$\boxed{A.1}$

$$\rho C_p \frac{DT}{Dt} = -(\underline{\nabla} \cdot q) - (\tau : \nabla v) - \left(\frac{\partial \ln P}{\partial \ln T}\right)\frac{DP}{Dt}$$

no convective transport : $\rho C_p \frac{DT}{Dt} = 0$

no viscous dissipation : $(\tau : \nabla v) = 0$

no pressure gradient : $\left(\frac{\partial \ln P}{\partial \ln T}\right)\frac{DP}{Dt} = 0$

then,

$$-(\underline{\nabla} \cdot q) = 0$$

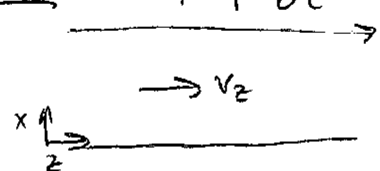
$$-\left(\frac{1}{r}\frac{\partial}{\partial r}(r q_r) + \frac{1}{r}\frac{\partial q_\theta}{\partial \theta} + \frac{\partial q_z}{\partial z}\right) = 0$$

no conduction in θ- or z-direction

$$\boxed{-\frac{1}{r}\frac{\partial}{\partial r}(r q_r) = 0}$$

$\boxed{A.7}$

$$\rho C_p \frac{DT}{Dt} = -(\nabla \cdot q) - (\tau : \nabla v) - \left(\frac{\partial \ln P}{\partial \ln T}\right)\frac{DP}{Dt}$$



LHS: $\rho C_p \frac{DT}{Dt} = \rho C_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z}\right)$

$\frac{\partial T}{\partial t} = 0$ no time dependence

$v_x = v_y = 0$ velocity only in z-direction

$\frac{\partial T}{\partial z} = 0$ Temp depends only on X

$\therefore \rho C_p \frac{DT}{Dt} = 0$

RHS terms:

* $\left(\frac{\partial \ln \rho}{\partial \ln T}\right)\frac{DP}{Dt} = 0$ (ρ constant)

* $-(\nabla \cdot q) = -\left[\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}\right]$

$\frac{\partial q_y}{\partial y} = \frac{\partial q_z}{\partial z} = 0$ conduction only in x-direction

$-(\nabla \cdot q) = -\frac{\partial q_x}{\partial x}$

* $(\tau : \nabla v) = \tau_{xx}\left(\frac{\partial v_x}{\partial x}\right) + \tau_{xy}\left(\frac{\partial v_x}{\partial y}\right) + \tau_{xz}\left(\frac{\partial v_x}{\partial z}\right) + \tau_{yx}\left(\frac{\partial v_y}{\partial x}\right)$

$+ \tau_{yy}\left(\frac{\partial v_y}{\partial y}\right) + \tau_{yz}\left(\frac{\partial v_y}{\partial z}\right) + \tau_{zx}\left(\frac{\partial v_z}{\partial x}\right) + \tau_{zy}\left(\frac{\partial v_z}{\partial y}\right) + \tau_{zz}\left(\frac{\partial v_z}{\partial z}\right)$

Velocity in z-direction only & depends only on x

$\therefore$ $\frac{\partial v_z}{\partial x}$ is only non-zero term

$$-(\tau : \nabla v) = -\tau_{zx}\left(\frac{\partial v_z}{\partial x}\right) = -\left(-\mu\left[\underbrace{\frac{\partial v_x}{\partial z}}_{0} + \frac{\partial v_z}{\partial x}\right]\right.$$

$$-(\tau : \nabla v) = \mu \frac{\partial v_z}{\partial x}$$

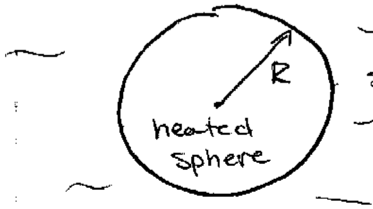
$\therefore$ total balance reduced to:

$$-\frac{\partial q_x}{\partial x} + \mu \frac{\partial v_z}{\partial x}\frac{\partial v_z}{\partial x} = 0$$

$$-q_x + \mu v_z \frac{\partial v_z}{\partial x} = C_1$$

$$\boxed{k\frac{\partial T}{\partial x} + \mu v_z \frac{dv_z}{dx} = C_1}$$

B.1 $$\rho C_p \frac{DT}{Dt} = -(\nabla \cdot q) - (\tau : \nabla v) - \left(\frac{\partial \ln \rho}{\partial \ln T}\right)\frac{DP}{Dt}$$



