# Link Modeling with Joint Fading and Distance Uncertainty

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***Abstract*— We introduce and discuss a novel link model that incorporates both uncertainty in the fading coefficients and the node distances for ad hoc networks with randomly placed nodes. The main result is the complete distribution of the received power for a transmission between a node and its $n$-th nearest neighbor. Several applications of the proposed fading model are discussed, including connectivity, opportunistic communication, and localization.**

## I. Introduction

In wireless networks, distances have a strong impact on the signal strengths and the signal-to-noise-and-interference ratios (SINRs), and, consequently, on the quality of the links. In addition, given a transmitter-receiver distance $d$, the path loss may deviate significantly from the expected value obtained from a large-scale path loss model, usually of the form $d^\alpha$, a phenomenon referred to as fading. Fading is commonly considered by the physical layer community, and distances in ad hoc networks are normally taken into account by researchers focusing on the higher layers, typically the network layer. It is essential that both effects are considered jointly to obtain an accurate link model. Hence the objective of this paper is to propose a novel link model that is based on uncertainty on both the fading state of the channel and the distance between transmitter and receiver.

We focus on flat small-scale fading, in particular of the Rayleigh type, and on networks whose nodes are distributed according to a homogeneous Poisson point process (PPP). These models have two advantages; they are analytically tractable on the one hand, and they constitute worst or extreme cases on the other hand, in the sense that most fading models are more benign than Rayleigh fading, and all (homogeneous) point processes have a smaller entropy than the PPP. For the large-scale path loss, we employ the common power law mentioned above, well aware of its shortcoming at small distances [1], [2].

### A. Static and Dynamic Fading Models

*a) Disk and threshold models:* In research on ad hoc networks, two channel or link models are prevalent: the *disk* or *protocol model* and the so-called *physical model* [3]. Both are entirely deterministic. The *disk model* suffers from three serious weaknesses: (1) It ignores the accumulated interference from a potentially large number of distant interferers[1]; (2) it wrongly suggests that the packet reception probabilities decrease if all nodes scale their power by the same factor; (3) it completely ignores the stochastic nature of the wireless channel induced by noise and fading, which leads to inaccurate analyses [4][2] and completely prevents the assessment of diversity techniques [5]. The inaccuracy of the disk model has also been emphasized in [6]–[10] and was demonstrated experimentally [11]–[15].

The *physical model* is based on the more realistic assumption that a certain SINR level $s$ is needed for successful reception. It treats interference as noise and is still deterministic, as the noise is modeled by its variance, and no fading is considered. Therefore the physical model may more accurately be denoted as a *deterministic threshold model*.

*b) Threshold models with fading:* Deterministic threshold models may model the channel with sufficient accuracy for certain types of networks. Most networks, however, exhibit some type of fading [16]–[18], *i.e.*, a stochastic variation in the received signal power that may be caused by multipath propagation, scattering, or obstruction. Fading is a *spatial phenomenon*, *i.e.*, the fading level depends on the *position* of a node and only varies over time if the transmitting and/or receiving nodes (or objects in their surroundings) move. To make this distinction, we use the terms *static* and *dynamic fading*. In static fading, only one channel realization is observed over time (infinite coherence time), whereas a dynamic fading process is usually ergodic. Static fading occurs in networks with static nodes that are placed in rich-scattering environments. A prominent example are sensor networks.

Fig. 1 shows a typical situation in a network with Rayleigh fading and a path loss exponent of $\alpha = 2$. In an $16\times16$ square,

[1] This is particularly important for path loss exponents near two. In fact, for $\alpha = 2$ the interference diverges as the number of nodes grows, and even nearest-neighbor communication becomes impossible, which is not at all apparent from the disk model.

[2] In this paper, it is shown that deterministic models give much more optimistic results than the ones considering fading.

$64^2$ nodes are placed in a regular square grid. The figure indicates which nodes can communicate directly with the base station in the center, if for successful communication a path gain of $s = 0.1$ is required. It is apparent that the successful receivers do not form a simple disk. For comparison, a disk of radius $r = 1/\sqrt{s}$ (the transmission radius in a disk model) is also drawn. Rayleigh fading permits transmissions over substantially larger distances than $r$.

