Delay characterization of random multi-hop networks

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Abstract

We consider a network where each route consists of a source, a number of relays and a destination at a finite distance, and the locations of the sources and relays are determined according to independent Poisson point processes. Given a TDMA/ALOHA medium access control (MAC) protocol, our objective is to determine a relay selection strategy such the mean end-to-end delay in a typical route is minimized. Towards this goal, we analyze an idealized network model where all routes have the same number of hops, the same distance per hop and their own dedicated relays. Using a combination of tools from queueing theory and stochastic geometry, the mean end-to-end delay and throughput are evaluated for a typical route. In the case of backlogged sources, we find that the delay is minimized if the first hop is much longer than the remaining hops, while in the case of non-backlogged sources, if all hops are equidistant. The respective optimal numbers of hops scale sublinearly and linearly with the source-destination distance. Simulating the original network scenario confirms that the analytical results are accurate, provided that an adequate number of relays is present in the network.

Index Terms

End-to-end delay, throughput, multi-hop, relay, Poisson point process, interference

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I. INTRODUCTION

One of the main design issues in multi-hop wireless networks is determining the number of hops between the source of information and the final destination [1]. For a given transmission power, a smaller hopping distance results in a larger received signal-to-interference-and-noise-ratio (SINR), which implies higher reliability and/or a higher transmission rate over a single hop. However, as argued in [1], this does not necessarily translate to an end-to-end performance benefit, e.g., in terms of delay: each node that is added between the source of information and the final destination is also the cause of additional delay, since a packet has to be decoded, re-encoded and wait in the queue, before it is transmitted to the next node. Moreover, if the intermediate nodes can transmit only one at a time, e.g., to avoid intra-route interference, the throughput might suffer as well. Along the line of thought in [2], a meaningful performance analysis and the design of multi-hop ad hoc networks should be carried out with end-to-end constraints on delay and reliability in mind.

This paper combines tools from stochastic geometry and queueing theory, in order to conduct an analytical study of the end-to-end delay performance in multi-hop random networks. We consider a network where each route consists of a source, a number of relays and a destination at a finite distance, and the locations of the sources and relays are determined according to independent Poisson point processes. Nodes are equipped with queues to accommodate the randomness in the delivery of a packet, which is determined by the level of the signal-to-interference-ratio (SIR) over each hop. Given a TDMA/ALOHA MAC protocol, we first study an idealized network model where all routes have the same number of hops, the same distance per hop and their own dedicated relays. Assuming that a stationary regime exists for this network, we analytically evaluate the mean end-to-end delay and the throughput for backlogged and non-backlogged sources. A key point in our analysis is taking into account that the interference level over each hop, i.e., the density of transmitting nodes, depends on the packet success probabilities and vice versa. The derivation of closed-form expressions permits the optimization of the mean end-to-end delay with respect to the number and placement of the relays. The benefits of this relay selection - or routing - strategy are then verified in the original network via simulation.

A. Related work

Excluding the literature on capacity scaling laws [3], previous work on random multi-hop networks considered a single hop of a typical route, with the implicit assumption that the destination lies at an infinite distance from its source [4]–[8] (also see [9] for a survey and a list of references). Single-hop metrics that are related to a performance benefit at the end-to-end level were devised and evaluated. Such metrics were: the expected progress [4], i.e., the product (packet success probability) $\times$ (hop length), for a given spatial density of transmitters, which reflects the trade-off between reliability and hopping distance; the transmission capacity [6], [8], i.e., the maximum density of transmitters allowed under a constraint on the success probability, for a given hop length; and the spatial density of progress [7], i.e., the product (spatial density of successful transmissions) $\times$ (hop length) which is a logical combination of the previous two metrics. A central assumption in [4]–[8] is that the locations of the transmitters constitute a Poisson random process on the plane. This enables an analytic characterization of the SINR statistics, hence the derivation of analytical results that demonstrate the effect of the channel (fading, interference and noise) and physical layer parameters on the above network-wide metrics.

A second body of work [10]–[13] is, in a sense, complementary to the first. A well-defined route is considered, where the distance to the final destination and the number of intermediate nodes, or relays, are specified. However, the impact of interference from other transmissions in the network is not taken into account. Assuming a channel model with path-loss, fading and noise, and no delay constraints, [11], [12] determined the end-to-end rate, i.e., the minimum achievable rate over all hops, when a TDMA-access protocol is employed. Alternatively, under a given delay constraint, [13] specified the number of hops and the rate allocation among them, such that the total power consumption is minimized. A similar problem was studied in [10], under an end-to-end success probability requirement.

To the best of our knowledge, [14] was the first to consider the multi-hop problem taking into account the interference present in a random network. Given an ALOHA medium access control (MAC) scheme, an opportunistic routing strategy was studied, according to which the next node in a route is dynamically selected based on its channel characteristics and its distance to the final destination. However, the characterization of the end-to-end delay was mainly based on simulations and interference was modeled in the network assuming all relays are backlogged.

B. Contributions

If the sources are backlogged, we find that the delay is minimized if the first hop is much larger than the remaining hops, e.g., in a delay-optimized three-hop route, the first hop covers half the total distance. On the other hand, in the non-backlogged case, the delay is minimized for equidistant hops. We demonstrate that the respective optimal numbers of hops scale sublinearly and linearly with the source-destination distance. We also discuss stability issues in a multi-hop random network and derive sufficient conditions on the medium access probability (MAP), and the traffic load in the case of non-backlogged sources, for the rate stability of the network.

Simulations show that, when the relays of each route are selected out of a random population, the analytical results are accurate, provided that the density of the relay process is sufficiently large.

The rest of the paper is organized as follows. Section II introduces the system model. In Section III, we develop our mathematical framework in order to evaluate the mean delay and discuss stability issues in a multi-hop random network. Numerical examples and simulation results are presented in Section IV, and our conclusions are summarized in Section V. A list of symbols commonly used throughout the paper is given in Table I.

II. System model

A. Network setting

We consider a network that consists of an infinite number of sources, relays and destinations, referred to indiscriminately as *nodes* (see Fig. 1). Each source has its own destination at a distance R and random orientation. The locations of the sources and the relays are drawn independently according to spatially homogeneous Poisson processes of densities λ and λ_r, respectively. A *route* or *flow* in the network consists of a source, a finite number of relays - whose sole function is to forward the packets that originate at the source - and a final destination.

Each node has an infinite queue, where packets that are received from the previous node in the route can be stored in a first-in, first-out fashion. Time is divided into packet slots. In a route with N hops, the source “receives” a packet from an upper layer of the protocol stack every N slots with probability p_a; if the sources are assumed to be backlogged, p_a is equal to one. Within a route, a TDMA/ALOHA protocol is observed, according to which a node is given the opportunity to transmit every N slots with a certain MAP. In the assigned TDMA slot, a source node is allowed to transmit with probability p, while a relay is allowed to transmit with probability p_r. A packet is received successfully by a node if the SINR in that slot is above a target threshold. If it is not, the transmitting node is informed via an ideal feedback channel and the packet remains at the head of its queue, at least for the next N slots, until it gets another opportunity to transmit [1].

The network operation starts at some arbitrary time, with an arbitrary number of packets in each queue. For convenience, we assume that different routes are synchronized at the slot level. However, to avoid an artificial periodic behavior, we assume that, for each route, the node which is first allowed to transmit is randomly selected among the source and the relays with probability $1/N$. Since the population of relays is common to all sources, there is a possibility that, at a given slot, a relay has to be used, either for transmission or reception, by more than one routes. We assume that the conflict is resolved arbitrarily.

B. Idealized network setting

The analysis of the network model described above is complicated by the fact that the number of relays and their placement differ across routes, as well as the fact that routes might intersect. In order to simplify the model, we let each route in the network have the same number of hops N, the same distance per hop $r_n,\ n = 1, \dots, N$, and its own set of relays. Under this idealization, node n of a typical route denotes the receiving node at hop n (node 0 denotes the source).

The design guidelines that stem from the analysis of the simplified model are then used to route packets in a simulation of the original realistic network.

