INTERFACIAL WAVE BEHAVIOR IN OIL-WATER CHANNEL FLOWS: PROSPECTS FOR A GENERAL UNDERSTANDING

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ABSTRACT

Oil-water pressure driven channel flow is examined as a model for general two-layer flows where interfacial disturbances are important. The goal is to develop sufficient understanding of this system so that the utility and limitations of linear and nonlinear theories can be known *a priori*. Experiments show that sometimes linear stabilty is useful at predicting the steady or dominant evolving waves. However in other situations there is no agreement between the linearly fastest growing wave and the spectral peak. An interesting preliminary result is that the bifurcation to interfacial waves is supercritical for all conditions that were studied for an oil-water channel flow, gas-liquid channel flow and two-liquid Couette flow. However, three different mechanisms are dominant for each of these three situations.

INTRODUCTION

All facets of energy production and distribution involve processes that contain multi-fluid flows. Hydrocarbon production and transportation, energy exchange devices such as condensers are common examples. Further, there is extensive use of multiphase reactors for chemical production in which hydrodynamic instabilities can alter the chemical product distribution.

Unfortunately, even for the simplest class of two-fluid flows, stratified, it is not possible to predict the nature of the instabilities that occur and the eventual downstream behavior. While it is in principle possible to perform a linear stability analysis of the base flow (the complication being the possible presence of turbulence), linear theory tells information only about the initial instability. This is sufficient to predict if any disturbances will grow and their initial nature. If the waves grow and then saturate at small amplitude without changing wavelength, then only the saturation process involves nonlinear effects. However, often, the initial waves form subharmonics[1,2], overtones [1] or even interact with wavelengths much longer than the fundamental [1].

Because of this (for general flows), the next level of theory, nonlinear analysis, cannot be done without looking at what happens in the experimental system. For example, nonlinear analysis is often based on the long wave assumptions [3,4]. For falling films the waves often evolve to long wavelengths, (even if the initial instability is short) so that this approach works. However, for two-layer systems where gravity is stabilizing, the positive linear growth rate region is often bounded away from 0 wavenumber [5,6]. Further, waves may remain of moderate wavenumber even though the linear growth rate is positive down to 0 wavenumber[7]. Thus long wave theory is not of general applicability for two-layer systems.

Blennerhassett [8] Renardy and Renardy [9] have formulated the interfacial problem as a weakly nonlinear expansion and derived equations for the amplitude of the one or two dominant modes. Sangalli et al. [5] showed that for a two-layer Couette flow that experiments agree quite well with theory. Thus the eigenfunction expansion and center manifold projection approach works for this system. However, a single Stuart Landau equation [8,9] will be valid only for situations when a single short-wave mode is dominant. Formation of subharmonic or other modes cannot be predicted and this single equation does not tell when a different one is required. (At least when the bifurcation is supercritical which it is for all interfacial systems that we have done calculations for.) Thus at the present time the weakly nonlinear problem is not well understood.

A serious need exists for a well defined system to study to sort out as many of this issues as possible. The gas-liquid channel flow does not conveniently exhibit a useful range of behavior for two-layer laminar flows. We have found that an oil-water system has several features that make it appropriate for study. First is a significant range of laminar flow. Second is that there are two distinct modes that lead to interfacial waves and which travel at different speed. Third, these two modes can be simultaneously neutrally stable. Further there is another condition where the long wave region and the short wave region of the same mode can be simultaneously neutrally stable. Thus we can study conditions where two distinct wave modes can be generated and then interact and other conditions where the same mode, but at different wavelengths, interact. Therefor we have an experimental system that allows a broad range of different nonlinear interactions.

In this paper, preliminary results from this study are given. We have found the linear stability theory predicts the onset well. Further, both waves lead to observable interfacial disturbances. (which is not the case for gas-liquid flows). Nonlinear theory suggests that even though the bifurcation seems to always be supercritical, the mechanism responsible for stabilization is different in different ranges.