For the speeds occurring in an ad hoc network, a dynamic fading process varies relatively slowly over time, so the coherence time usually equals or exceeds the packet transmission time. For moderate levels of mobility, in particular in TDMA or frequency hopping channels, a *block fading* model [19] may be justified, meaning that the channel stays constant during the transmission of a packet but changes between transmissions.

For this class of *stochastic threshold models*, the reception probability for a certain link distance $d$ is given by

$$p_r(d) = \mathbb{P}[\gamma(d) \geqslant s] = \mathbb{P}\left[\frac{GP_t d^{-\alpha}}{N+I} \geqslant s\right], \tag{1}$$

where $\gamma$ is the SINR, $P_t$ is the transmit power, $\alpha$ the path loss exponent, $G$ a constant depending upon antenna gain and wavelength[3], $N$ the noise variance, and $I$ the total interference. The purpose of this paper is to draw attention to the fact that the distance $d$, often assumed to be known, is itself subject to uncertainty and thus needs to be modeled as a random variable $D$. The reception probability is then given by the expectation of (1) with respect to $D$, *i.e.*,

$$p_r = \mathbb{E}_D\left[\mathbb{P}[\gamma(D) \geqslant s \mid D]\right] \tag{2}$$

Note that we restrict ourselves to the interference-free case. In the case of Rayleigh fading, noise and interference can be treated independently [20], so an interference analysis would simply yield an additional factor in the reception probability.

### B. Node Distribution: The Poisson Point Process

A well accepted model for the node distribution[4] is the homogeneous *Poisson point process* of intensity $\lambda$. For the simplicity of our exposition, we will focus on infinite networks, and without loss of generality, we can assume $\lambda = 1$ (scale-invariance). From the Poisson property, the following result can immediately be derived [21]: For a 2-dimensional network, the distance $D_n$ between a node and its $n$-th neighbor has the generalized gamma probability density function (pdf)

$$f_{D_n}(r) = e^{-\pi r^2}\frac{2(\pi r^2)^n}{r(n-1)!}, \tag{3}$$

In particular, the distance to the nearest neighbor is Rayleigh distributed with mean $1/2$, and the squared ordered distances $D_n^2$ are Erlang with parameter $\pi$, *i.e.*, $\mathbb{E}[D_n^2] = n/\pi$.

Fig. 2 shows the same situation as in Fig. 1, but for an actual PPP of intensity 1 rather than a grid arrangement.

[3]In general, $G \ll d^\alpha$, but since the constants do not matter for what we are doing, we will assume $G = 1$.

[4]In particular, if nodes move around randomly and independently, or if sensor nodes are deployed from an airplane in large quantities.

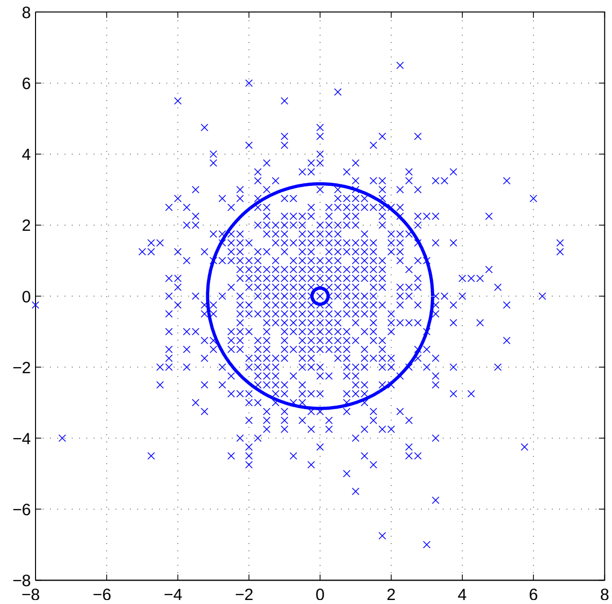


Fig. 1. From $64^2$ nodes arranged in a grid of size $16 \times 16$, this plot shows the ones that can communicate directly with a node situated at the origin in a Rayleigh fading environment and a path loss exponent of $\alpha = 2$. The required path gain for a node to be reachable is $s = 0.1$. The radius of the disk is chosen such that the same number of nodes could be reached under the disk model, *i.e.*, $r = 1/\sqrt{s} \approx 3.16$. Clearly, with Rayleigh fading, much further nodes can be reached than under the disk model.