C. Physical layer

The channel between any two nodes at distance r includes Rayleigh fading and path-loss according to the law r^{-b}, where $b > 2$ is the path-loss exponent. Given the MAC scheme, we assume that the coherence time of the fading coefficient is at least (most) equal to the duration a slot (N slots). We also assume an interference-limited setting, i.e., thermal noise is considered

[1]This implies that retransmissions are not given priority.

negligible and disregarded, and that all nodes have the same transmit power, which is normalized to one. The signal-to-interference-ratio (SIR) at the n^{th} hop of a typical route is therefore

$$\mathrm{SIR}_n = \frac{A_n r_n^{-b}}{\sum_{i\in\Phi_n} B_i d_i^{-b}},\ \ n = 1,\dots,N \tag{1}$$

where r_n is the distance of the n^{th} hop; A_n is the fading coefficient between the transmitting and receiving node and exponentially distributed with unit mean; Φ_n is the process of interfering nodes over hop n; d_i is the distance between the interfering node i and the receiver, and B_i is the respective fading coefficient, also exponentially distributed with unit mean.

A packet is successfully received when the SIR is above a target threshold θ [2]. The following result provides an expression for the probability of successful reception over the n^{th} hop, p_n^s, when Φ_n is a homogeneous Poisson point process.

Lemma 1 (Corollary 3.2 [7]): Given the definition of SIR_n in (1) and that Φ_n is a homogeneous Poisson point process with density λ_n

$$p_n^s = \mathbb{P}(\mathrm{SIR}_n > \theta) = \mathrm{e}^{-\lambda_n c r_n^2}, \tag{2}$$

where $c = \Gamma(1+2/b)\Gamma(1-2/b)\pi\theta^{2/b}$ and $\Gamma(x),\ x > 0,$ is the gamma function.
The physical meaning of (2) is that the success probability is equal to the probability that a disk of area $\lambda_n c r_n^2$ around the intended receiver is free of interfering nodes.

D. *Network metrics*

Our metric of interest is the mean end-to-end delay[3] D, i.e., the mean total time (in slots) that it takes a packet to travel from the source to the destination in a typical route. Assuming negligible propagation times, D is the sum of the mean *waiting times* and *service times* along the queues of the route. The waiting time at a given node is measured starting from the moment a packet arrives at that node's queue, till it becomes the head-of-line packet. The service time is measured from the moment a packet reaches the head of the queue, till it is successfully received by the next node, and incorporates the access delay associated with the MAC protocol.

In addition to the delay, we are interested in evaluating the overall route throughput RT, defined as the expected number of packets successfully delivered to the destination per slot over a typical route. The respective network throughput is then defined as $\mathrm{NT} = \lambda\mathrm{RT}$.

[2]This implies that the receiver regards interference as noise.

[3]Henceforth, the terms "mean delay" and "delay" will be used interchangeably.

III. Analysis

To begin our analysis, we assume that, under certain conditions which are discussed in Section III-C, the queues approach a stationary behavior in the long run. Moreover, due to the randomness and symmetry present in our network model, we assume that, in this stationary regime, successful packet transmissions over the n^{th} hop of the typical route are *independent* events that occur with probability p_n^s. The latter assumption allows us to consider the set of interfering nodes over each hop as a homogeneous Poisson point process. Using (2) then leads to a set of fixed-point equations that provide solutions for the success probabilities $\{p_n^s\}_{n=1}^N$. Note that, in spirit, this approach is similar to that introduced in [15], in the context of analyzing the performance of the IEEE 802.11 protocol for wireless local area networks.

In the first part of this section, we use tools from queueing theory in order to express the mean delay over a typical route as a function of p, p_r, p_a and $\{p_n^s\}_{n=1}^N$. In the second part, we evaluate $\{p_n^s\}_{n=1}^N$.

A. Queueing analysis

We examine the backlogged and non-backlogged source cases separately.

1) Backlogged sources: The mean waiting and service times at a given relay are functions of the packet arrival and departure probabilities to and from that relay. Assume that we are looking at the queue of the first relay. A packet arrival occurs at the end of slot $mN,\ m \in \mathbb{Z}$, if the source transmitted in that slot and the transmission was successful. The probability of this event is therefore $p_I = pp_1^s$. Similarly, at the beginning of slot $1+mN$ - provided that the queue is not empty - a packet departs from the head of the queue with probability $p_O = p_r p_2^s$. This procedure is repeated every N slots.

The queue of the relay is modeled as a Random Walk [16], whose state is the number of packets in the queue at the end of slot mN. The transition probability from state k to state $k+1$, $k \geq 1$, is $p_I(1-p_O)$, while, from state $k+1$ to state k, it is $p_O(1-p_I)$, for $k \geq 0$. Since in state 0 the queue is empty, the transition probability from state 0 to state 1 is simply p_I. Assuming that the queue is stable, the steady state probability of being in state k, π_k, is found to be

$$\pi_k = \frac{p_I}{p_O(1-p_I)} \cdot \left(\frac{p_I(1-p_O)}{p_O(1-p_I)}\right)^{k-1} \cdot \pi_0,\ \ k \geq 1, \tag{3}$$

where $\pi_0 = 1 - \frac{p_I}{p_O}$. Obviously, a necessary condition for the stability of the queue is $p_I < p_O$.

The mean waiting time at the first relay, Q_1, can be computed by Little's theorem, as the average queue size, excluding the head-of-line packet, divided by the arrival rate [16], in this case p_I/N. Using (3), after some calculations we find that

$$Q_1 = N\frac{p_I}{p_O}\frac{1-p_O}{p_O-p_I}.$$

It is also straightforward to show that the service time for the head-of-line packet is

$$H_1 = N\left(\frac{1}{p_O}-1\right)+1. \tag{4}$$

The total time in the queue of the first relay is

$$Q_1+H_1 = N\frac{p_I}{p_O}\frac{1-p_O}{p_O-p_I}+\frac{N}{p_O}-N+1 = N\frac{1-p_I}{p_O-p_I}-N+1. \tag{5}$$

If the queue at the first relay is stable, i.e., $pp_1^s < p_r p_2^s$, the packet arrival probability to the second relay is pp_1^s. In fact, as long as all intermediate queues are stable, i.e., $pp_1^s < p_r p_n^s$, $n = 2, \ldots, N$, the packet arrival probability to relay n is pp_1^s. As a result, we can calculate the total time in the queue of relay n by setting $p_O = p_r p_n^s$ in (5).

Adding the service time at the source and the total times at the queues of $N-1$ relays, the mean end-to-end delay for a packet at the head of the source queue, $D_{\rm b}$, is found to be [4]

$$D_{\rm b} = \frac{N}{pp_1^s}+N\sum_{n=2}^{N}\frac{1-p_r p_n^s}{p_r p_n^s - pp_1^s}, \tag{6}$$

The route throughput is determined as follows: a packet is received every N slots by the final destination with probability pp_1^s. Hence, $\mathrm{RT}_{\rm b} = pp_1^s/N$.

2) Non-backlogged sources: If the sources are not backlogged, then, provided that all queues are stable (including the source queue), the packet arrival probability to all queues is p_a and $p_a < \min\{pp_1^s, p_r p_2^s, \ldots, p_r p_N^s\}$. Following a similar reasoning to the backlogged source case, the mean end-to-end delay for a packet that arrives at the end of the source queue, $D_{\rm nb}$, is

$$D_{\rm nb} = N\frac{1-p_a}{pp_1^s-p_a}+N\sum_{n=2}^{N}\frac{1-p_r p_n^s}{p_r p_n^s - p_a} \tag{7}$$

and the route throughput is $\mathrm{RT}_{\rm nb} = p_a/N$.

[4]Since the sources are backlogged, the only meaningful way to define the end-to-end delay is for a packet at the head of the source queue.

B. Evaluation of packet success probabilities

We now determine the packet success probabilities $\{p_n^s\}_{n=1}^N$, which are required for the evaluation of the mean delay in (6) and (7).

The definition of the Lambert function is needed in the subsequent analysis and is stated here.

Definition 1: The Lambert function, $y = W(x)$, is the unique solution to the equation $y\mathrm{e}^y = x$, where $x \geq -\mathrm{e}^{-1}$ and $y \geq -1$.

The following lemmas are also useful and are stated without proof.

Lemma 2: The function $f(x) = x\mathrm{e}^{-\mu x},\ \mu > 0, x \geq 0$, is continuous, with a unique maximum $f_o = (\mu \mathrm{e})^{-1}$ at $x_o = \mu^{-1}$. There are two solutions to the equation $y = f(x),\ 0 \leq y < f_o$, and the smaller of the two is $x_1 = -W(-\mu y)/\mu < \mu^{-1}$.

Lemma 3: The function $g(x) = x\mathrm{e}^{\mu x},\ \mu > 0, x \geq 0$, is continuous and strictly increasing. The equation $y = g(x),\ \mu > 0,\ x, y \geq 0$, has a unique solution $x_1 = W(\mu y)/\mu$.