EXPERIMENTAL SYSTEM

Figure 1 shows a schematic of the oil-water channel that is used for the experiments. Data are obtained from visual and video observations and from conductance probes. We are currently working on an optical technique for measuring the interface tracings to replace the conductance probes. The fluids are water, with Sodium Silicate added to improve its ability to wet the Plexiglas® channel and a light hydrocarbon oil with a density of 0.88 g/cm^3 and a viscosity of 17.8 cP. More details about the flow system and its construction are included in a thesis by McKee[10].

THEORY

Theoretical analysis for this system includes (temporal) linear stability analysis for a two-layer laminar flow that has been completely formulated in papers by Yih[11]or Blennerhassett [8]. The problem is solved numerically with Chebyshev-Tau spectral technique [12] using a scheme devised by Gardner et al.[13]. It is difficult to get accurate results in the long wave region so the results are compared to the analytical long wave solution of Blennerhassett [8]. Strictly speaking, the channel flow geometry is a convective situation and should require a spatial stability formulation. However, this is much more complicated to program and lengthy to compute and we have found that satisfactory results can be obtained from the temporal stability problem with a Gaster transformation if necessary.

Nonlinear analysis for the two-layer problem has been formulated with a multiple scales technique by Blennerhassett [8] and an eigenfunction, center manifold approach by Renardy and Renardy [9]. We use the basic approach of Renardy and Renardy [9] except that the individual contributions to the coefficient β, in the Stuart-Landau equation

