Chapter 1
Transition to Three-dimensional Waves in Cocurrent Gas-liquid Flows *

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Abstract

The transition from two-dimensional to three-dimensional waves on the interface of a gas-liquid flow in a horizontal channel is studied experimentally. It is found that there are two different mechanisms for this transition. For sufficiently thin films or sufficiently low liquid Reynolds number, the transition occurs by localized defects becoming large enough to disrupt the wave field. For deeper layers, the transition occurs by formation of oblique modes that can form a regular herringbone pattern which becomes irregular with increasing shear. Linear stability analysis, through the use of a Squire transformation, indicates that the observed transverse modes are unstable. However neither linear analysis nor a set of weakly-viscous, weakly-nonlinear mode interaction equations can predict the preferred wavelength or explain why there are different mechanisms for thin and thick films.

1 Introduction

Owing to its application to production and transport of hydrocarbon mixtures, multiphase contactors and reactors and evolving designs for nuclear energy devices, there is a continuing interest in conduit flows of gas-liquid and liquid-liquid mixtures. For a general case, these flows will have at least 6 dimensionless parameters and have been shown [19] and [10] to exhibit 5 to 7 different geometrical (and dynamical) configurations which are called flow regimes. The simplest method for predicting flow regimes is by use of a flow map such as the popular one by Mandhane et.al [19]. However, this is a dimensional plot and as expected, if the some variable is changed, the boundaries move [1] and [17]. While it seems to be possible to get a slightly more general plot using Froude number coordinates [8] and [14], it is clear that the prediction of flow regimes, which is the most important issue in design of multiphase devices, cannot be done with simple flow maps.

Because of this there have been considerable theoretical efforts at describing flow regimes and the first order process variables, pressure drop and holdup (i.e., volumetric fraction of liquid in the pipe). Two rather distinct approaches which originate in the "natural" limits of the flow regime maps have emerged. At sufficiently sedate conditions, if gravity is present (because the gas and liquid have different densities), the flow will be stratified. Starting at this limit and increasing the gas flow there is a transition to interfacial waves. At still higher gas velocities the flow may become annular where there is a liquid layer on the wall and a core of gas and entrained droplets. For these regimes the phases are generally distinct

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so that a convenient limit on which to base theory is a separated flow. Large perturbations of this are roll waves and ultimately slugs that can form from interfacial waves [6] and [9]. At severe flow conditions there will also be considerable atomization that appears to originate with large waves. Conversely, the other natural limit is a "homogeneous" flow in the dispersed regime which occurs at either high liquid with low gas, or high gas and relatively high liquid. The homogeneous flow can vary from liquid continuous with bubbles to gas continuous with liquid drops (although there will be a liquid wall film if the liquid wets the wall). Perturbations of this limit are bubble coalescence into the bubbles between slugs and possibly liquid coalescencing into waves. The dominant approach to describing dispersed flow is the so-called "two-fluid model" where averaged equations are written for the two phases and a number of regime dependent closure relations used to describe the flow. Ishii (1990) gives a complete review of this work which continues at a rapid pace.

The engineering work on separated flows is based on "one-dimensional" (i.e. averaged normal to the interface) equations. This includes linear stability of the complete equations, searches for regions of real characteristics for the inviscid version of these, and analytical and numerical studies of the nonlinear versions of these equations. Barnea and Taitel [3] give an up to date account of this work which endeavors primarily to predict the transitions from stratified to wavy, slug and annular flow. However, it should be noted that the one-dimensional equations do not match the solutions from the differential equations in the long wave limit [15] which would seem to be the only possible place that they could match. Thus the use of the instability of one-dimensional equations to predict flow regimes causes some fundamental concern about what is really being predicted.

The mechanism that is assumed to cause flow regime transitions is the formation of long wavelength waves that grow to sufficient height to disrupt stratified flow. Fan et al. [9] show that this may be somewhat of a simplification but Kuru et al. [14] note that typical data for slugs occur under conditions of unstable long waves. Thus correct prediction of the onset of long waves should be an important question in two-phase flows. Further, it is well known [2] that short and moderate wavelength waves on the interface cause dramatically increased drag which raises the pressure drop and ultimately causes a change in the relative depths of the two phases (i.e., holdup). This is important to the issue of long wave stability because [14] show that the long wave boundary is very sensitive to the base state of the flow.

This paper examines the short and moderate wave field on the surface of a stratified flow. The interface of a gas-liquid channel flow has been observed across the initial transition to conditions severe enough to cause strong three-dimensionality in the waves. This work extends our previous study, [21] that included only the region where the films are thin. Video imaging reveals that for sufficiently low liquid thicknesses, the onset of transverse variation seems to be through localized defects which become more frequent as the forcing (gas flow rate) is increased. For thicker layers, the transition occurs globally though formation of oblique modes that have a well-defined angle to the flow direction and a flow direction wavenumber close to the value of the fundamental. The mechanism for the selection of the value of the transverse wavelength cannot be obtained from a Squire's transformation and a simple (weakly-viscous) nonlinear model also provides no predictions of transverse wavelength. This suggests that the wavelength is determined as a boundary effect. Furthermore, none of our linear or nonlinear efforts have been able to explain why defects dominate at small liquid flows and transverse modes at higher liquid flows.

2 Description of experiments and results

Experiments were conducted in a horizontal rectangular channel that is 2.54 cm high, 30.5 cm wide and 6.5 m long. More details of the channel and experimental techniques are given in theses by [5] and [13] The gas was air and the liquid was a glycerin-water solution that had viscosity of either 4 or 10 cP. Wire conductance probes were used to get a continuous time series for the measurements and an Ekta-Pro video imaging system was used to acquire wave images. A white incandescent light, inclined about 30° above the channel, parallel to the flow direction, was found to give the best uniformity. Waves are visualized because the liquid and gas have different refractive indices. Further details of the imaging experiment and procedure are given in a thesis [15].

Figure 1 shows a wave regime map for the experiments taken at 3 m from the channel inlet. The liquid viscosity is about 10 cP. The neutral stability curve is calculated for a two-layer laminar flow using the complete differential equations and boundary conditions [4] with a tau-spectral technique [23]. This curve is likely to be somewhat inaccurate because the gas flow is turbulent and the calculations are for a laminar flow. A more important error is that close to neutral stability for a horizontal flow, the liquid depth may not be constant with flow length thus there will be a slight hydraulic gradient along the flow. Because the linear growth curves increase quickly on the scale of this diagram, the true location of the neutral curve is likely to be just below the "—" symbols that denote no observed waves. For Re_L, the liquid Reynolds number (defined as volumetric flow per unit width/kinematic viscosity), less than about 38, the first observed waves have nearly-perfect parallel crests, but there are always some imperfections present.