Once again, the backlogged and non-backlogged source cases are presented separately.

1) Backlogged sources: Under the assumption that packet success events are independent across routes, the queue of relay n of the typical route is not empty at the beginning of its assigned slot with probability $pp_1^s/(p_r p_n^s)$, hence the relay transmits a packet with probability $pp_1^s/(p_r p_n^s) \cdot p_r = pp_1^s/p_n^s$. As a result the interfering nodes over hop n constitute a homogeneous Poisson point process with density $\lambda_n = \frac{\lambda p}{N} + \frac{\lambda p}{N} \sum_{k=2}^N \frac{p_1^s}{p_k^s}$ where the first term represents the transmitting sources, while the second term represents the total population of transmitting relays. From Lemma 1, p_n^s satisfies the fixed-point equation

$$p_n^s = \mathrm{e}^{-\frac{\lambda}{N} p \left(1 + \sum_{k=2}^N \frac{p_1^s}{p_k^s}\right) c r_n^2},\ n = 1, \ldots, N. \tag{8}$$

Eq. (8) illustrates the coupling effect between concurrent transmissions in a wireless network. The packet success probabilities depend on the interference level in the network and vice versa.

The solution of (8) over $\{p_n^s\}_{n=1}^N$ for general values of $r_1, \ldots, r_N$ seems complicated. Proposition 1 provides the solutions for the packet success probabilities in the setting $r_2 = \cdots = r_N$, i.e., when hops $2, \ldots, N$ are equidistant.

Proposition 1: Let $r_2 = \cdots = r_N$. Given that the relay queues are stable, the packet success probabilities are given by

$$p_n^s = \mathrm{e}^{-\frac{\lambda}{N} p \left(1 + (N-1)\frac{p_1^s}{p_2^s}\right) c r_n^2},\ n = 1, 2 \tag{9}$$

$$p_2^s = \cdots = p_N^s.$$

Furthermore

$$\frac{p_1^s}{p_2^s} = \begin{cases} \dfrac{W\left(\frac{\lambda}{N}(N-1)pc(r_1^2-r_2^2)\mathrm{e}^{-\frac{\lambda}{N}pc(r_1^2-r_2^2)}\right)}{\frac{\lambda}{N}(N-1)pc(r_1^2-r_2^2)} & r_1 \neq r_2 \\ 1 & r_1 = r_2. \end{cases} \tag{10}$$

Proof: Setting $p_2^s = \cdots = p_N^s$, (9) results directly from (8). Dividing the two equations in (9) and rearranging terms

$$\frac{p_1^s}{p_2^s}\mathrm{e}^{\frac{\lambda}{N}(N-1)pc(r_1^2-r_2^2)\frac{p_1^s}{p_2^s}} = \mathrm{e}^{-\frac{\lambda}{N}pc(r_1^2-r_2^2)}. \tag{11}$$

We have the following cases.

- $r_1 > r_2$: by Lemma 3, (11) has a unique solution with respect to p_1^s/p_2^s, given by (10).
- $r_1 = r_2$: setting $r_1 = r_2$ in (9), it follows that $p_1^s = p_2^s = \mathrm{e}^{-\lambda pc r_1^2}$.
- $r_1 < r_2$: by Lemma 2, (11) has a solution with respect to p_1^s/p_2^s if and only if

$$p\mathrm{e}^{-\frac{\lambda}{N}pc(r_1^2-r_2^2)} \leq \frac{1}{\frac{\lambda}{N}(N-1)c(r_2^2-r_1^2)\mathrm{e}}.$$

However, this condition holds if the relay queues are stable, which we show as follows. Assuming that all the relay queues in the network are backlogged, a necessary requirement for stability is that

$$p\mathrm{e}^{-\frac{\lambda}{N}(p+(N-1)p_r)cr_1^2} < p_r\mathrm{e}^{-\frac{\lambda}{N}(p+(N-1)p_r)cr_2^2}.$$

This implies that

$$p\mathrm{e}^{-\frac{\lambda}{N}pc(r_1^2-r_2^2)} < p_r\mathrm{e}^{-\frac{\lambda}{N}(N-1)c(r_2^2-r_1^2)p_r} \leq \frac{1}{\frac{\lambda}{N}(N-1)c(r_2^2-r_1^2)\mathrm{e}}.$$

By Lemma 2, the smaller of the two solutions of (11) with respect to p_1^s/p_2^s is given by (10). We have ignored the other solution on the basis that it is decreasing in r_2.

■

Substituting (9) and (10) in (6), we can evaluate D_b. Concluding the analysis of the backlogged sources case, we now show that setting $r_2 = \cdots = r_N$ is of significance in minimizing D_b.

Proposition 2: The mean delay, given by (6), is minimized when the relays are placed on the line between the source and the destination and $r_2 = \cdots = r_N = \frac{R-r_1}{N-1}$.

Proof: By pairwise division and summation of the expressions in (8), we obtain

$$\sum_{n=2}^{N}\frac{p_1^s}{p_n^s} = \sum_{k=2}^{N}\mathrm{e}^{-\frac{\lambda}{N}p\left(1+\sum_{n=2}^{N}\frac{p_1^s}{p_n^s}\right)c(r_1^2-r_k^2)}. \tag{12}$$

Fix r_1 and define $z_n = \frac{r_n^2}{r_1^2} < 1,\ n = 2, \dots, N$. The function $h(z_2, \dots, z_n) \triangleq \sum_{n=2}^{N} \frac{p_1^s}{p_n^s}$ is strictly increasing in $\{z_n\}$. Indeed, differentiating (12) with respect to z_k and rearranging terms, we obtain

$$\frac{\partial h}{\partial z_k}\left(1 + \frac{\lambda}{N} pcr_1^2 \sum_{n=2}^{N} (1 - z_n) \mathrm{e}^{-\frac{\lambda}{N} p(1+h)cr_1^2(1-z_n)}\right) = \frac{\lambda}{N} p(1+h)cr_1^2 \mathrm{e}^{-\frac{\lambda}{N} p(1+h)cr_1^2(1-z_k)}.$$

Since the right hand side term and the left hand side term in the parenthesis are positive, it follows that $\frac{\partial h}{\partial x_k} > 0$.

From (8), it follows that, given r_1, $\{p_n^s\}_{n=1}^N$ are decreasing functions of $r_2, \dots, r_N$. Since D_b in (6) is a sum of identical decreasing functions of $p_2^s, \dots, p_N^s$, then, given R, D_b is minimized if the relays are placed on the line between source and destination and $r_2 = \dots = r_N$. ■

2) Non-backlogged sources: Similarly to the backlogged scenario, the probability that a stable source queue is not empty is $p_a/(pp_1^s)$, so the probability that a source transmits is p_a/p_1^s. The corresponding probabilities for relay n are $p_a/(p_r p_n^s)$ and p_a/p_n^s. The density of interferers at any hop is therefore $\lambda_n = \frac{\lambda}{N} \sum_{k=1}^{N} \frac{p_a}{p_k^s}$, and, by Lemma 1, the success probability over hop n is

$$p_n^s = \mathrm{e}^{-\frac{\lambda}{N} \sum_{k=1}^{N} \frac{p_a}{p_k^s} cr_n^2},\ n = 1, \dots, N. \tag{13}$$

Following a similar reasoning to Proposition 2, we can show that the delay is minimized if the relays are placed on the line between the source and the destination and *all* hops are equidistant. For equidistant hops, the solution of (13) is given by the following proposition.

Proposition 3: Let $r_1 = \dots = r_N$. Given that the queues are stable, the packet success probabilities are given by

$$\begin{aligned} p_1^s &= -\frac{\lambda p_a cr_1^2}{W(-\lambda p_a cr_1^2)} = \mathrm{e}^{W(-\lambda p_a cr_1^2)} \\ p_1^s &= p_2^s = \dots = p_N^s. \end{aligned} \tag{14}$$

Proof: Setting $r_1 = \dots = r_N$ in (13) results in

$$p_1^s = \dots = p_N^s = \mathrm{e}^{-\lambda \frac{p_a}{p_1^s} cr_1^2}. \tag{15}$$

By Lemma 3, (15) has a solution with respect to $1/p_1^s$, if $\frac{1}{\lambda p_a cr_1^2 \mathrm{e}} \geq 1$ or $p_a \leq \frac{1}{\lambda cr_1^2 \mathrm{e}}$. This condition is satisfied if the queues are stable. Indeed, assuming the sources are backlogged, a necessary condition for the stability of the source queues is $p_a < p\mathrm{e}^{-\lambda pcr_1^2}$. Since $p\mathrm{e}^{-\lambda pcr_1^2} \leq \frac{1}{\lambda cr_1^2 \mathrm{e}}$, it follows that $p_a \leq \frac{1}{\lambda cr_1^2 \mathrm{e}}$. By Lemma 3, (14) is the solution to (15). The other solution is ignored on the basis that it is increasing in r_1. ■

The mean delay can now be computed by substituting (14) in (7).

