c. By equating the dimensionless times, we get

$$\frac{\alpha_1 t_1}{R_1^2} = \frac{\alpha_2 t_2}{R_2^2}$$

or

$$\alpha_2 = \alpha_1\left(\frac{t_1}{t_2}\right) = 0.573\left(\frac{1}{2}\right) = 0.287$$

d. The partial differential equation from which Fig. 12.1-3 was constructed is

$$\frac{\partial T}{\partial t} = \alpha \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right)$$

12A.2 Comparison of the two slab solutions for short times

According to Figure 12.1-1, at $\alpha t/b^2 = 0.01$ and $y/b = 0.9$

$$\frac{T - T_0}{T_1 - T_0} \approx 0.46$$

where y is the distance from the mid-plane of the slab.

Next we use Fig. 4.1-1, which can be interpreted as a plot of $(T - T_0)/(T_1 - T_0)$ vs $y'/\sqrt{4\alpha t}$, where $y' = b - y$ is the distance from the wall. We then get

$$\frac{y'}{\sqrt{4\alpha t}} = \frac{1}{2}\frac{(1-0.9)}{\sqrt{\alpha t/b^2}} = \frac{1}{2}$$

Then from Fig. 4.1-1 we find

$$\frac{T - T_0}{T_1 - T_0} \approx 0.48$$

Hence the use of the combination of variables solution introduces an error of about 4%. Smaller errors occur at smaller values of the dimensionless time $\alpha t/b^2$.

12B.4 Heat transfer from a wall to a falling film (short contact time limit)

a. From Eq. 2.2-18, we get

$$v_z = v_{z,\max}\left[1-(x/\delta)^2\right] = v_{z,\max}\left[1-(1-(y/\delta))^2\right]$$
$$= v_{z,\max}\left[1-1+2(y/\delta)+(y/\delta)^2\right] \to 2v_{z,\max}(y/\delta)$$

this last expression is good in the vicinity of the wall, where the quadratic term can be neglected.

b. Equation 12B.4-2 presupposes that the heat conduction in the z direction can be neglected relative to the heat convection in the z direction. In addition, laminar, nonrippling flow is assumed.

c. The fictitious boundary condition at an infinite distance from the wall may be used instead of the boundary condition at a distance δ from the wall, since for short contact times the fluid is heated over a very short distance y. Therefore the inifinite boundary condition can be expected to be adequate.

d. Equation 12B.4-3 can be written as $y(\partial\Theta/\partial z) = \beta(\partial^2\Theta/\partial y^2)$. Next we have to convert the derivatives to derivatives with respect to the dimensionless variable η:

$$\frac{\partial\Theta}{\partial z} = \frac{d\Theta}{d\eta}\frac{\partial\eta}{\partial z} = \frac{d\Theta}{d\eta}\frac{y}{\sqrt[3]{9\beta z}}\left(-\frac{1}{3z}\right)$$

$$\frac{\partial\Theta}{\partial y} = \frac{d\Theta}{d\eta}\frac{\partial\eta}{\partial y} = \frac{d\Theta}{d\eta}\frac{1}{\sqrt[3]{9\beta z}}; \quad \frac{\partial^2\Theta}{\partial y^2} = \frac{d}{d\eta}\left(\frac{d\Theta}{d\eta}\frac{1}{\sqrt[3]{9\beta z}}\right)\frac{\partial\eta}{\partial y} = \frac{d^2\Theta}{d\eta^2}\left(\frac{1}{\sqrt[3]{9\beta z}}\right)^2$$

When these relations are substituted into the partial differential equation and use is made of the defining equation for η we get Eq. 12B.4-7.

e. When we set $d\Theta/d\eta = p$, we get $dp/d\eta + 3\eta^2 p = 0$, which is first-order and separable, and the solution is given in the book. The next integration gives

$$\Theta = C_1\int_0^{\eta} e^{-\bar{\eta}^3} d\bar{\eta} + C_2$$