Figure 2a gives an example of this region where there is some distortion of the parallel crests in the top right corner of the picture. Video taken in this region shows that there is also weak modulation of the wave field such that waves appear to increase and decrease in amplitude. Modulation related to either side bands or low frequency mode interactions over a range of conditions similar to the present experiment was also found by [12]. As Re_G is increased, localized defects in the wave field become more pronounced. Figure 2b, c, and d show examples of the different types of defects. In general, each type of defect can occur at all of the conditions; the main effect of increased Re_G is to increase the density or rate of occurrence of defects. It is interesting that the overall modulation decreases as Re_G increases so that defects are the most important feature of the wave field. Another important point is that defects move (usually) faster than wave crests so that the wave field is always evolving. This is consistent with the idea that defects will move with the group velocity. Typical group velocity and phase velocity plots are shown in figure 3. It is seen that they are comparable, but not identical, over much of the range of the waves. Our observations are consistent with the hypothesis that defects originate in the entrance section as noise that causes imperfections in the waves. At sufficient shear, they grow large enough to disrupt the uniform waves. It is likely that there are imperfections in all wave fields, even at very long evolution times and distances, that could become defects with more interfacial shear.

Strikingly different behavior occurs for Re_L greater than about 40. At a Re_G value just large enough to make waves visible, the waves have a parallel crests but there is a regular pattern superimposed. If Re_G is increased slightly, this pattern is a herringbone [18] an example of which is shown in figure 4a. When viewed on video, waves at this condition are not quite "frozen" but the field is much more steady than for thinner layers. The basic pattern is present for the remaining 3.5 m of the flow system. Measurements of the wave field with wire probes, that give more resolution than video but provide only a local measurement at single point, show that a series of overtones are present in the wave field. However, there is no evidence of a subharmonic as seen by Liu et al.[18] for a falling film. The 2-D (i.e., two-dimensional) FFT of the image of the wave field shown in figure 4b, indicates that the wave field is comprised of a fundamental and an oblique mode that has nearly the same flow direction wavenumber and a transverse wave number of about 1/2 this value. There are actually two oblique modes at about 10° from the flow direction, as seen in the wave field reconstruction, shown in figure 4c. The FFT does not show this because the windowing function that we used, cut the edges too quickly and the wave in the center of the picture is asymmetric. As Re_G is increased, the herringbone pattern becomes irregular and then breaks down entirely. Figure 5 shows images for this range. Also note that as Re_G is increased, the wave field exhibits an increasing spatial evolution rate.

It is not possible to explore all of the parameters that govern interfacial waves but similar experiments were done at 4 cP. Again there was a thin film region now at $Re_L < 100$ where localized defects occurred; these also increase in rate of occurrence as Re_G increases. In the thick film region there was a range where almost perfect 2-D waves appeared. As Re_G was increased, a rather abrupt change to a fully 3-D wave field (similar to figure 5b) occurred. However, the basic classification that the transition occurs through localized defects at low Re_L and by a global mechanism for sufficiently large Re_L is believed to be the same for 4 cP as for the 10 cP experiments.

3 Discussion

3.1 Linear Theory

The primary questions regarding formation of transverse disturbances raised by the data are: what selects the wavelength of the oblique modes and why are there two distinct regions of behavior? To address the first question, it can been seen from figure 1 that herringbone waves and defects occur some distance above neutral stability so it is not surprising for either conditions that transverse variation occurs which could be caused by oblique modes. Blennerhassett [4] has shown that Squire's transformation can be applied to a two-layer flow. The stability boundary from this analysis is shown on figure 4b. The transverse mode for the conditions of figure 4a is seen to be linearly unstable. While this is expected and it may even be gratifying, it does not tell how the transverse wavelength is chosen. The maximum growth rate is for a parallel disturbance and growth decreases monotonically as the angle is increased from this value. The measured transverse wavenumber has a lower growth than any modes with lower values of the transverse wavenumber. Consequently, a linear analysis of this problem does not tell how the transverse mode is selected (other than it is unstable).

The Squire transformation for the thin film region likewise provides little useful information. There is again a range of unstable transverse modes but this does not seem important because localized defects are not a manifestation of global transverse modes and thus they have no well-defined transverse wavelength.

The only other issue that linear theory could be expected to resolve would be to explain the reason for the different behavior for thin versus thick regions. Figure 6 shows families of phase speed and growth curves close to the transitions typical of the high Re_L and low Re_L regions. The interface velocity is subtracted from the speed curves to provide some normalization. There is less speed variation with wavenumber for the thinner films than the thicker films. Also, there can be a long wave with the same speed as the fundamental for $Re_L = 20$ in the range of 10/m (wavelength, $\lambda = 62$ cm). For $Re_L = 60$, there is more dispersion and the only chance of a resonant wavenumber occurs at wavenumber $\sim$ 1/m ($\lambda = 6.2$m). The growth curves are also similar for the regions with the high Re_L having a larger unstable region and much less of a well at low wavenumber. The long wave stability boundary shows no apparent relation to the observed behavior. While there are some quantitative differences for the two regions, there does not seem to be any qualitative difference that could explain the different types of transverse variation. Thus linear stability apparently tells nothing that can distinguish the two regimes.