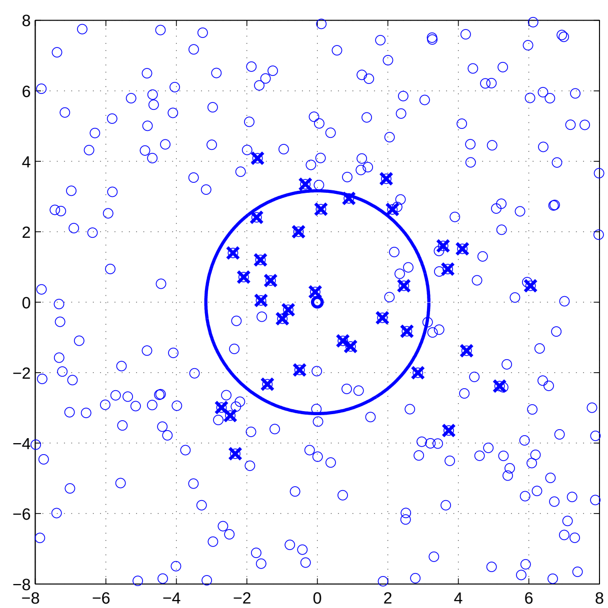


Fig. 2. Same situation as in Fig. 1, but for an actual Poisson point process of intensity 1. The reachable nodes are indicated by a bold $\times$. The expected number of nodes in the fading and non-fading cases are equal.

## II. A Joint Link Model for $n$-th Nearest-Neighbor Communication

### A. Distribution of Path Gain

In this section we determine and discuss the path gain from a node to its $n$-th nearest neighbor, or, equivalently, the received power at a node if its $n$-th nearest neighbor transmits at unit power. The main result is the following theorem:

**Theorem 1** *Consider a node in a Rayleigh fading network whose nodes are distributed according to a Poisson point process in $\mathbb{R}^2$ with intensity 1. Let $Q_n$ denote the (power) path gain between the node and its $n$-th nearest neighbor for a path loss exponent of 2. The cumulative density function (cdf) of $Q_n$ is*

$$F_{Q_n}(x) = 1 - \frac{\pi^n}{(\pi + x)^n} \,. \tag{4}$$

*Proof:* Given the distance $D_n$, the path gain $Q_n$ is exponentially distributed with mean $D_n^{-2}$ due to the Rayleigh fading assumption, and $D_n^2$ is Erlang as mentioned previously. Let $A := D_n^2$, and denote the fading random variable (exponentially distributed with mean 1) by $Q^f$. We obtain

$$\begin{aligned} F_{Q_n}(x) &= \mathbb{P}[Q^f < Ax] && (5)\\ &= \mathbb{E}_A[1 - e^{-Ax}] && (6)\\ &= \int_0^\infty (1 - e^{ax}) \left( \frac{\pi^n a^{n-1}}{\Gamma(n)} e^{-\pi a} \right) \mathrm{d}a && (7)\\ &= 1 - \frac{\pi^n}{(\pi + x)^n} \,. && (8) \end{aligned}$$

□

Note that this result can also be obtained by calculating the distribution of the ratio $Q_n^f / Q_n^d$, *i.e.*, the ratio of the exponential fading part and the Erlang distance part. In particular, for $n = 1$, (4) is the cdf of the ratio of two exponential random variables whose means have a ratio $\pi$. The pdf is

$$f_{Q_n}(x) = \frac{n\pi^n}{(\pi + x)^{n+1}} \,, \tag{9}$$

and the first and second moments are

$$\mathbb{E}[Q_n] = \frac{\pi}{n-1} \quad \text{for } n > 1 \,, \tag{10}$$

$$\mathbb{E}[Q_n^2] = \frac{2\pi^2}{(n-1)(n-2)} \quad \text{for } n > 2 \,. \tag{11}$$

Generally, given $n$, the highest existing (finite) moment is $\mathbb{E}[Q_n^{n-1}] = \pi^{n-1}$. The variance is decreasing quickly: $\mathrm{Var}(Q_n) = O(1/n^2)$.