C. Discussion on stability

In Sections III-A and III-B, we evaluated the mean end-to-end delay over a typical route, based on the assumptions that the queues approach a stationary behavior and that packet successes are independent across routes. The latter assumption is justified when the probabilities of random access p and p_r are sufficiently small [17]

Determining if and under what conditions the network attains a stationary regime, i.e., it is *dynamically stable* [7], is a complicated problem, as transmissions interfere with each other, creating couplings between different queues. However, we can determine conditions under which the network is *rate stable*, i.e., the queue backlogs remain finite over time. Rate stability is a necessary condition for dynamic stability of the network [18].

1) Backlogged sources: The following proposition establishes a necessary condition for rate stability when $p = p_r$.

Proposition 4: Assume $p = p_r$. If the relay queues are rate stable, then $r_1 > r_n$, $n = 2, \ldots, N$.

Proof: Let all the relay queues in the network be backlogged. If the relay queues are rate stable, then, from (2), it must be that

$$p\mathrm{e}^{-\frac{\lambda}{N}(p+(N-1)p_r)cr_1^2} < p_r\mathrm{e}^{-\frac{\lambda}{N}(p+(N-1)p_r)cr_n^2}, \ n = 2, \ldots, N,$$

or, equivalently, $r_1 > r_n$, $n = 2, \ldots, N$, since $p = p_r$. ■

This condition can also be deduced by demanding that the denominators in (6) are greater than zero. We now propose a sufficient condition for rate stability.

Proposition 5: Assume $p = p_r$. The relay queues are rate stable if $r_1 > \sqrt{N} r_n$, $n = 2, \ldots, N$.

Proof: Since the sources are backlogged, the success probability in the first hop is the largest when no potential interfering relay has a packet to transmit. From (2), an upper bound to the success probability in the first hop is thus $\bar{p}_1^s = \mathrm{e}^{-\frac{\lambda}{N}pcr_1^2}$. In contrast, the success probability in the second hop is the smallest when all nodes that are potential interferers in the second hop have packets to transmit. In this case, the density of interferers is $\frac{N-1}{N}\lambda p + \frac{1}{N}\lambda p = \lambda p$ and a lower bound to the success probability in the second hop is $\underline{p}_2^s = \mathrm{e}^{-\lambda pcr_2^2}$. A sufficient condition for the stability of the first relay queue is therefore $p\bar{p}_1^s < p_r\underline{p}_2^s$ or simply $\bar{p}_1^s < \underline{p}_2^s$. Defining $\underline{p}_n^s = \mathrm{e}^{-\lambda pcr_n^2}$ as the lower bound to the success probability of hop n, $n = 2, \ldots, N$, a set of sufficient conditions for the stability of *all* relay queues is

$$\bar{p}_1^s < \underline{p}_n^s \Leftrightarrow \mathrm{e}^{-\frac{\lambda}{N}pcr_1^2} < \mathrm{e}^{-\lambda pcr_n^2} \Leftrightarrow r_1 > \sqrt{N} r_n, n = 2, \ldots, N.$$

■

Since packets arrive to the queues of the initially empty relays with positive probability, the packet arrival probability to the backlogged relays progressively decreases due to increasing interference. For this reason, it appears that Proposition 5 places a strict requirement on the distance of the first hop and may be far from necessary. We conjecture that the condition in Proposition 4 is also sufficient for the rate stability of the relay queues, which is corroborated by our simulation results in Section IV.

We now let $r_2 = \cdots = r_N$ and, in a similar manner to Proposition 5, derive sufficient conditions on p and p_r such that the queues are rate stable for any value of r_1.

Proposition 6: Let $r_2 = \cdots = r_N$. The relay queues are rate stable if

$$p < \bar{p} \triangleq \begin{cases} \dfrac{W\left(\frac{r_2^2 - r_1^2}{(N-1) r_2^2 \mathrm{e}}\right)}{\frac{\lambda}{N} c (r_2^2 - r_1^2)} & r_1 \leq \sqrt{N} r_2,\ \frac{\lambda(N-1)cr_2^2}{N} \geq 1 \quad \text{(16a)} \\ \dfrac{W\left(\frac{\lambda}{N} c (r_2^2 - r_1^2) \mathrm{e}^{-\frac{\lambda(N-1)cr_2^2}{N}}\right)}{\frac{\lambda}{N} c (r_2^2 - r_1^2)} & r_1 \leq \sqrt{N} r_2,\ \frac{\lambda(N-1)cr_2^2}{N} < 1 \quad \text{(16b)} \\ 1 & r_1 > \sqrt{N} r_2 \quad \text{(16c)} \end{cases}$$

and

$$\underline{p}_r < p_r < \min\left\{ \frac{1}{\frac{\lambda(N-1)cr_2^2}{N}}, 1 \right\}, \tag{17}$$

where

$$\underline{p}_r \triangleq \frac{W\left(-\frac{\lambda(N-1)cr_2^2}{N} p \mathrm{e}^{-\frac{\lambda}{N} pc(r_1^2 - r_2^2)}\right)}{-\frac{\lambda(N-1)cr_2^2}{N}}. \tag{18}$$

The limit $\lim_{r_1 \to r_2} \bar{p}$ exists and provides the upper bound on p.

Proof: See Appendix A. ■

Proposition 6 provides an upper bound on p such that rate stability is ensured for $r_1 \leq \sqrt{N} r_2$. Additionally, the upper bound on p_r in (17) ensures that the relays do not create excessive interference by transmitting too often, thus causing the packet departure probability from the typical relay queue to become less than the packet arrival probability. The following proposition sheds more light on the relation between $\underline{p}_r$ and p.

Proposition 7: If $r_1 \leq \sqrt{N} r_2$, then $\underline{p}_r \geq p$, with the equality occuring at $r_1 = \sqrt{N} r_2$. If $r_1 > \sqrt{N} r_2$, then $\underline{p}_r < p$.

Proof: See Appendix B. ■

In other words, if the distance of the first hop is larger than $\sqrt{N}r_2$, a p_r smaller than p stabilizes the relay queues. On the other hand, if $r_1 \leq \sqrt{N}r_2$, rate stability is guaranteed if the relays transmit with a probability p_r which is larger than p.

2) Non-backlogged sources: Following a similar reasoning to the backlogged sources case, we establish rate stability conditions for the source and relay queues in the network, when $p = p_r$ (the case $p \neq p_r$ can be easily handled, but is omitted for the sake of brevity).

Proposition 8: Let p_a be the packet arrival probability to the sources every N slots, and $p = p_r$. The queues are rate stable if and only if

$$p_a < \min_{n=1,\ldots,N} \left\{ p\mathrm{e}^{-\lambda pcr_n^2} \right\}. \tag{19}$$

Proof: Similarly to Propositions 5 and 6, (19) results by demanding that p_a is smaller than the packet departure probability from any node in a typical route, when all queues in the network are backlogged. ■

Assuming that the relays are placed on the line between the source and the destination (19) implies that, for rate stability, a strict upper bound to p_a is $\bar{p}_a \triangleq p\mathrm{e}^{-\lambda pc\frac{R^2}{N^2}}$. A strict upper bound to the network throughput is therefore

$$\overline{\mathrm{NT}}_{\mathrm{nb}} = \frac{\lambda \bar{p}_a}{N} = \frac{\lambda p}{N}\mathrm{e}^{-\lambda pc\frac{R^2}{N^2}}. \tag{20}$$

This function can be optimized over N. Assuming that N can take any positive value, we easily find that $\overline{\mathrm{NT}}_{\mathrm{nb,max}} = \sqrt{\frac{\lambda p}{2\mathrm{ce}R^2}}$ and $N_{\mathrm{opt}} = \sqrt{2\lambda pc}R$. It is meaningful to now ask what is the number of hops that minimizes the delay when the system operates at a given percentage of $\overline{\mathrm{NT}}_{\mathrm{nb,max}}$. We address this issue via a numerical example in the next section.