3.2 Nonlinear Theory

Given the steadiness and extent of the herringbone pattern, it could be conjectured that the fundamental and oblique modes are interacting rather strongly and are perhaps nearly resonant. To accurately predict behavior one would have to use a rigorous reduction of the full Navier-Stokes equations in three dimensions [20] and simulate the interface. This simulation is beyond the present study. Here we describe the results, which were not conclusive, of a very simple weakly-viscous model. In this model the linear evolution of wave amplitude is obtained from the complete linear problem and the nonlinear interactions are obtained from an inviscid formulation. The interaction coefficients were obtained from the boundary conditions and potential flow using the procedure described by [7]. Terms up to only quadratic order were obtained because the cubic inviscid terms are "conservative" and we know from our analysis of the complete weakly-nonlinear problem [22] that cubic terms are non-conservative and quite important. Thus this model could be valid only when cubic terms are not important. The form of the model is

$$\frac{d}{dt}A_{1,0} = \Lambda_{1,0}A_{1,0} + P_1 A_{2,0}A^*_{1,0},$$

$$\frac{d}{dt}A_{2,0} = \Lambda_{2,0}A_{2,0} + P_2 A_{1,-l}A_{1,+l} + P_3 A^2_{1,0},$$

$$\frac{d}{dt}A_{1,+l} = \Lambda_{1,+l}A_{1,+l} + P_4 A_{2,0}A^*_{1,-l} + P_5 A_{2,+2l}A^*_{1,+l},$$

$$\frac{d}{dt}A_{1,-l} = \Lambda_{1,-l}A_{1,-l} + P_6 A_{2,0}A^*_{1,+l} + P_7 A_{2,-2l}A^*_{1,-l},$$

$$\frac{d}{dt}A_{2,+2l} = \Lambda_{2,+2l}A_{2,+2l} + P_8 A^2_{1,+l},$$

$$\frac{d}{dt}A_{2,-2l} = \Lambda_{2,-2l}A_{2,-2l} + P_9 A^2_{1,-l}, \tag{1}$$

In these equations $A_{i,l}$ is the complex wave amplitude, the first subscript is the flow direction wavenumber, the second subscript is the transverse wavenumber, $\Lambda_{i,l}$ is the linear growth and speed of mode i (obtained using the full linear problem and Squire's transformation if necessary), P_l are the interaction coefficients obtained from inviscid theory. Note that for an inviscid model, the interaction coefficients are purely imaginary. The modes were chosen because a parallel fundamental and $\pm l$ oblique modes were observed to make the herringbone pattern. An overtone for each of these modes (which is assumed linearly stable) was added to stabilize the system. There was no evidence of subharmonics or low frequency mode interactions. It is certain that an argument could be made for including more modes and for the need to have cubic terms. Further, this inviscid model may be reasonably valid for $Re_L \sim 100$ but it will definitely breakdown as Re_L approaches $O(10)$, and the results will become even less reliable. Thus equations [1] cannot provide any reliable predictions of the difference between the thick film and thin film regions.

Equations (1) were explored by integrating forward in time. Kuru [15] gives several cases and many more were tried. These calculations show that the long time amplitude of the $A_{1,l}$ mode is determined primarily by its linear growth rate. Thus for a smaller value of l, the growth is larger and the eventual amplitude is larger. The interaction coefficients are large enough to cause oscillatory energy transfer between the modes; the amplitudes usually oscillated from the initial conditions before settling down to their final amplitudes. The results of the integration of (1) are that the preferred transverse mode is the one with the largest growth rate, which is the one with the lowest wavenumber. For conditions in the range of the experiments, no exact resonances were found between the fundamental and the oblique modes.

It is clear that more theoretical work needs to be done for transverse variation in two-layer interfacial systems. Further analysis of the exact equations [20] seems to be a good starting point because very good agreement has been obtained in a two-dimensional system using the same formalism [22]. However, it is our strong suspicion that the presence of side walls is playing an important role in choosing the wavelengths that appear and probably in promoting transverse disturbances. In a two-layer Couette experiment [22] one bounding edge is air and the other is Mercury so that there is effectively little stress at the edges of the flow. In this experiment, two-dimensional waves persist far above neutral stability such that the parameter $(U_p - U_{pcritical})/U_{pcritical}$, where U_p is the plate speed of the Couette flow and $U_{pcritical}$ is the value of the onset of waves, is greater than unity. For the channel flow experiments, an equivalent parameter is $(Re_G$ - $Re_{Gcritical})/Re_{Gcritical}$. For conditions where this parameter is about 0.3, there is often significant transverse variation. Thus the presence of side walls may have a profound influence on both qualitative and quantitative wave behavior.

4 Conclusions

Video and visual imaging of interfacial waves in a gas-liquid flow at conditions slightly above to neutral show that transverse variations can arise as either localized defects or a global appearance of oblique modes. The defects are favored at sufficiently low liquid Reynolds numbers (or thin films). At higher Reynolds numbers, a distinct herringbone pattern comprised of a mode traveling parallel to the flow direction and two oblique modes that have the same flow direction wavenumber as the fundamental and are oriented at about 100 from the flow direction, are observed. A Squire transformation shows that the oblique modes should be linearly unstable. Neither linear stability theory nor a crude nonlinear model for the waves explains why different behavior exists for different Re_L or predicts the wavelength of the oblique modes when they appear. It is our belief that mode selection is determined to a large extent by side wall interaction with the waves and the flow field. Further, it is likely that the herringbone patterns are formed by some sort of resonance. For thinner films, such resonances do not exist to aid in the generation of transverse modes. For these conditions the first transverse irregularities that appear are defects that originate with inlet noise that has been amplified enough to disrupt regular waves.