For the differential entropy $h(Q_n) := \mathbb{E}[-\ln f_{Q_n}(Q_n)]$, we obtain

$$h(Q_n) = \frac{n+1}{n} + \ln\left(\frac{\pi}{n}\right) \,, \tag{12}$$

which is (as expected due to the decreasing variance) monotonically decreasing with increasing $n$.

In the Appendix, we give some remarks on the path gain distribution for path loss exponents other than 2 and for 3-dimensional networks.

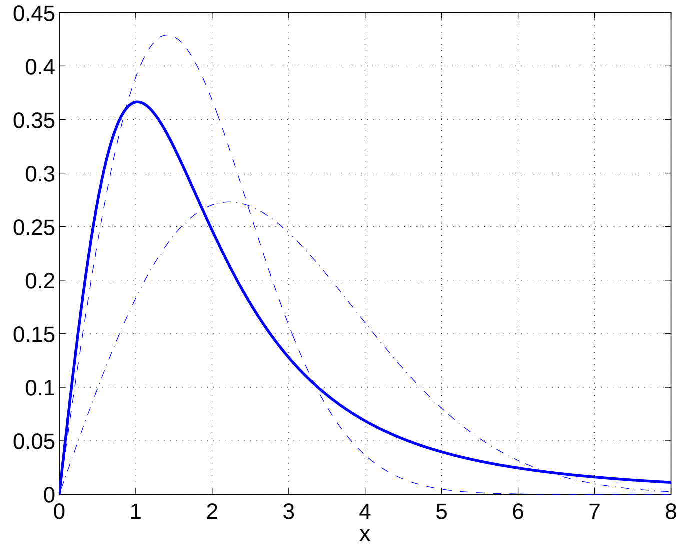


Fig. 3. Pdf of the joint amplitude fading process $H$ (solid line) and Rayleigh pdfs with mean $\sqrt{\pi}$ (dashed) and mean $\pi^{3/2}/2$ (dotted, same mean as the joint process).

### B. Nearest-neighbor Communication

Communication to the nearest neighbor is of particular importance, so we discuss this case in more detail here. With $Q := Q_1$, we have for cdf and pdf:

$$F_Q(x) = \frac{x}{\pi + x} \,; \qquad f_Q(x) = \frac{\pi}{(\pi + x)^2} \,. \tag{13}$$

The pdf of the amplitude $H := \sqrt{Q}$ is

$$f_H(x) = \frac{\mathrm{d}}{\mathrm{d}x} \mathbb{P}[\sqrt{Q} < x] = \frac{2x\pi}{(\pi + x^2)^2} \,, \tag{14}$$

with mean $\mathbb{E}[H] = \pi^{3/2}/2$. In Fig. 3, this joint fading process is compared with two Rayleigh pdf curves, one with mean $\sqrt{\pi}$ and the other with the same mean as $H$. As can be seen, the main difference is the long tail.

The fact that the expected path gain $\mathbb{E}[Q]$ diverges may not be desirable. It is a consequence of the singularity of the path loss model at $d = 0$ and the fact that the PPP allows points to be arbitrarily close. While the latter fact is a consequence of the PPP assumption, the first one is clearly impractical for small distances. There is certainly a cap on the path gain, say $\hat{g}$, which results in $Q = Q^f \min(\hat{g}, D^{-2})$. The same calculation as before in this case yields the cdf

$$\mathbb{P}[Q < x] = \frac{x}{\pi + x} e^{-(\pi + x)/\hat{g}} + 1 - e^{-x/\hat{g}} \,, \tag{15}$$

and the mean received power is

$$\mathbb{E}[Q] = \hat{g}(1 - e^{-\pi/\hat{g}}) + \pi \Gamma_{\mathrm{ic}}(0, \frac{\pi}{\hat{g}}) \,, \tag{16}$$

where $\Gamma_{\mathrm{ic}}$ is the incomplete gamma function[5]. As expected, $\lim_{\hat{g} \to \infty} \mathbb{E}[Q] = \infty$. From this more realistic joint power