IV. Numerical results

In this section, we present the analytical results of Section III, along with simulation results, for $\lambda = 10^{-4}$ sources/m^2, $b = 4$ and $p = p_r = 0.05$ (unless otherwise stated). The desired placement of the relays is on the line between the source and the destination, with $r_2 = \cdots = r_N = \frac{R-r_1}{N-1}$.

The simulated network size is 5000×5000 m^2, which on the average yields 2500 sources (and routes). Since our analysis determines the mean delay and throughput of a typical route, in the simulation, these metrics are evaluated as averages over time, over different routes, as well as over different network realizations. The duration of a simulation for a given network and the number of realizations are chosen in order to provide adequate statistical confidence. In order to minimize the impact of edge effects, metrics are collected only for routes whose sources are situated inside an inner square of size 1100×1100 m^2.

A. Backlogged sources

Given the desired placement of the relays on the line between the source and the destination, routing in each hop is performed by selecting among the relays that are between the respective desired point and the destination, the one that is closest to that point. This constraint ensures that, if the desired point satisfies the stability constraint, the position of the selected relay will do so as well, as it is even farther away from the source. If the last relay found is closer to the destination than r_2, then transmission is made directly to the destination. As a result, routes with a smaller number of hops than that theoretically specified may exist in the network. The average end-to-end delay is calculated over those routes as well.

We first explore the impact of the number of hops on the mean delay and the network throughput. In Figs. 2 and 3, $D_{\rm b}$ and the respective $\rm NT_b$ are plotted vs. R for different numbers of hops. For each N, the distance of the first hop r_1 is determined such that $D_{\rm b}$ is minimized. For each R, there is an optimal number of hops that minimizes $D_{\rm b}$ and the distance between the switching points (values of R for which N and $N+1$ hops yield the same delay) progressively increases. This points to a sublinear relationship between the optimal N and R (Fig. 8). Note that the optimal total service time, shown in a dashed line, provides a rather tight lower bound to the optimal end-to-end delay. This shows that, for optimally placed relays and selected N, the waiting time at each queue is small compared to the service time.

The simulation results, shown with points, correspond to a relay density $\lambda_r = 2(N-1)\lambda$. We observe that, in the range of R for which a given number of hops is optimum, the match between the theoretical and simulated delay values is satisfactory. As R increases, the dicrepancy between theory and simulation for small N becomes worse. This is due to the fact that the average number of crossings between routes increases, so bottlenecks are created at the relays which are used by more than one routes. We confirm that the discrepancy decreases with increasing relay density in Fig. 4. A related observation is that the theoretical network throughput values in Fig. 3 match the simulation ones quite well even for increasing R; the reason being that the routes with bottlenecks get very few packets across to the final destination, hence have a minimal contribution to the total end-to-end throughput.

In Fig. 5, we examine the sensitivity of the delay to the placement of the first relay, for three different pairs of values (R, N). The curves correspond to the theoretical delay obtained by (6). The simulation confirms the validity of the analysis for a relay density $\lambda_r = 2(N-1)\lambda$. The discrepancy at $r_1 = 450$ m for $R = 500$ m is caused by the existence of a large number of two-hop routes in the network, for which the delay is very large, as the - only - relay is placed very close to the destination. The optimal distance of the first hop for $R = 500$ m and $N = 3$ is approximately 250 m, i.e., half the total distance. As $r_1 \to R$, $p_2^s \to 1$ and the service time at the source dominates the delay. On the other hand, as $r_1 \to R/N$, i.e., all hops tend to become equidistant, $p_2^s - p_1^s \to 0$ and the queueing delay at the relays obtains very large values, e.g., an r_1 approximately 100 m short of the optimal value results in a large delay penalty.

In Figs. 6 and 7, we set $R = 800$ m and examine the sufficient rate stability conditions of Proposition 6. The lower and upper bounds on p_r and p, respectively, are shown in Fig. 6. Note that, for $r_1 \lesseqgtr \sqrt{N} r_2 = 320$ m, $\underline{p}_r \gtreqless p$, as indicated by Proposition 7. The respective delay curves as a function of p, along with simulation results, are shown in Fig. 7. For each curve, the value of p_r is set equal to the maximum value of $\underline{p}_r$ over p in Fig. 6. As p approaches zero, D_{b} is dominated by the service time at the source and tends to infinity. For $p > 0$, the sensitivity of the delay to p depends on the relay placement, i.e., for $r_1 = 100$ m, only a small range of p values around 0.005 yields a delay below 2000 slots. As the relay is moved farther away from the source, this range becomes wider and, for $r_1 \geq 320$ m, the delay varies less with p. Note that, for all values of r_1, D_{b} is finite as $p \to \bar{p}$, which is also verified via the simulation; this provides an indication that the sufficient conditions of Proposition 6 are stringent.

An issue of interest is to determine how D_{b} scales with R. From Fig. 2, we obtain the number of hops that minimizes D_{b} for each value of R and plot it as a function of R in Fig. 8. It is seen that this number scales *sublinearly* with R; specifically, we find that $N \approx \eta_1 R^{0.74}$, where $\eta_1 > 0$ is determined by the sublinear fit. Likewise, in Fig. 9, the optimal distance for the first hop also scales sublinearly with R and we find that $r_1 \approx \eta_2 R^{0.43},\ \eta_2 > 0$. In Fig. 10, we plot the optimal success probabilities vs. R and note that, as R increases, p_1^s progressively decreases, while p_2^s approaches a constant value $\tilde{p}_2^s \approx 0.55$. As a result, solving (9) over p_1^s for large values of R, we have the following approximation for p_1^s

$$p_1^s \approx \tilde{p}_2^s \frac{W\left(\frac{\lambda p c r_1^2}{\tilde{p}_2^s} \mathrm{e}^{-\frac{\lambda p c}{N} r_1^2}\right)}{\lambda p c r_1^2}.$$

From (6), the end-to-end delay can thus be approximated as

$$D_{\mathrm{b}} \approx \frac{\lambda c r_1^2 N}{\tilde{p}_2^s W\left(\frac{\lambda p c r_1^2}{\tilde{p}_2^s} \mathrm{e}^{-\frac{\lambda p c}{N} r_1^2}\right)} + N^2 \left(\frac{1}{p_r \tilde{p}_2^s} - 1\right). \quad (21)$$

Taking into account that $p_r \ll 1$, we have

$$D_{\mathrm{b}} \approx \frac{\lambda c \eta_1 \eta_2^2 R^{1.6}}{W\left(\frac{\eta_3 \eta_1}{\tilde{p}_2^s} R^{0.74}\right)} + \frac{\eta_1^2 R^{1.48}}{p_r}, \quad (22)$$

where $\eta_3 \triangleq \frac{\lambda p c r_1^2}{N} \mathrm{e}^{-\frac{\lambda p c r_1^2}{N}} \leq \mathrm{e}^{-1}$. Since $W(x) = \mathcal{O}(\ln x)$ for $x \to \infty$, it also holds that $W(x) = \mathcal{O}(x)$. Moreover, $W(x) = \Omega(1)$. As a result, the first term in (22) is $\Omega(R^{0.86})$ and $\mathcal{O}(R^{1.6})$. The superlinear scaling of D_{b} with R in (22) is a result of the scaling of the optimal r_1 with R, as well as the TDMA nature of the MAC protocol, i.e., the constraint that a node is allowed to transmit only once every N slots. These scaling results indicate that the MAC protocol is tailored to a small number of hops; as R increases, intra-route spatial reuse should be considered, a scenario which is beyond the scope of this paper. As a final side note, since R is present in every expression in the product $\lambda c R^2$, for a given R, we also conclude that the optimal number of hops scales with $(\lambda c)^{0.37}$.

B. Non-backlogged sources

In Fig. 11, we consider a range of R from 50 m to 1200 m and equidistantly placed relays. For each R, the number of hops and traffic load p_a that minimize D_{nb} are found, when $\mathrm{NT}_{\mathrm{nb}}$ is a certain fraction η of $\overline{\mathrm{NT}}_{\mathrm{nb,max}}$, i.e., $\frac{\lambda p_a}{N} = \eta \sqrt{\frac{\lambda p}{2 \mathrm{ce} R^2}}$. The delay is then plotted vs. the optimal number of hops, for $\eta = 0.25, 0.5 0.75$. The plot confirms that operating the network at a smaller throughput results in a smaller delay and a smaller number of hops (delay-throughput trade-off). In Fig. 8, the optimal number of hops is shown for a larger range of R and $\eta = 0.75$; in contrast to the backlogged case, the scaling is linear. This implies that the optimal p_a, thus the packet success probabilities in (15), are approximately constant over R. Taking these trends into account in (7) we can show that D_{nb} scales with N^2 (or R^2). This behavior is the result of the linear scaling of the optimal N with R, combined with the TDMA character of the MAC protocol.