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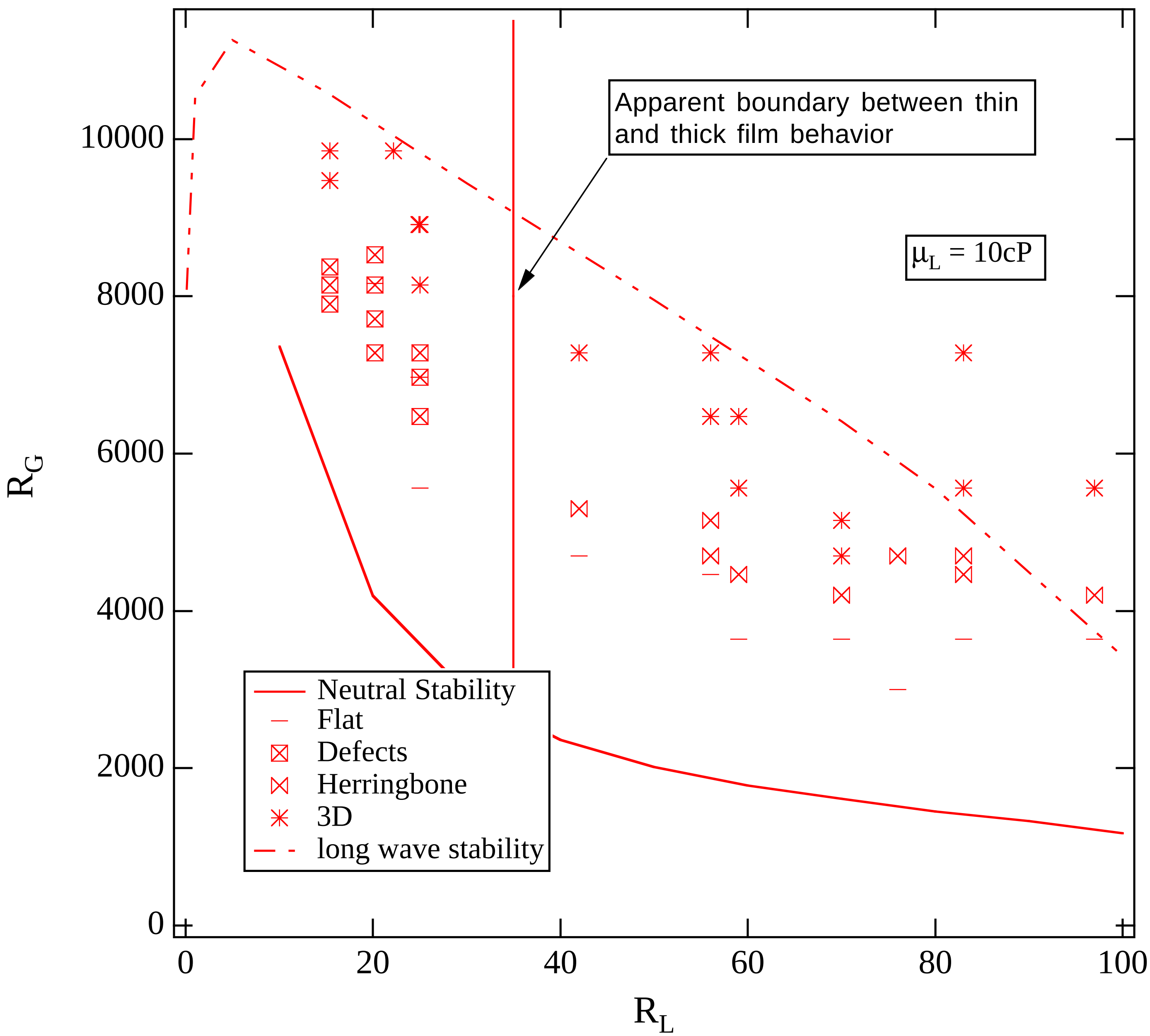
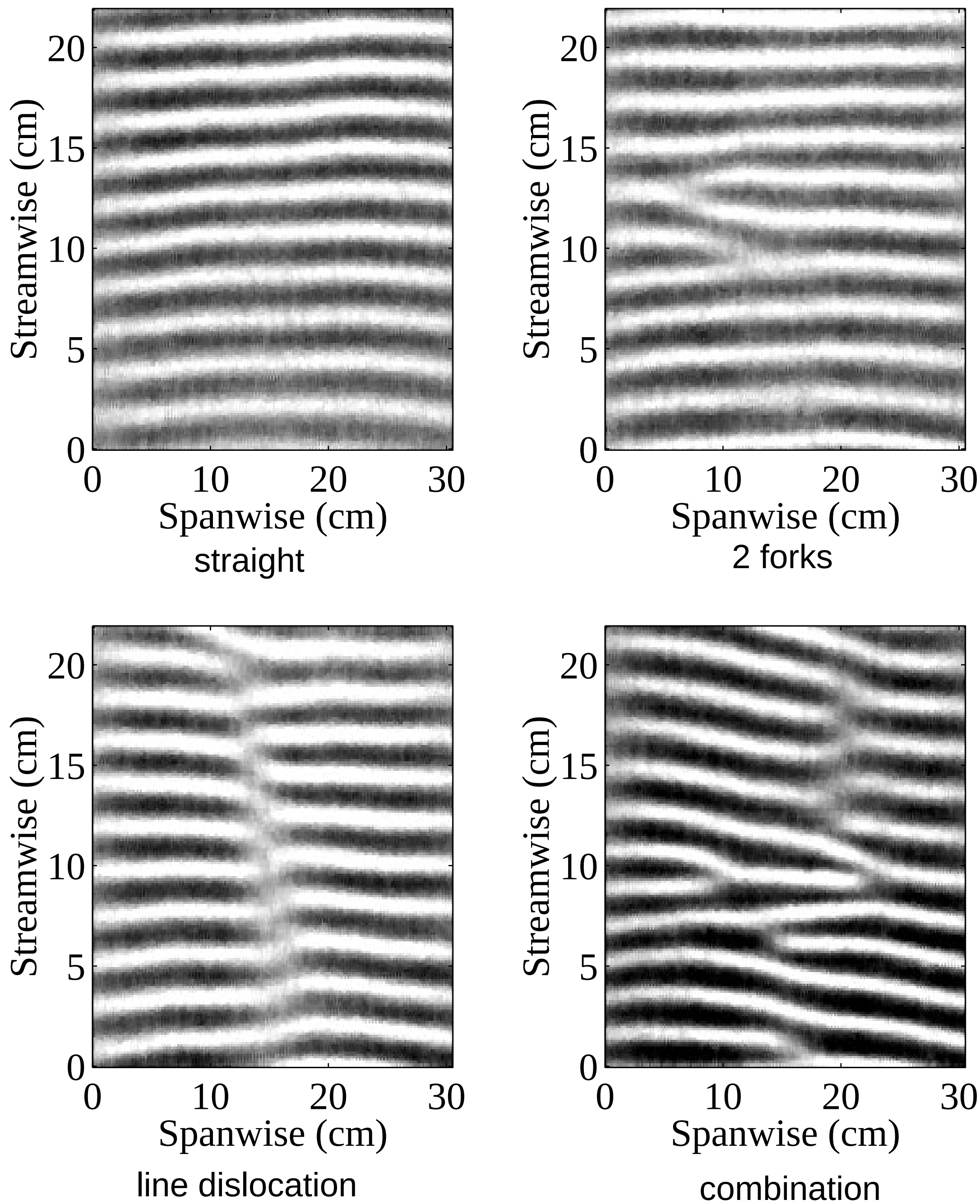


Figure 1. Wave map for gas-liquid channel flow. The different wave types are shown along with the interfacial mode neutral stability curve and the conditions at which long waves become unstable.