$$\frac{\partial A}{\partial t} = L(\lambda)\,A + \beta\,|A|^2 A \qquad [1]$$

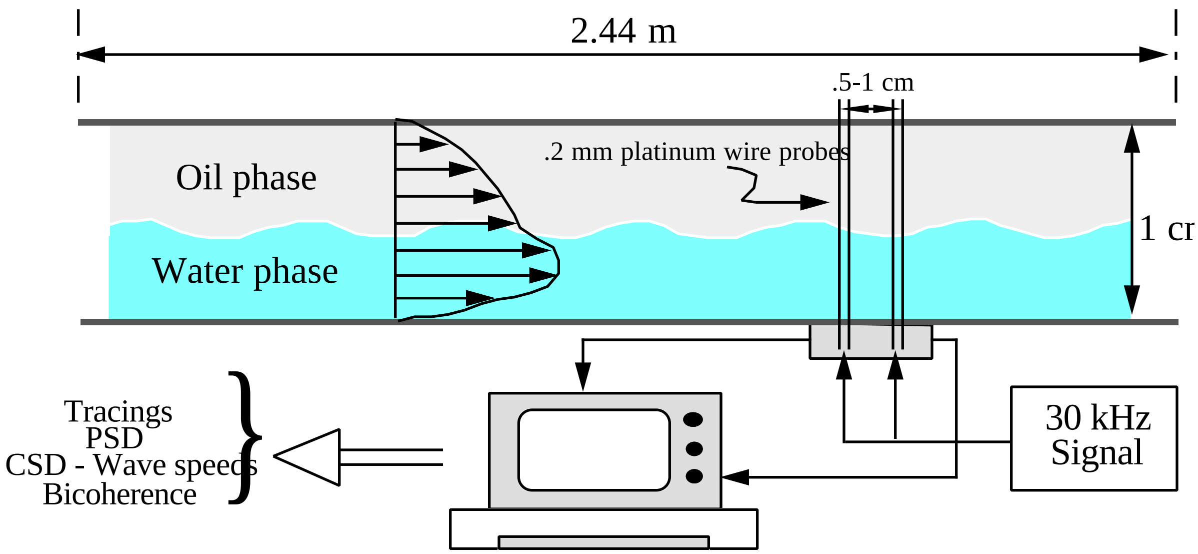

FIGURE 1. OIL-WATER CHANNEL FOR STUDYING INTERFACIAL WAVES

are separated into a quadratic contribution, a cubic self interaction and a cubic interaction with the mean flow (see [6]) for details. This equation tells the nature (i.e., super or subcritical) of the initial bifurcation from a steady base state if the neutral wave number occurs away from 0 wavenumber. If the flow is unstable to long waves, then the neutral wavenumber is 0 and this analysis is not valid. A further difficulty is that even when the initial bifurcation is supercritical, at sufficient forcing, the region of unstable modes and the growth rates get large enough for other modes to occur -- even though the amplitude/wavelength ratio of any mode remains small.

RESULTS

The linear stability diagram should predict the initial transition behavior. Figure 2 shows a plot in Re_{oil} versus Re_{water} of the long wave and short wave stability boundaries. At two points, the different wave types are simultaneously neutrally stable. Thus there are regions where two distinct modes are simultaneously linearly unstable and (by change of the inclination angle of the flow) can have a variable growth rate ratio. It is important to examine the nature of these modes. Figure 3 shows the growth and speed plots for conditions close to the crossing point at low Re_o. The three most unstable modes are plotted. Because an individual mode would be expected to vary smoothly with wavenumber (particularly on the speed plot) it is clear that the same mode is not most unstable always. The mode that is unstable at long wavenumber is distinct from the mode that is unstable at intermediate wavenumbers.

The disturbance eigen function is plotted in figure 4 as a contour plot (for one complete wave) to show what the different modes look like. It is seen that the low k disturbance has its water part centered in the water (lower) phase. However, there is a second unconnected (weaker) part that starts in the oil but is centered at the interface. The high k disturbance has the oil part of the disturbance centered at the interface connected with a skewed part in the water phase. As the water phase travels much faster than the oil, this could be the reason that the high k disturbance has a

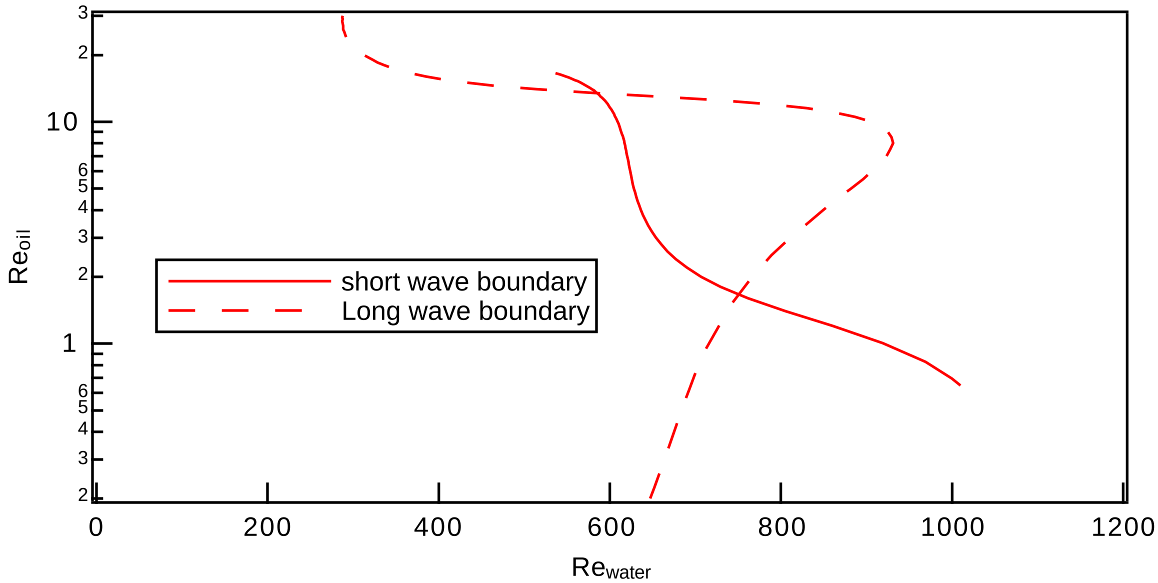

Figure 2. Plot of neutral stability boundaries for long and short waves.