[5]More specifically, the *upper* incomplete gamma function as defined in Maple or Mathematica, *i.e.*, $\Gamma_{\mathrm{ic}}(a, 0) = \Gamma(a)$. Note that Matlab's `gammainc` function is the *normalized lower* incomplete gamma function, and the arguments are flipped: $\Gamma_{\mathrm{ic}}(a, z) = \Gamma(a)(1 - \mathtt{gammainc}(z, a))$.

fading process, the corresponding amplitude fading process can be derived.

For the $n$-th nearest neighbors with larger $n$, mean and variance are finite (see (11)) since the probability of small distances decreases.

## III. Applications

In this section we discuss a few applications of the link model. We will focus on the simple path loss model where there is no cap on the path gain.

### A. Connectivity

In accordance with the stochastic threshold model, we that a certain path gain $s$ (or more) is needed for successful communication. Denote two nodes to be connected if they can communicate under this model.

**Corollary 1** *Let $I_n$ be the event that the node under consideration is disconnected from all its $n$ nearest neighbors. Under the same assumptions as in the theorem,*

$$\mathbb{P}[I_n] = \prod_{k=1}^{n} \left(1 - \frac{\pi^k}{(\pi+s)^k}\right) . \tag{17}$$

*The probability $\mathbb{P}[I_\infty]$ of being completely isolated is*

$$\mathbb{P}[I_\infty] = \left(\frac{\theta_1'(0,\sqrt{b})}{2b^{1/8}}\right)^{1/3} , \tag{18}$$

*where $b := \pi/(\pi+s)$ and $\theta_1'(z,q)$ is the derivative (with respect to $z$) of the first Jacobi Elliptic function[6]. $\mathbb{P}[I_\infty]$ is bounded by*

$$1 - \frac{\pi}{\pi+s} - \frac{\pi^2}{(\pi+s)^2} < \mathbb{P}[I_\infty] < e^{-\pi/s} . \tag{19}$$

*Proof:* The link to node $n$ is in outage with probability $F_{Q_n}(s)$ given in (4). All neighbors are independent, so the probability of being disconnected from the nearest $n$ of them is (17). For $n \to \infty$, (18) is a known identity for this infinite product. To obtain the upper bound on $\mathbb{P}[I_\infty]$, we use the logarithmic inequality $\ln(1-x) < -x$, which yields

$$\ln \mathbb{P}[I_\infty] < -\sum_{k=1}^{\infty} \frac{\pi^k}{(\pi+s)^k} = -\pi/s . \tag{20}$$

The lower bound follows from a direct expansion of (17) and truncating after the second term. Similarly, a Taylor expansion of (18) yields $\mathbb{P}[I_\infty] = 1 - b - b^2 + O(b^5)$. □

Fig. 4 shows the probabilities $\mathbb{P}[I_n]$ as a function of $n$ for $s = 1$ and $s = 3$. Note that irrespective of $s$, $\mathbb{P}[I_n]$ converges to $\mathbb{P}[I_\infty]$ after taking into consideration at most $n = 10$ nodes. Fig. 5 displays $\mathbb{P}[I_\infty]$ as a function of $s$, together with the bounds (19). As can be seen, the bounds are quite tight, in particular at higher $s$. In a static fading scenario, $\mathbb{P}[I_\infty]$ is the probability that a node is completely isolated for all times. In a dynamic (block) fading case, it is the probability that nobody can be reached in that particular time slot.