V. Concluding remarks

We have conducted an analytical and simulation study of random, interference-limited, multi-hop networks. Our approach consisted of two steps: first, studying an idealized network model and deriving expressions for the mean end-to-end delay and throughput over a typical route, using tools from basic queueing theory and stochastic geometry; then, employing the obtained design insights to perform routing in a realistic network environment, where the relays are picked from a random population. We determined the number of relays and their placement such that the mean delay is minimized and derived conditions for the rate stability of the network. The simulation results confirmed the validity of the analysis for moderate to high relay densities and illustrated the usefulness of the design guidelines in minimizing the mean end-to-end delay.

A variety of interesting problems remain open, such as extending the analysis to accommodate varying source-destination distances, intra-route spatial reuse or variable rate transmission.

Appendix

A. Proof of Proposition 6

Following the line of thought in the proof of Proposition 5, a sufficient condition for the rate stability of the relay queues is

$$p\bar{p}_1^s < p_r \underline{p}_2^s \Leftrightarrow p\mathrm{e}^{-\frac{\lambda}{N}pcr_1^2} < p_r\mathrm{e}^{-\frac{\lambda}{N}(p+(N-1)p_r)cr_2^2} \Leftrightarrow p\mathrm{e}^{-\frac{\lambda}{N}pc(r_1^2-r_2^2)} < p_r\mathrm{e}^{-\frac{\lambda}{N}(N-1)cr_2^2p_r}. \tag{23}$$

The proof involves making use of Lemmas 2 and 3, in order to determine a set of acceptable values for p and p_r, such that (23) is satisfied. The function on the right hand side of (23) is $f(p_r)$ of Lemma 2, with $\mu = \frac{\lambda}{N}(N-1)cr_2^2$. By Lemma 2, (23) is possible only if

$$p\mathrm{e}^{-\frac{\lambda}{N}c(r_1^2-r_2^2)p} < \max_{p_r} f(p_r) = \frac{1}{\frac{\lambda}{N}(N-1)cr_2^2\mathrm{e}}. \tag{24}$$

We take the following cases.

- $r_1 < r_2$: let $g(p) = p\mathrm{e}^{\frac{\lambda}{N}c(r_2^2-r_1^2)p}$. By Lemma 3, (24) holds if (16a) is satisfied.
- $r_1 = r_2$: by (24), it immediately follows that $p < \frac{1}{\frac{\lambda}{N}(N-1)cr_2^2\mathrm{e}}$. This is also the limit of (16a) for $r_1 \to r_2$, which can be shown using the Lambert function property $W(x) = x\mathrm{e}^{-W(x)}$.
- $r_2 < r_1 \leq \sqrt{N}r_2$: let $f(p) = p\mathrm{e}^{-\frac{\lambda}{N}c(r_1^2-r_2^2)p}$. For this range of r_1

$$r_1 \leq \sqrt{N}r_2 \Leftrightarrow \frac{1}{\frac{\lambda}{N}c(r_1^2-r_2^2)\mathrm{e}} \geq \frac{1}{\frac{\lambda}{N}(N-1)cr_2^2\mathrm{e}} \Leftrightarrow \max_p f(p) \geq \frac{1}{\frac{\lambda}{N}(N-1)cr_2^2\mathrm{e}}. \tag{25}$$

By Lemma 2, $f(p) = \frac{1}{\frac{\lambda}{N}(N-1)cr_2^2 \mathrm{e}}$ has two solutions; a sufficient condition for (24) to hold is p being smaller than the solution given by Lemma 2. This condition is identical to (16a).

- $r_1 > \sqrt{N} r_2$: following the same steps as in (25), $f(p) < \frac{1}{\frac{\lambda}{N}(N-1)cr_2^2 \mathrm{e}}$ for any p. Since p is a probability, it suffices that $p < 1$.

We have determined sufficient constraints on p such that (24) is satisfied. By Lemma 2, a sufficient condition for (23) to hold is (17). We now need to explore if, for $\frac{\lambda(N-1)cr_2^2}{N} < 1$, there are additional constraints on p, such that $\underline{p}_r < 1$ is ensured or

$$W\left(-\frac{\lambda(N-1)cr_2^2}{N} p \mathrm{e}^{-\frac{\lambda}{N}pc(r_1^2 - r_2^2)}\right) > -\frac{\lambda(N-1)cr_2^2}{N} \Leftrightarrow p\mathrm{e}^{-\frac{\lambda}{N}pc(r_1^2 - r_2^2)} < \mathrm{e}^{-\frac{\lambda(N-1)cr_2^2}{N}}. \tag{26}$$

If $r_1 \neq r_2$, then, by Lemmas 2 and 3, (26) holds if (16b) is satisfied while, if $r_1 = r_2$, it holds if $p < \mathrm{e}^{-\frac{\lambda(N-1)cr_2^2}{N}}$, which is also the limit of the bound in (16b) when $r_1 \to r_2$. By comparing the arguments of the Lambert function in (16b) and (16a), we can show that (16b) is a more stringent condition. Moreover, by the property $W(x) = x\mathrm{e}^{-W(x)}$, the bound in (16b) is an increasing function of r_1. Since, for $r_1 = \sqrt{N} r_2$, $W\left(\frac{\lambda}{N}c(r_2^2 - Nr_2^2)\mathrm{e}^{-\frac{\lambda(N-1)cr_2^2}{N}}\right) = \frac{\lambda}{N}c(r_2^2 - Nr_2^2)$, it follows that, for $r_1 \geq \sqrt{N} r_2$, (26) holds for all $p < 1$ and (16c) is established.

B. *Proof of Proposition 7*

From (18), $\underline{p}_r$ can be written as $\underline{p}_r = \delta p$, where $\delta \triangleq \frac{W(\beta\gamma)}{\beta}$, $\beta \triangleq -\frac{\lambda}{N}(N-1)cr_2^2 p$ and $\gamma \triangleq \mathrm{e}^{-\frac{\lambda}{N}pc(r_1^2 - r_2^2)}$. We want to show that $\delta \geq 1$, when $r_1 \leq \sqrt{N} r_2$, with the equality occuring at $r_1 = \sqrt{N} r_2$, and $\delta < 1$, when $r_1 > \sqrt{N} r_2$. We take the following cases.

- $r_1 \leq \sqrt{N} r_2$ and $r_1 \neq r_2$: The following statements are equivalent due to Definition 1.

$$\delta = \frac{W(\beta\gamma)}{\beta} > 1 \Leftrightarrow W(\beta\gamma) < \beta \Leftrightarrow \beta\gamma < \beta \mathrm{e}^{\beta} \Leftrightarrow \gamma > \mathrm{e}^{\beta}. \tag{27}$$

  However, the last statement is true, as $\mathrm{e}^{-\frac{\lambda}{N}pc(r_1^2 - r_2^2)} \geq \mathrm{e}^{-\frac{\lambda}{N}(N-1)cr_2^2 p} \Leftrightarrow r_1^2 \leq N r_2^2$, with equality occuring at $r_1 = \sqrt{N} r_2$.

- $r_1 > \sqrt{N} r_2$: If $\beta < -1$, then $\delta = \frac{W(\beta\gamma)}{\beta} < 1$ as $W(\beta\gamma) \in [-1, 0)$. On the other hand, if $\beta \geq -1$, then, following the same steps as in the previous bullet, we can show that $\delta < 1$.
- $r_1 = r_2$: From (23), we can see that p and p_r must satisfy the condition $p_r > p\mathrm{e}^{\frac{\lambda}{N}(N-1)cr_2^2 p_r}$. As the exponential factor is greater than one, it follows that $\underline{p}_r = \delta p$, with $\delta > 1$.