suspended in motionless fluid

NO convection

$$\rho C_p \frac{DT}{Dt} = 0$$

$$(\tau : \nabla v) = 0$$

constant ρ, P $\quad \frac{\partial \ln \rho}{\partial \ln T}\frac{DP}{Dt} = 0$

then total balance reduced to

$$-(\nabla \cdot q) = 0$$

in spherical coordinates:

$$-\left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 q_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(q_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial q_\phi}{\partial \phi}\right] = 0$$

q in r-direction only

$$\therefore \quad \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 q_r) = 0$$

$$\boxed{\frac{\partial}{\partial r}(r^2 q_r) = 0}$$

B.6 $$\rho C_p \frac{DT}{Dt} = -(\nabla \cdot q) - (\tau : \nabla v) - \left(\frac{\partial \ln \rho}{\partial \ln T}\right)\frac{DP}{Dt}$$

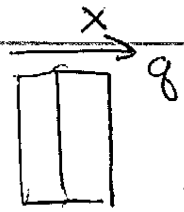
No flow

$$\rho C_p \frac{DT}{Dt} = 0$$

$$(\tau : \nabla v) = 0$$

Const. $\rho, P \Rightarrow \left(\frac{\partial \ln \rho}{\partial \ln T}\right)\frac{DP}{Dt} = 0$

$$\therefore \quad -(\nabla \cdot q) = 0$$

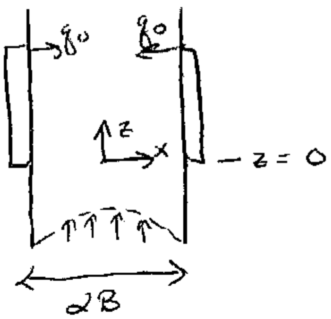


conduction is in x-direction only

so $-\left(\frac{\partial q_x}{\partial x} + \cancel{\frac{\partial q_y}{\partial y}} + \cancel{\frac{\partial q_z}{\partial z}}\right) = 0$

$$\boxed{-\frac{\partial q_x}{\partial x} = 0}$$

B.7



$$\rho C_p \frac{DT}{Dt} = -(\nabla \cdot q) - (\tau : \nabla v) - \left(\frac{\partial \ln \rho}{\partial \ln T}\right)\frac{DP}{Dt}$$

LHS:

$$\rho C_p \frac{DT}{Dt} = \rho C_p \left[\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z}\right]$$

T is time independent

$v_x = v_y = 0$

$$\therefore \rho C_p \frac{DT}{Dt} = \rho C_p \frac{\partial T}{\partial z} v_z$$

RHS:

$(\tau : \nabla v) = 0$ (problem statement)

$\left(\frac{\partial \ln \rho}{\partial \ln T}\right)\frac{DP}{Dt} = 0$ ρ const.

$$-(\nabla \cdot q) = -\left[\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}\right]$$

$\frac{\partial q_y}{\partial y} = 0$ (no conduction in y-direction)

$\frac{\partial q_z}{\partial z} = 0$ (problem statement – neglect axial heat conduction term)

then,

$$\boxed{\rho C_p v_z \frac{\partial T}{\partial z} = -\frac{\partial q_x}{\partial x}}$$

B.15

$$\rho C_p \frac{DT}{Dt} = -(\nabla \cdot q) - (\tau : \nabla v) - \left(\frac{\partial \ln \rho}{\partial \ln T}\right)\frac{DP}{Dt} + S_e$$

no convective transport: $\rho C_p \frac{DT}{Dt} = 0$

$(\tau : \nabla v) = 0$

Constant Pressure, $\frac{DP}{Dt} = 0$

then,

$$(\nabla \cdot q) = S_e$$

$$\left[\frac{1}{r}\frac{\partial}{\partial r}(r q_r) + \frac{1}{r}\frac{\partial q_\theta}{\partial \theta} + \frac{\partial q_z}{\partial z}\right] = S_e$$

Temp gradient in r-direction only,

$$\frac{1}{r}\frac{\partial}{\partial r}(r q_r) = S_e$$

$$\boxed{\frac{\partial}{\partial r}(r q_r) = S_e r}$$