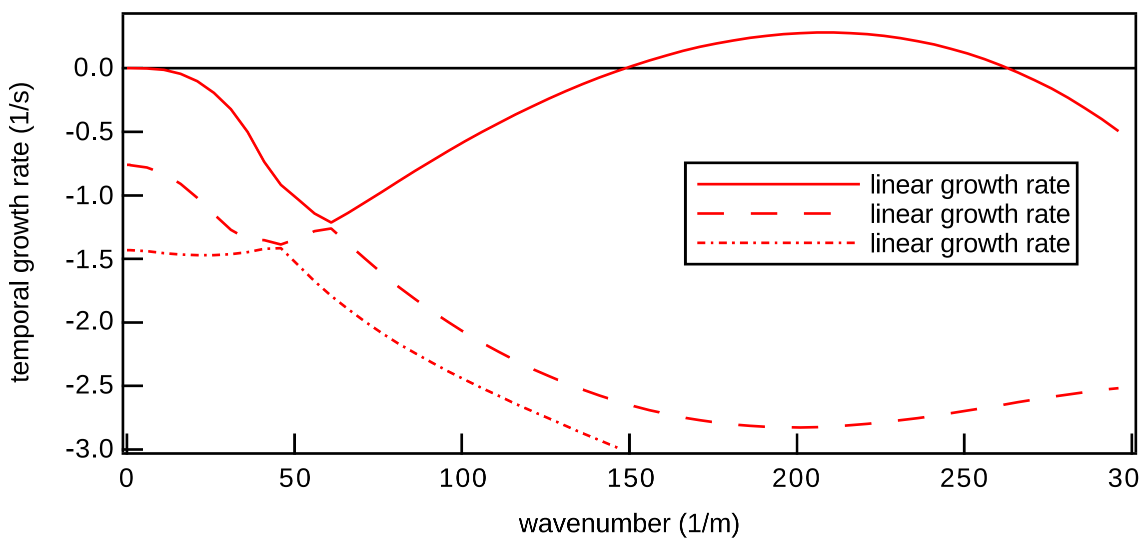

Figure 3a. Plot of growth curves for first three modes, Re_o = 1.8, Re_w = 770

much higher phase velocity. It should be noted that neither of these disturbances has the "pure" form of interfacial and internal modes as described by Yiantsios and Higgins[14]. Experiments show that both of them lead to interfacial waves. This is in contrast to situations for say, gas-liquid flows where a gas phase internal mode can be unstable, and perhaps be associated with turbulence, but not cause interface waves.