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TABLE I: Commonly used symbols

Symbol	Meaning
p	source medium access probability (MAP)
p_r	relay MAP
p_a	probability of packet arrival at the source every N slots
λ	density of sources
λ_r	density of relays
R	source-destination distance
N	number of hops
p_n^s	packet success probability over hop n
r_n	distance of hop n

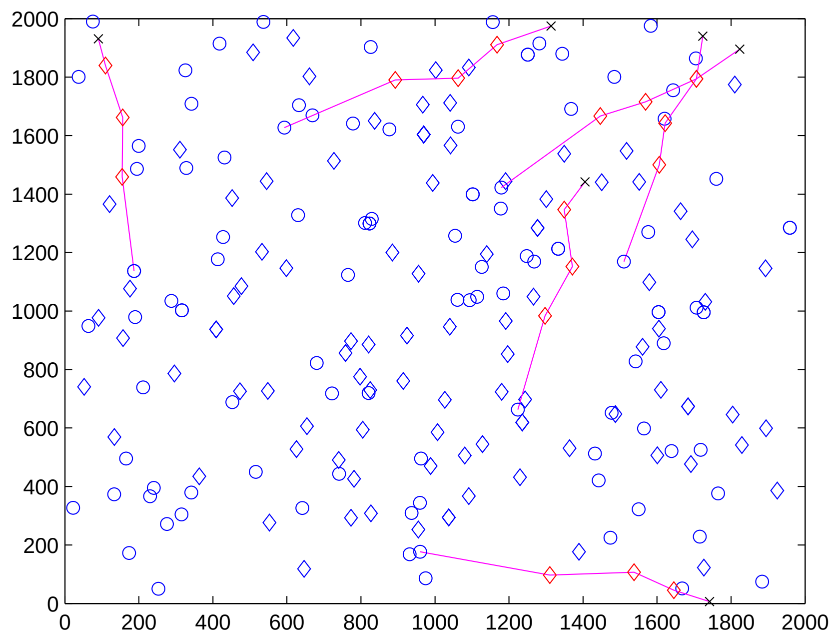

Fig. 1: A network realization where 10% of the nodes are shown ($\lambda = 10^{-4}$ sources/m^2, $\lambda_r = 3\lambda$). Sources are denoted by circles, relays by diamonds and destinations by $\times$'s. The distance to the destination is $R = 800$ m and the number of hops is $N = 4$. In order to avoid cluttering the figure, only a few routes are shown.

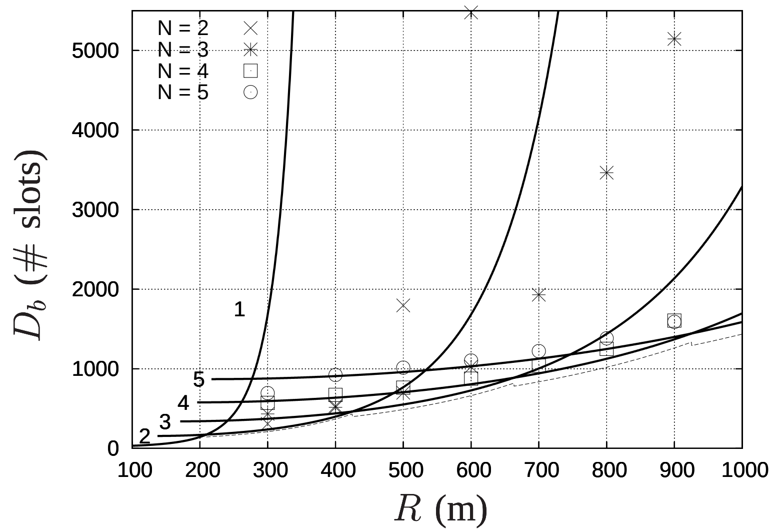

Fig. 2: D_b vs. R, for $N = 1, \ldots, 5$. The dashed line shows the optimal end-to-end service time. Simulation points are shown for $\lambda_r = 2(N-1)\lambda$. ($\lambda = 10^{-4}$ sources/m^2, $b = 4$, $p = p_r = 0.05$)

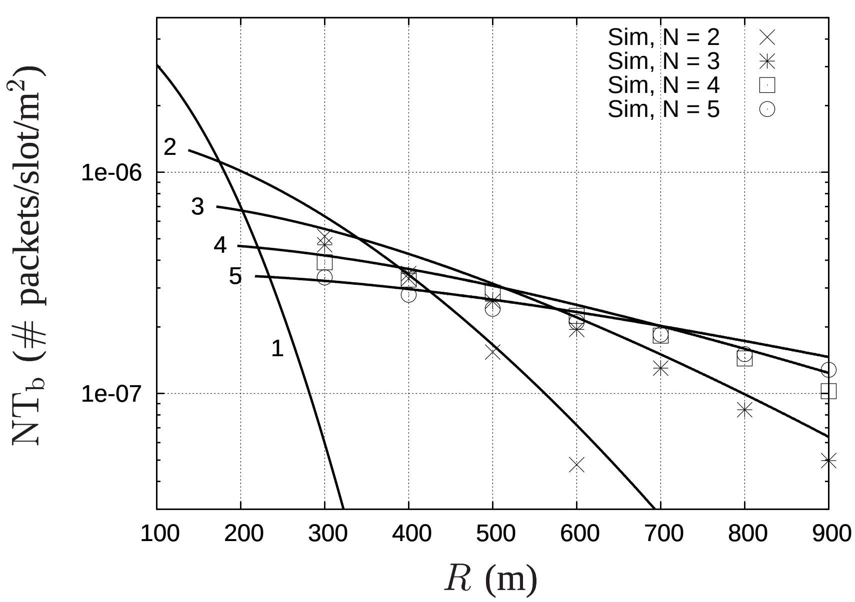

Fig. 3: $\mathrm{NT_b}$ vs. R for $N = 1, \ldots, 5$. Simulation points are shown for $\lambda_r = 2(N-1)\lambda$. ($\lambda = 10^{-4}$ sources/m^2, $b = 4$, $p = p_r = 0.05$)

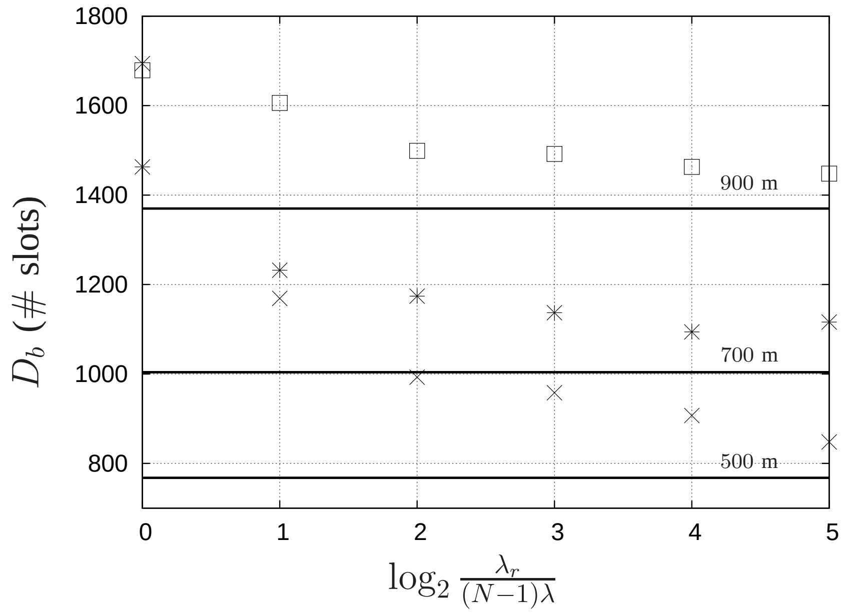

Fig. 4: D_{b} vs. $\frac{\lambda_r}{(N-1)\lambda}$, for $R = 500$ m ($N = 2$), $R = 700$ m ($N = 3$) and $R = 900$ m ($N = 4$). Simulation results are shown with points. ($\lambda = 10^{-4}$ sources/m^2, $b = 4$, $p = p_r = 0.05$)

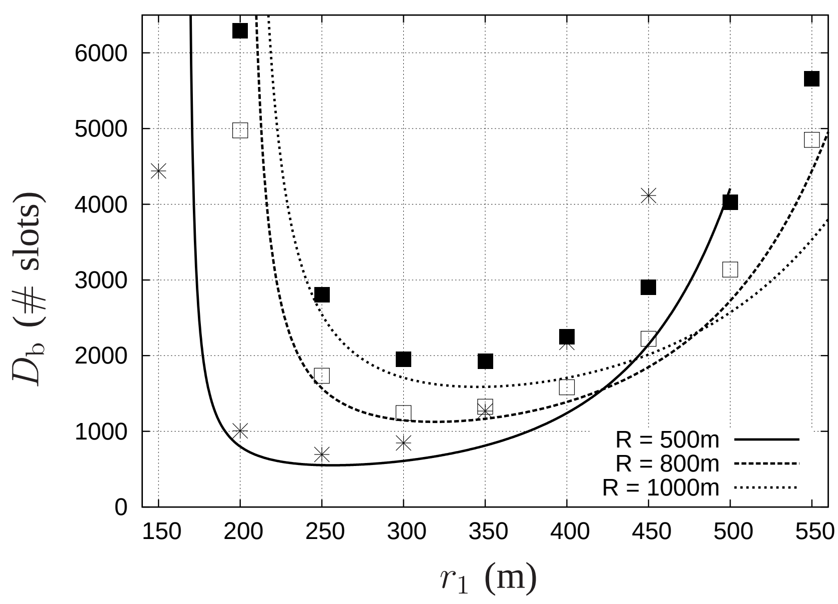

Fig. 5: D_{b} vs. r_1, for $R = 500, 800, 1000$ m ($N = 3, 4, 5$ respectively). Simulation points are shown for $\lambda_r = 2(N-1)\lambda$. ($\lambda = 10^{-4}$ sources/m^2, $b = 4$, $p = p_r = 0.05$)