[6]Mathematica: `EllipticThetaPrime[1,z,q]`

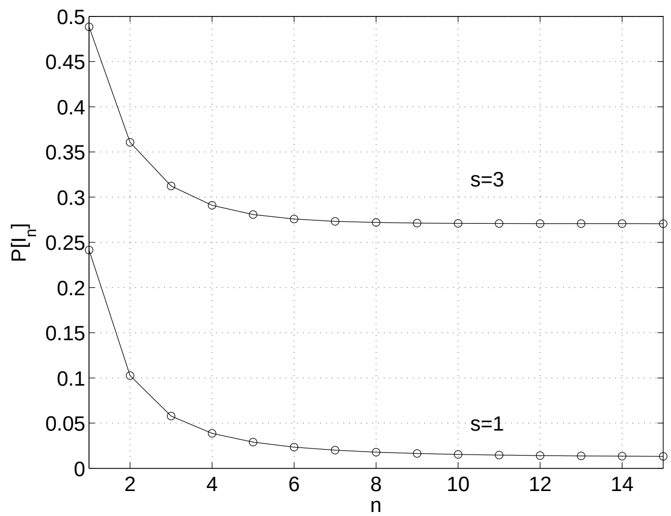


Fig. 4. Probability $\mathbb{P}[I_n]$ of being disconnected from the $n$ nearest neighbors for SNR threshold $s = 1, 3$.

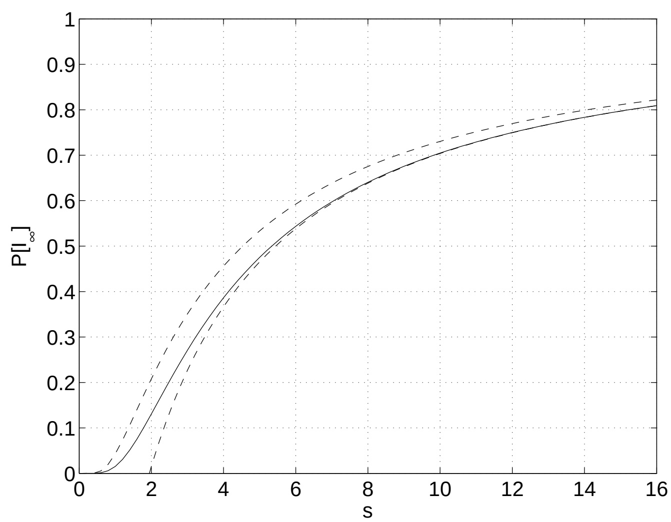


Fig. 5. Probability $\mathbb{P}[I_\infty]$ of a node being completely isolated (solid) and bounds (19) (dashed) as a function of the threshold $s$.

### B. Opportunistic Transmission

Here we investigate the question how frequently it occurs that more distant nodes have better channels, *i.e.*, that "distance" with respect to signal strength does not coincide with geometrical distance. This problem is of interest in opportunistic transmission as well as RSSI-based localization.

**Corollary 2** *Let $\mathcal{M}_n := \{k \in \mathbb{Z} \mid k > -n,\ k \neq 0\}$. For the same assumptions as in the theorem,*

$$\mathbb{P}[Q_n < Q_{n+m}] = \frac{n}{2n+m} , \qquad m \in \mathcal{M}_n . \tag{21}$$

*Proof:* We have for $m \in \mathcal{M}_n$

$$\mathbb{P}[Q_n < Q_{n+m}] = \mathbb{E}\big[\mathbb{P}[Q_n < Q_{n+m} \mid Q_{n+m}]\big] \qquad (22)$$

$$= \int_0^\infty \left(1 - \frac{\pi^n}{(\pi+x)^n}\right) \frac{(n+m)\pi^{n+m}}{(\pi+x)^{n+m+1}} \mathrm{d}x \qquad (23)$$

$$= 1 - \frac{n+m}{2n+m} \int_0^\infty \underbrace{\frac{(2n+m)\pi^{2n+m}}{(\pi+x)^{2n+m+1}}}_{f_{Q_{2n+m}}(x)} \mathrm{d}x\,, \qquad (24)$$

so the last integral is 1. □

So, for any fixed $m \in \mathcal{M}_n$,

$$\lim_{n\to\infty} \mathbb{P}[Q_n < Q_{n+m}] = \frac{1}{2}\,, \qquad (25)$$

*i.e.*, the path gains become comparable. This is not surprising, since as $n \to \infty$, the difference of the distances to node $n$ and $n+m$ for any fixed $m$ goes to zero.