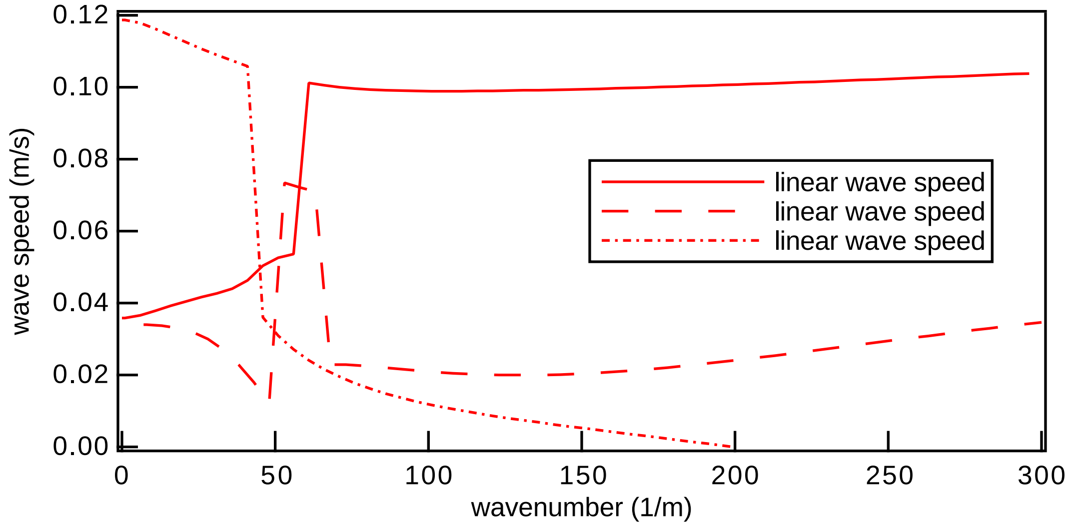

Figure 3b. Plot of wave speeds for first three modes, Re_o = 1.8, Re_w = 770

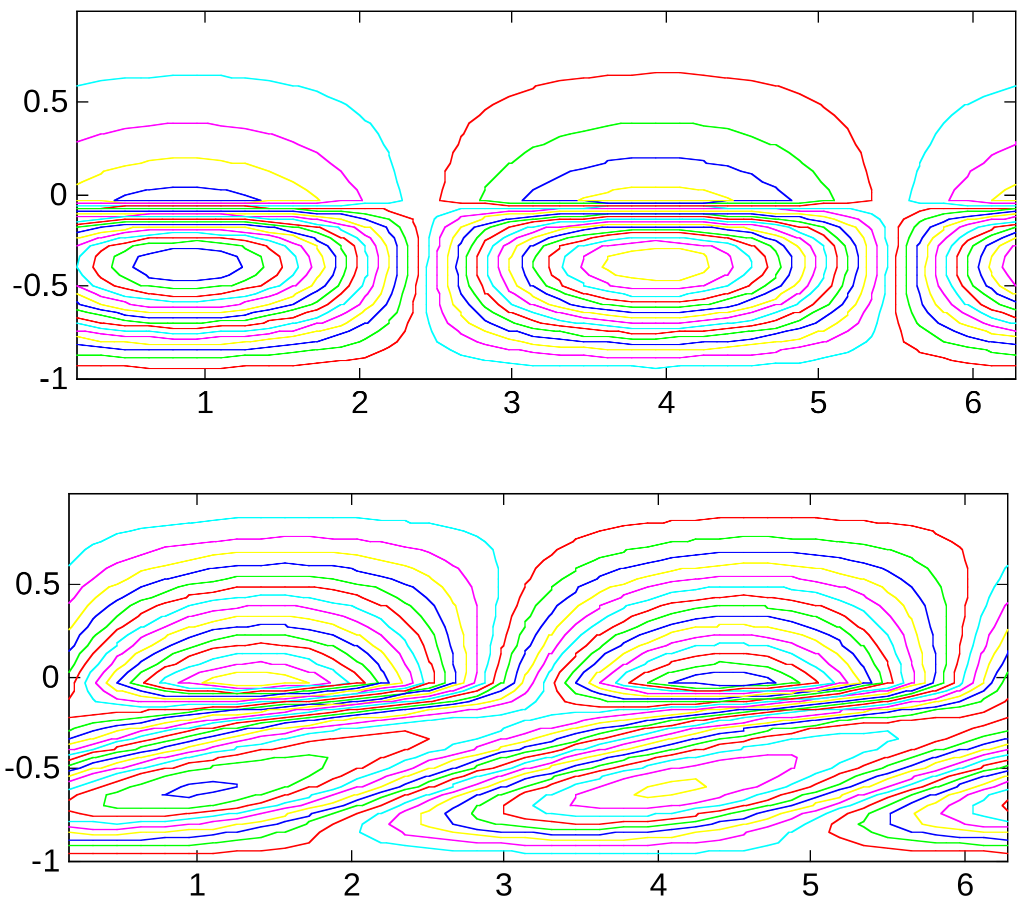

Figure 4. Disturbance eigen function at Re_o = 1.8, Re_w = 770. a. top: k=1/m
b. bottom: k=211/m.