Different types of defects observed in wave field



μ_L = 10.5 cP, R_L = 15.4
R_G = 7900, 8140, 8370, 8520

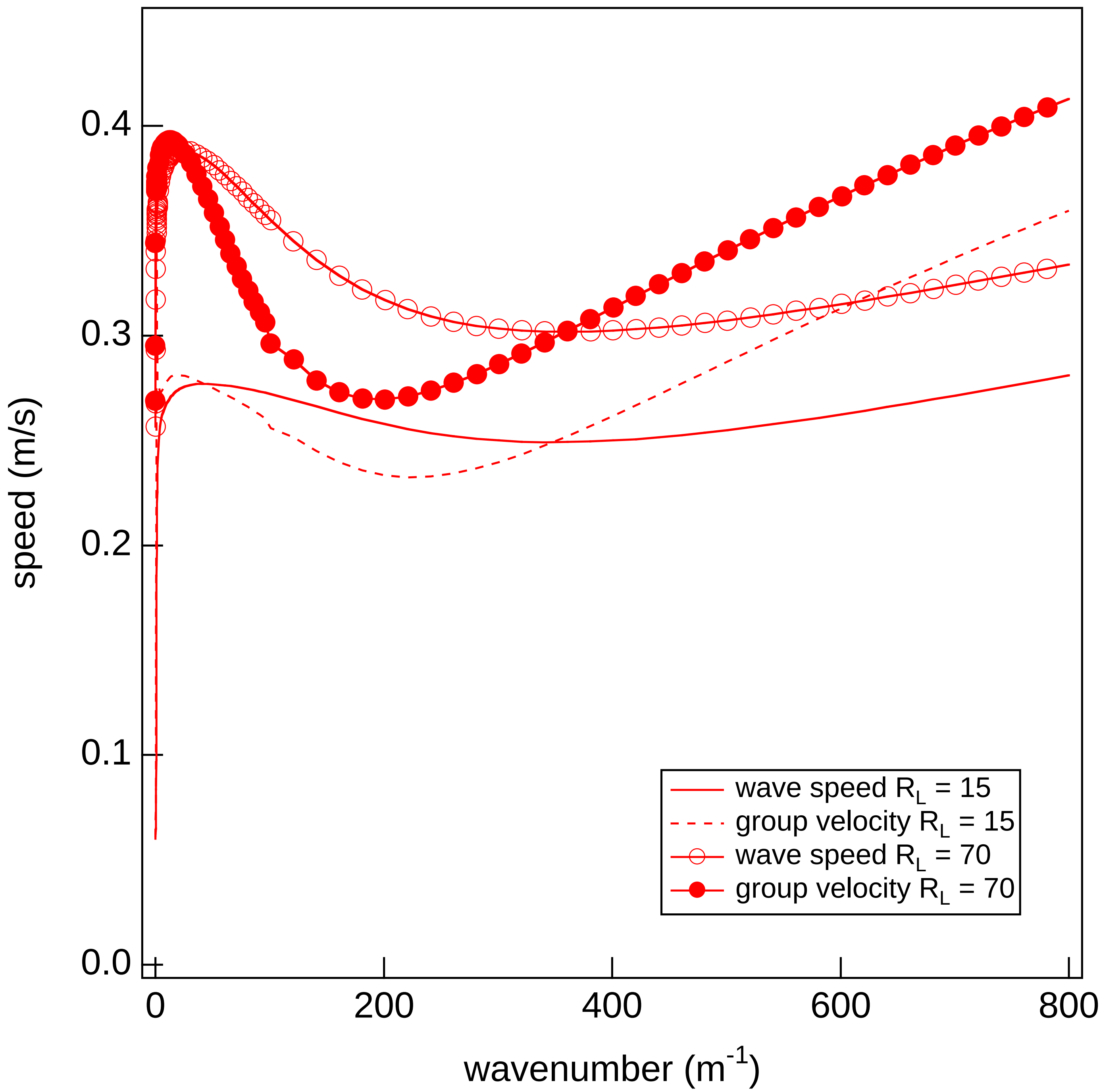
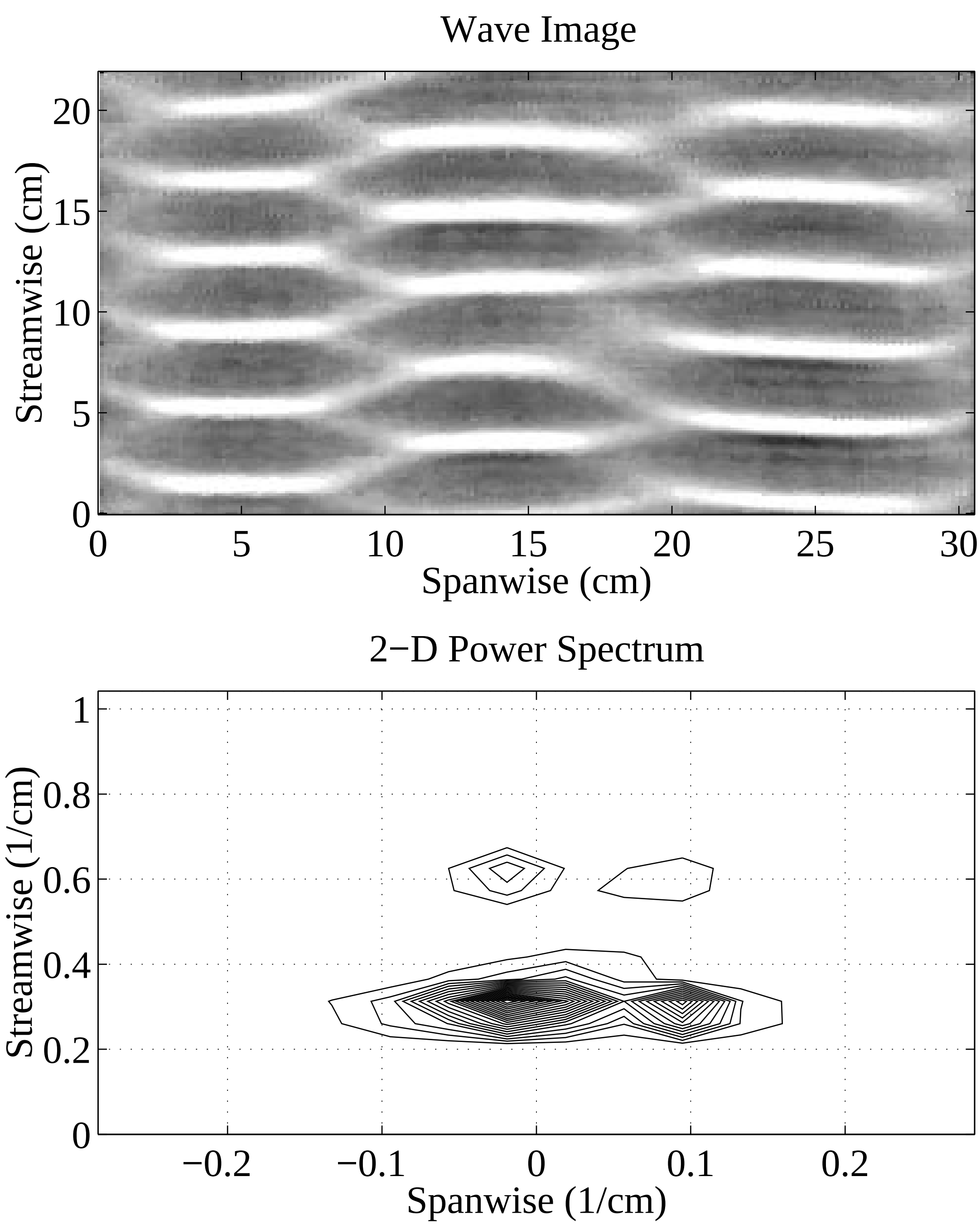