Note that the probability (21) must exhibit the symmetry property $\mathbb{P}[Q_n < Q_{n+m}] = 1 - \mathbb{P}[Q_{n+m} < Q_n]$. Indeed,

$$\frac{n}{2n+m} = 1 - \frac{n+m}{2(n+m)-m}\,. \qquad (26)$$

A more general question is what is the probability $P_{n,m}$ that the path gain to node $n$ is larger than all the path gains to nodes $n+1, n+2, \ldots, n+m$, *i.e.*, to determine

$$P_{n,m} = \mathbb{P}[Q_n > \max_{n<k\leqslant n+m}\{Q_k\}] \qquad (27)$$

$$= \mathbb{E}\bigg[\prod_{k=n+1}^{n+m} 1 - \left(\frac{\pi}{\pi+Q_n}\right)^k\bigg]\,. \qquad (28)$$

Finding a closed-form expression of this expectation for general $n, m$ seems elusive. For the case $m = 2$, we obtain without effort by expanding the product in (28)

$$\mathbb{P}[Q_n > \max\{Q_{n+1}, Q_{n+2}\}] = \frac{4n^2+11n+6}{12n^2+18n+6}\,, \qquad (29)$$

which shows that the probability that $Q_n$ is larger than both the closest two more distant nodes goes to $1/3$ as $n$ increases. If more nodes are considered, the probability will still be a rational function with rapidly increasing but equal numerator and denominator orders.

For the case $n \to \infty$, it can be shown that

$$\lim_{n\to\infty} P_{n,m} = \frac{1}{m+1}\,. \qquad (30)$$

Again, this result agrees with intuition, since as $n \to \infty$, the distances to nodes $n, n+1, \ldots, n+m$ all become equal, and each node in this group of $m+1$ nodes has the same probability of having the largest path gain.

### *C. Routing*

So far, we have assumed that a node may transmit to any reachable node, without any preference in direction. Clearly, if the goal is to communicate with a remote destination node rather than broadcasting, the next relay node should roughly be located in the direction of that destination, such that actual *progress* is made. We use the model suggested in [20], *i.e.*, we assume that only nodes within an angle $\pm\phi/2$ of the source-destination axis are possible relay nodes. The squared ordered distances $D_n^2$ in this case are still Erlang, but with parameter $\phi/2$. The cdf of $Q_n$ is obtained simply by replacing $\pi$ by $\phi/2$, *i.e.*,

$$F^\phi_{Q_n}(x) = 1 - \left(\frac{\phi/2}{\phi/2+x}\right)^n\,, \qquad (31)$$

and the mean number of nodes that can be reached within this sector $\phi$ is $\mathbb{E}[N^\phi] = \phi/(2s)$.

### *D. Localization and Channel State Information*

Here we present some thoughts of how the proposed model is to be interpreted in the context of training and localization.

If a receiving node measures the path gain of a transmission, what can we say about the distance? The path gain distribution permits an ML on the index of the node $\hat{n}$ that is transmitting: For a received power $x$:

$$\hat{n}(x) = \arg\max_n f_{Q_n}(x) \qquad (32)$$

The ML decision $\hat{n}$ is the following:

$$\hat{n}(x) = n \quad \Longleftrightarrow \quad \frac{\pi}{n} \leqslant x < \frac{\pi}{n-1} \qquad (33)$$

So, the ML decision is $\hat{n}(x) = \lceil \pi/x \rceil$. This is of course related to the fact $\mathbb{E}[D_n^2] = n/\pi$.

The purpose of training is to acquire knowledge of the channel state. In our framework, channel state information (CSI) includes the combined information on fading state and distance. From measuring RSSI, it is not possible to decide on the fading state $Q^f$, since a low path loss may just as well be from a short distance rather than good fading.