The situation is different at the long-short crossing point at higher Re_o. Plots of the disturbance function would show that both the long and short waves are the same qualitative shape, (similar to figure 3b) and that the speeds of the long and short waves are comparable.

The importance of developing an better understanding of the nonlinear processes that affect wave behavior is shown by reference to figure 5. In figure 5a, the linear growth curve has a peak close to 3 Hz and the wave data have a corresponding spectral peak. Thus the wavelength and speed of steady waves are close to the linear stability values. However, if the water Reynolds number is increased the situation is very different. Now the spectrum of figure 5b has no relation to the peak in the linear growth region but occurs at much smaller frequency. While the low frequency region is linearly unstable, there is no indication that the low mode will dominate completely with almost no evidence of short waves. Further, video images of this condition show no evidence that the short wave instability is reforming even though the growth rate is large enough that short waves would be expected to reform between the large waves.

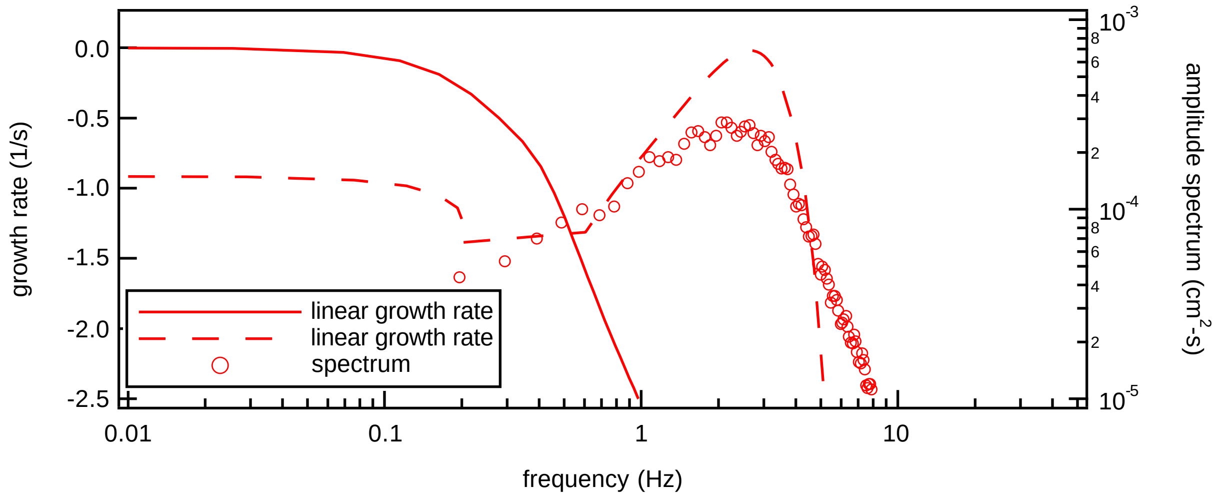

Figure 5a. Growth rate of two modes and measured spectrum at Re_o=3, Re_w = 700

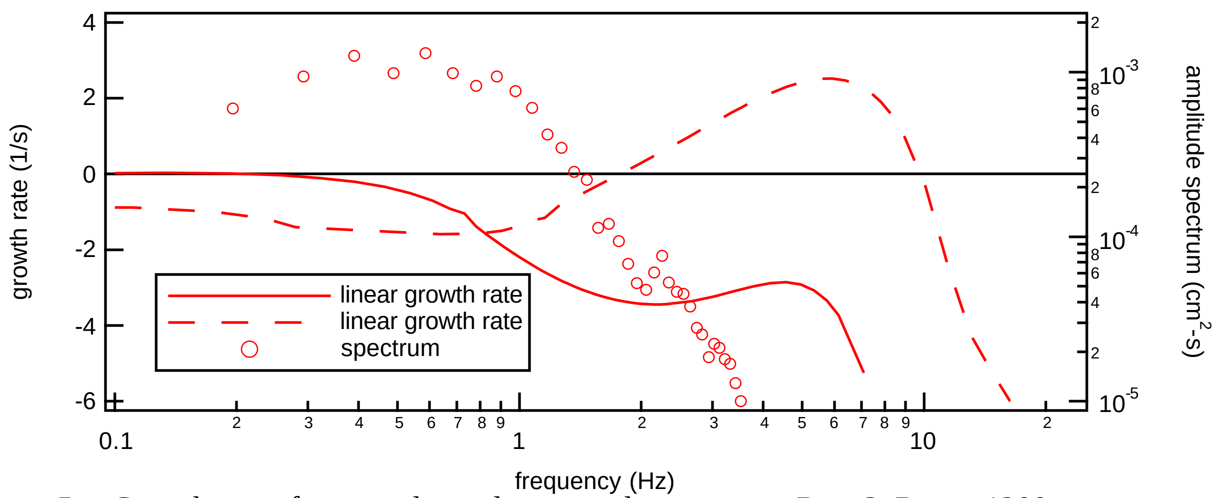

Figure 5a. Growth rate of two modes and measured spectrum at Re_o=3, Re_w = 1200

DISCUSSION

While this project is just beginning, there are several issues that have emerged which were not expected based on numerous previous studies of gas-liquid flows.

First the relative interaction rates of different wavelength modes with different speeds should be successfully determined by experiments in the vicinity of the two-mode crossing points. Preliminary indications are that close to the low mode crossing point there is no interaction between the modes. We are not yet sure about the region around the other crossing point. In either case, the two-mode weakly nonlinear theory is expected to be adequate to describe the process close to the neutral point.