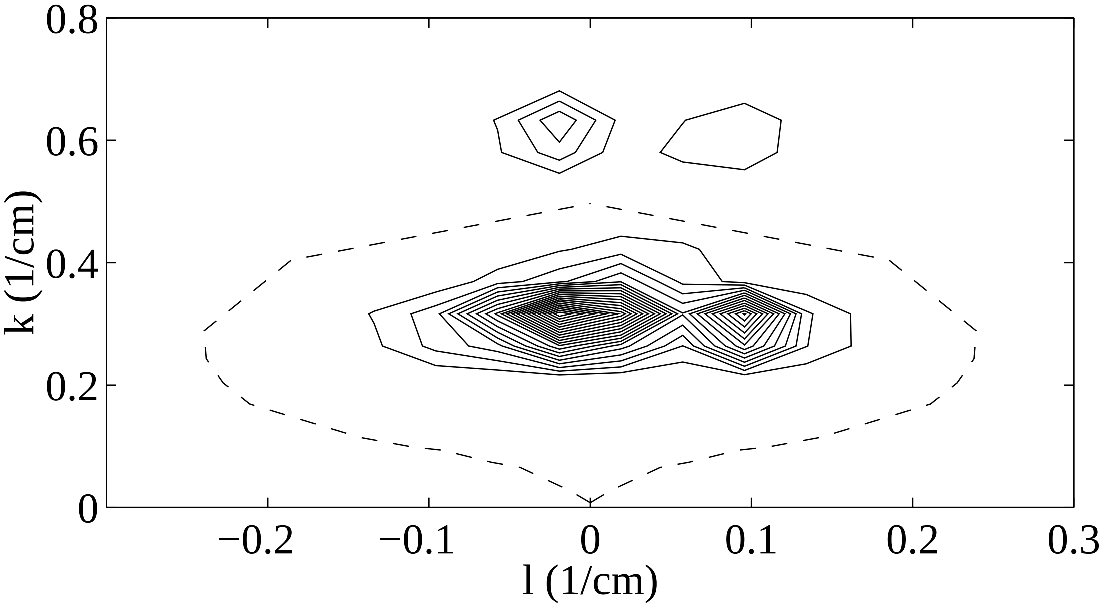
Figure 3. Group velocity and phase velocity for thin and thick films.

groupvel?