One way to disentangle actual fading state and distance is localization, which reduces the uncertainty in the link distance $D$, *i.e.*, to find an estimate $\hat{D}$ with smaller variance or differential entropy than $D$. For many node localization schemes, a Gaussian model $\mathcal{N}(d_0, \sigma^2)$ seems reasonable for $\hat{D}$, where $d_0$ denotes the actual distance and $\sigma^2$ the variance or residual uncertainty. This implies that localization turns Rayleigh distances into Ricean distances, where the Ricean $K$ factor depends monotonically on $\sigma^{-2}$.

Another important aspect is the coherence time. While the coherence time of the Rayleigh fading part is related to wavelength and velocity and may be relatively short, the coherence time of the distance component will be significanly larger in most cases. So the combined fading values will not be independent from packet to packet.

## IV. Concluding Remarks

We have proposed a link model that incorporates the two main types of uncertainty in the channels of wireless ad hoc networks, namely the fading state and the link distance. The model is characterized by the distribution of the path gain. Several applications are discussed: connectivity, opportunistic communication, and localization. Related to connectivity, it turns out that the expected number of nodes that can be reached is the same as in the non-fading case. However, the probability of a node being isolated is smaller.

The fading model is particularly simple in the case where the number of network dimensions equals the path loss exponent.

We expect the proposed model to provide better insight into the behavior of large ad hoc networks and to find many useful applications, *e.g.*, in throughput and outage analyses, connectivity, the design of flooding algorithms, and RSSI-based localization.

## APPENDIX

### A. Other Path Loss Exponents

From (3), the cdf of $D_n^\alpha$ is

$$F_{D_n^\alpha}(x) = 1 - \frac{\Gamma_{\text{ic}}(n, \pi r^{2/\alpha})}{\Gamma(n)} . \tag{34}$$

Thus we obtain for the cdf of $Q_n$, analogously to (8):

$$F_{Q_n}(x) = \int_0^\infty (1 - e^{-ax}) \left( \frac{2}{\alpha} \frac{\pi^n e^{-\pi r^{2/\alpha}} r^{2n/\alpha - 1}}{\Gamma(n)} \right) \tag{35}$$

For $\alpha = 4$, we find:

$$F_{Q_n}(x) = 1 - \frac{\pi^n x^{-\frac{1}{2} - \frac{n}{2}} A(n,x)}{2\Gamma(n)} \tag{36}$$

with

$$\begin{aligned} A(n,x) :=& \sqrt{x}\, \Gamma\left(\frac{n}{2}\right) {}_1\mathrm{F}_1\left(\frac{n}{2}, \frac{1}{2}, \frac{\pi^2}{4x}\right) \\ & - \pi \Gamma\left(\frac{1+n}{2}\right) {}_1\mathrm{F}_1\left(\frac{1+n}{2}, \frac{3}{2}, \frac{\pi^2}{4x}\right) , \end{aligned} \tag{37}$$

where ${}_1\mathrm{F}_1$ is the confluent hypergeometric function of the first kind[7].

For $\alpha = 3$, $F_{Q_n}(x)$ involves a general hypergeometric function ${}_2\mathrm{F}_2$.

### B. 3-Dimensional Networks

In the 3-dimensional case, the cdf of $D_n^\alpha$ is [21]

$$F_{D_n^\alpha}(x) = 1 - \frac{\Gamma_{\text{ic}}(n, \frac{4}{3}\pi r^{3/\alpha})}{\Gamma(n)} . \tag{38}$$

Similarly to the two-dimensional case, we obtain a particularly simple joint fading model when the path loss exponent equals the number of dimensions: For $\alpha = 3$, the cubed ordered distances are Erlang with parameter $3/(4\pi)$. So we obtain the distribution of the path gain simply by replacing $\pi$ by $4/(3\pi)$ in (4).

[7]Also called Kummer's function of the first kind. Mathematica: `Hypergeometric1F1[a,b,z]`.

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