An interesting issue emerges when the oil-water channel is compared to two other two-layer systems that we have studied. Sangalli et al.[6]studied a gas-liquid cocurrent flow close to neutral stability and found that weakly-nonlinear theory predicted the qualitative behavior of the system quite well. The results of most interest here are that the bifurcation of the short wave mode was always supercritical (for all cases calculated) and that the origin of the nonlinear stabilization was the cubic terms that originate in the boundary conditions except when the speeds of the fundamental and overtone were close to the same. In this case the system was nearly-resonant and efficient transfer of energy from the unstable fundamental to the stable overtone stabilized the system at amplitudes smaller than expected for cubic stabilization. A third mechanism [6] cubic interaction between the fundamental and the base flow was found to be unimportant. In contrast, for a two-layer Couette flow [7], we have calculated results for quite a few cases and found that the bifurcation is again supercritical and the overtone interaction is dominant even if the fundamental and overtone are not close to resonant. (In most cases the cubic interaction is destabilizing and the base state interaction is not significant.) Thus it is quite interesting that the oil-water channel flow should again exhibit a supercritical bifurcation for all tested conditions but, the dominant interaction is usually the cubic interaction between the fundamental and the mean flow. For the three different experiments available to us, there are three different mechanisms of stabilization. However, in all cases, cubic order stabilization is observed and the bifurcation is supercritical.

In light of these calculations a check of the calculation procedure as applied to two-layer flows is warranted (note that we had already reproduced the single phase subcritical behavior). For a gas - liquid flow it is possible to get a gas-phase internal mode to be neutrally stable before any interfacial modes occur by raising the gas Reynolds number above 6000-7000, while keeping the liquid one low. For $Re_G = 8000$, $Re_L = 2$, $\mu_L = 5$ cP, it was found that the gas mode appeared and was, as expected, subcritical. Thus the observed supercritical nature for two-layer flows is for the interfacial mode when the instability is bounded away from 0 wavenumber.