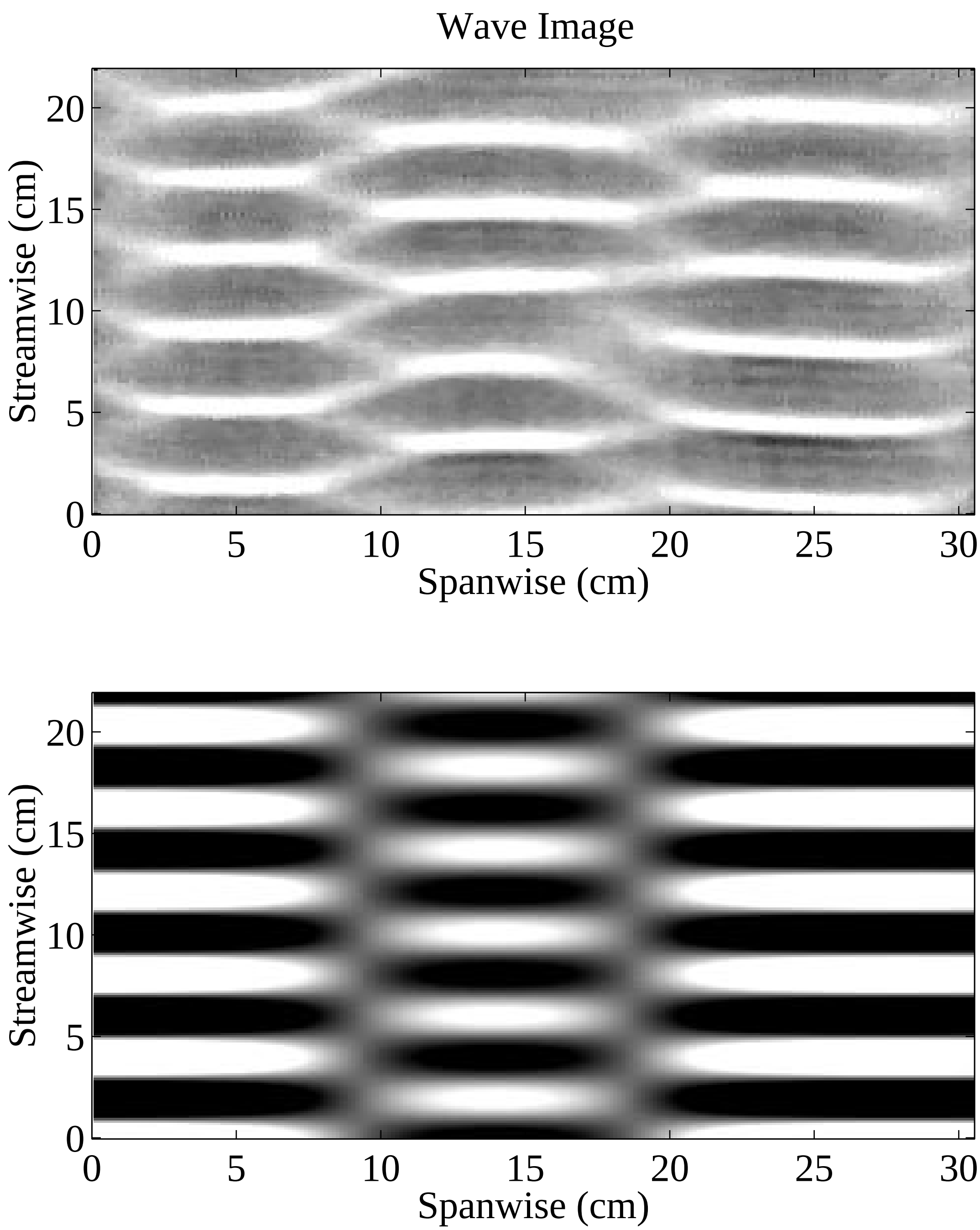
Global appearance of transverse modes



$R_L = 83$, $R_G = 4460$, $\mu_L = 9$ cP



0.8
0.6
0.4
0.2
0
k (1/cm)
−0.2
−0.1
0
0.1
0.2
0.3
l (1/cm)



Wave Image
Streamwise (cm)
Spanwise (cm)
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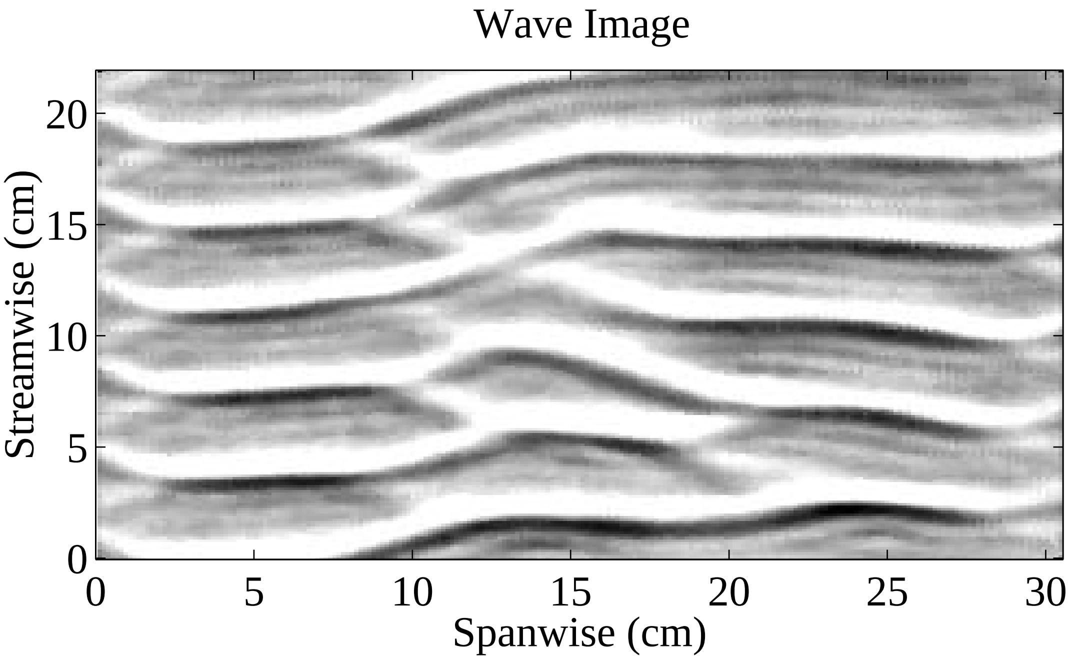
Streamwise (cm)
Spanwise (cm)


Wave Image
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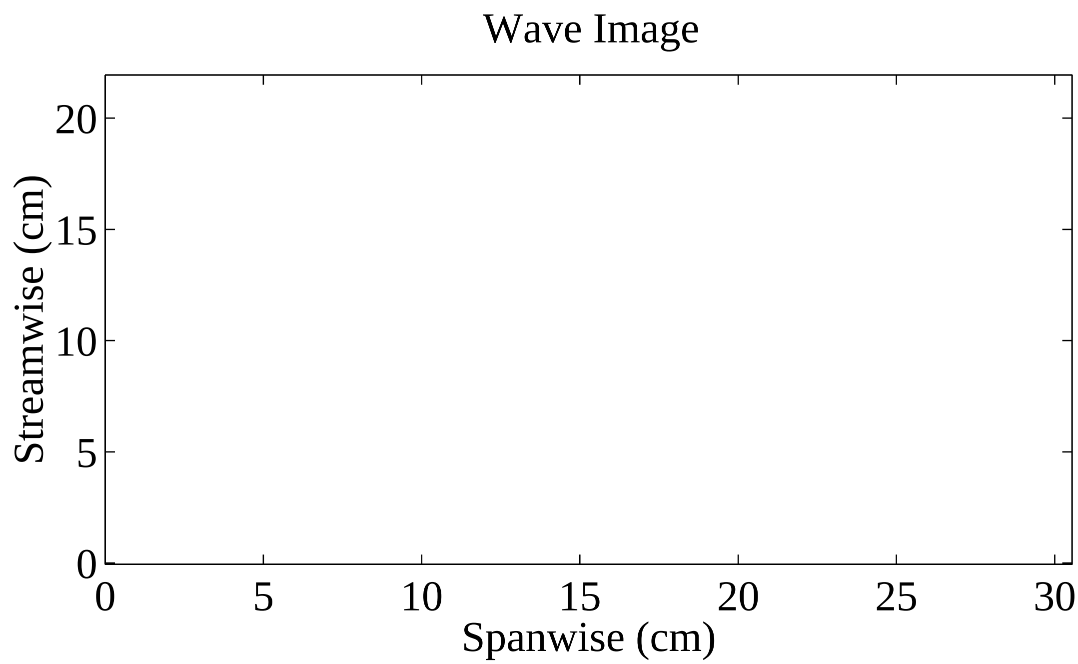
Wave Image
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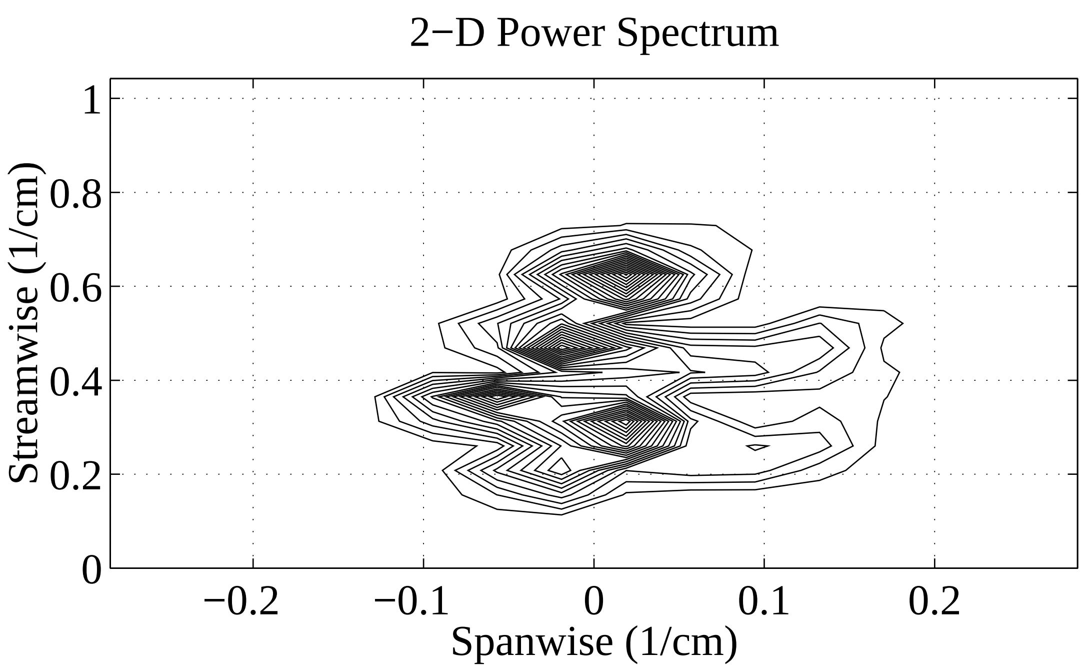
2−D Power Spectrum
Streamwise (1/cm)
1
0.8
0.6
0.4
0.2
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−0.2
−0.1
0
0.1
0.2
Spanwise (1/cm)