Of course it is not known if the bifurcation is supercritical of *all* conditions. It would be interesting to determine if this is the case. A proof seems impossible given the complexity of the system although perhaps worth looking for some restricted parameter ranges. Certainly we will continue to look for subcritical regions. If can find any, experiments will be done in this range to see what differences in behavior exist. Because this issue is so fascinating and we have not been able to find a subcritical region for the fundamental, calculations have been done to find how wavenumbers other than the peak would bifurcate if they were present. Interestingly, for the gas-liquid systems, most modes at wavenumbers less than the fundamental are subcritical. A way to simulate the behavior of these modes is to excite waves artificially with a paddle. For one set of flow conditions, paddling was done over the range of 1-10 Hz. In all cases no amplification of the paddled modes were observed and sufficiently far downstream the spectrum was identical to cases where no paddling was done. This result can not be considered conclusive, however the experiments do not seem consistent with the calculations and further work is needed.

One last issue that will require more study and probably a new formulation is how to deal with conditions far above criticality. For the conditions of figure 5b, the initial bifurcation was supercritical and the wave amplitude are never real large. However, once the growth curve has a sufficiently wide region of instability, it is clear that one or two amplitude equations will not be sufficient. Perhaps the weakly-nonlinear formulation of the Navier-Stokes equations can be solved by integration in time and space to produce results that predict the experiments.

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REFERENCES

1. L. A. Jurman, S. E. Deutsch, S. E. and M. J. McCready, "Interfacial mode interactions in horizontal gas-liquid flows," *J. Fluid Mech.* **238**, 187-219 (1992).
2. Z. Fan, F. Lusseyran, F. and T. J. Hanratty, "Initiation of slugs in horizontal gas-liquid flows, “*AIChE J.* **39** 1741-53 (1993).
3. B. S. Tilley, S. H. Davis and S. G. Bankoff, “Long-wave nonlinear stability of superposed fluids in an inclined channel,” Phys. Fluids **6**, 3906 (1994).
4. F. Charru and J. Fabre, *Phys. Fluids* , **6** , 1223 (1994).
5. M. Sangalli, C. T. Gallagher, D. T. Leighton, H. -C. Chang and M.J. McCready,"Finite amplitude wave evolution at the interface between two viscous fluids", *Phys. Rev. Let.* **75**, pp 77-80, (1995).
6. M. Sangalli, M. J. McCready and H. -C. Chang, "Stabilization mechanisms of short waves in gas-liquid flow", *Phys. Fluids*, **9,** pp 919-939, (1997).
7. C. T. Gallagher, M. J. McCready and D. T. Leighton, “Experimental investigation of a two-layer shearing instability in a cylindrical Couette cell", *Phys. Fluids*, **8**, pp 2385-2392, (1996).
8. P. J. Blennerhassett, “On the Generation of waves by wind,” *Proc. R. Soc. Lond*. A **298**, 451-494 (1980).
9. M. Renardy and Y. Renardy, “Derivation of amplitude equations and analysis of sideband instabilities in two-layer flows”, *Phys. Fluids*, **A 5**, 2738-2762 (1993).
10. William McKee, "An experimental study of interfacial waves in cocurrent oil-water flows" -- M.S. Thesis, University of Notre Dame, (1995).
11. C. S. Yih, “Instability due to viscosity stratification”, *J. Fluid Mech.* **27**, 337-352 (1967).
12. Y. Y. Su and B. Khomami, B., “Numerical Solution of Eigenvalue Problems Using Spectral Techniques”, *Journal of Computational Physics* **100**, 297-305, (1992) .
13. D. R. Gardner, S.A. Trogdon, and R.W. Douglass, “A modified tau spectral method that eliminates spurious eigenvalues”, *J. Comp. Phys.* **80**, 137 (1989).
14. S. G. Yiantsios and B. G. Higgins, “Linear stability of plane Poiseuille flow of two superposed fluids”, *Phys. Fluids* **31**, 3225-3238 (1988).