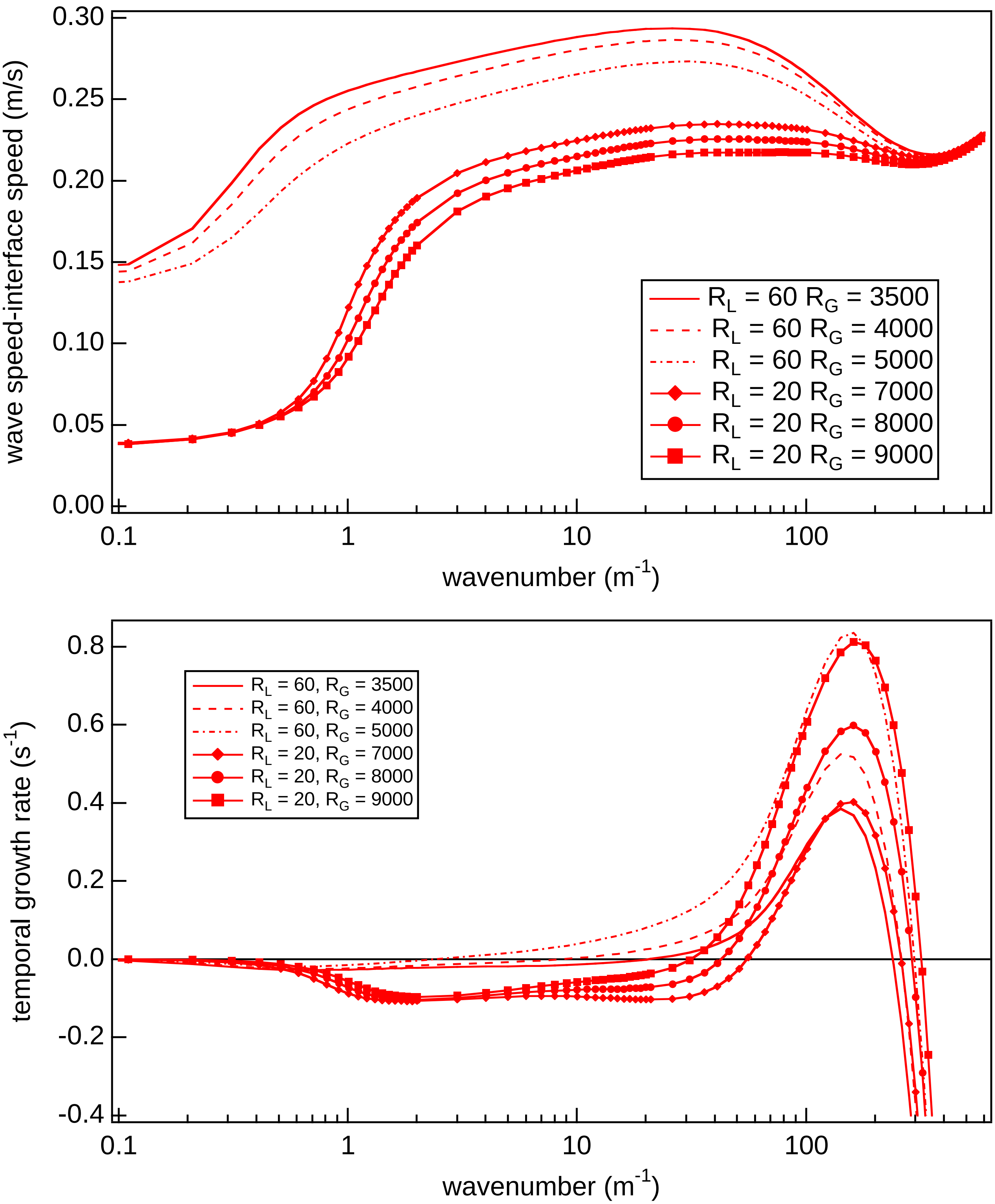

Figure 6. Linear growth curve and phase speed minus interface velocity for thin and thick film regions

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