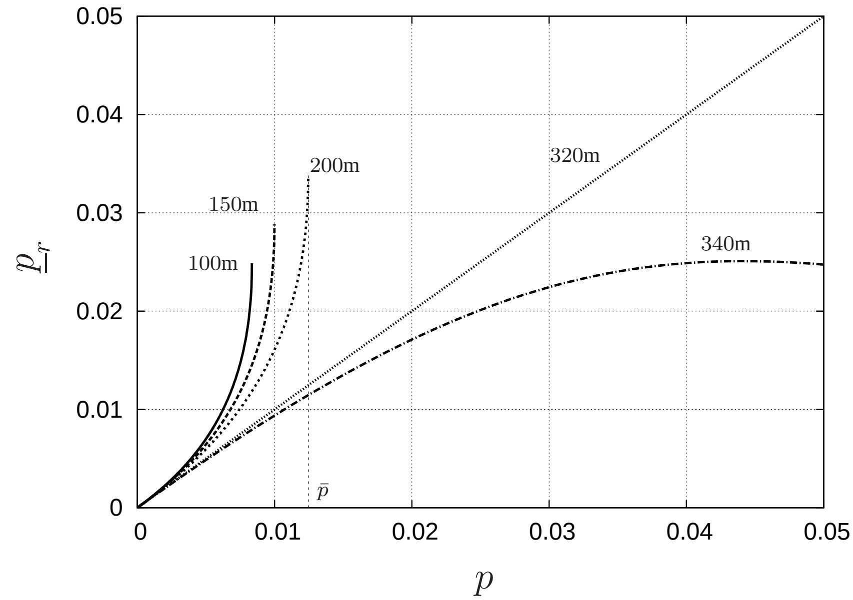

Fig. 6: $\underline{p}_r$ vs. p, for $R = 800$ m, $N = 4$ and various placements of the first relay r_1. ($\lambda = 10^{-4}$ sources/m^2, $b = 4$)

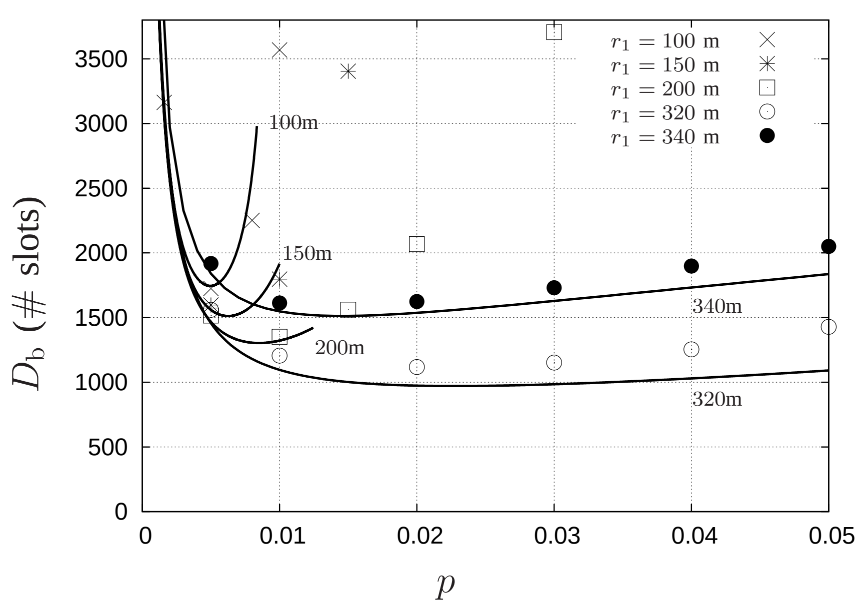

Fig. 7: D_{b} vs. p, for $R = 800$ m, $N = 4$ and various placements of the first relay r_1. Simulation points are shown for $\lambda_r = (N-1)\lambda$. ($\lambda = 10^{-4}$ sources/m^2, $b = 4$)

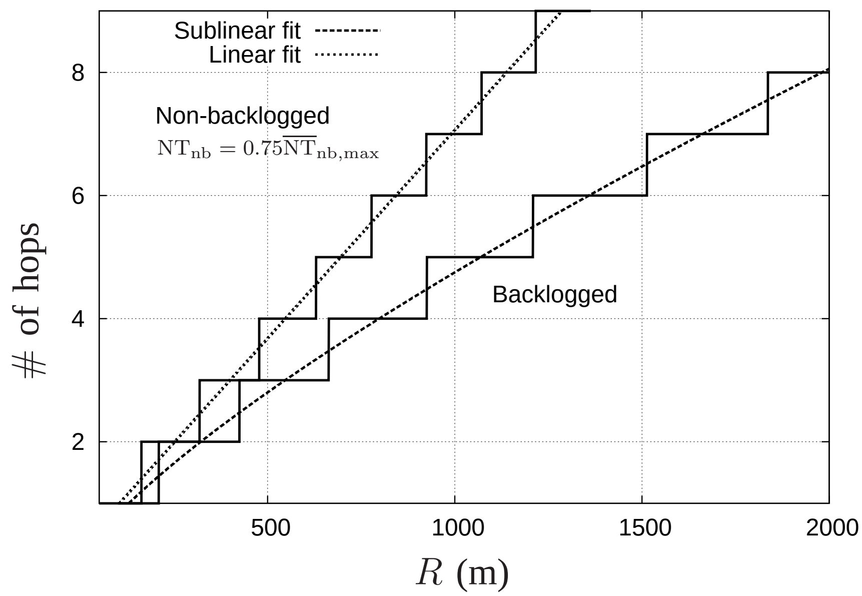

Fig. 8: Optimal number of hops vs. R for backlogged and non-backlogged sources. ($\lambda = 10^{-4}$ sources/m^2, $b = 4$, $p = p_r = 0.05$)

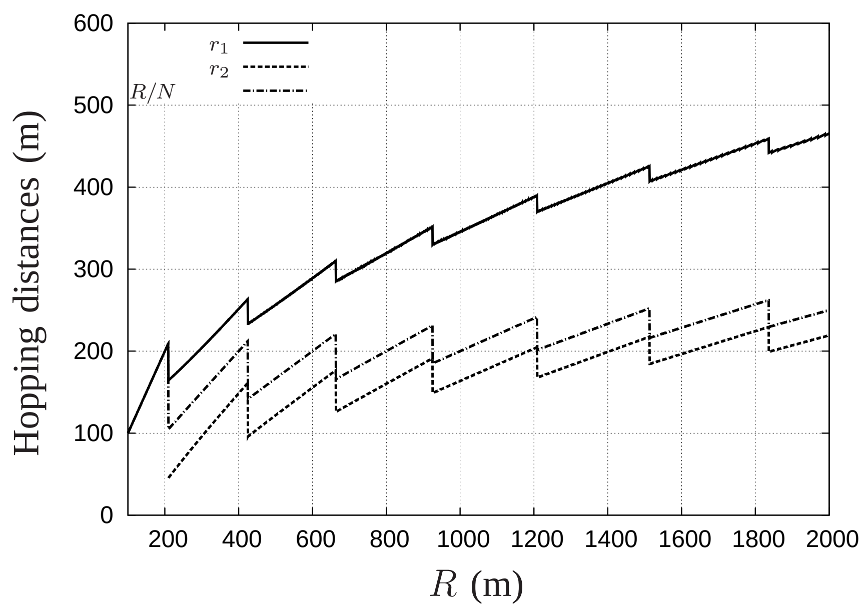

Fig. 9: Optimal hopping distances vs. R. ($\lambda = 10^{-4}$ sources/m^2, $b = 4$, $p = p_r = 0.05$)

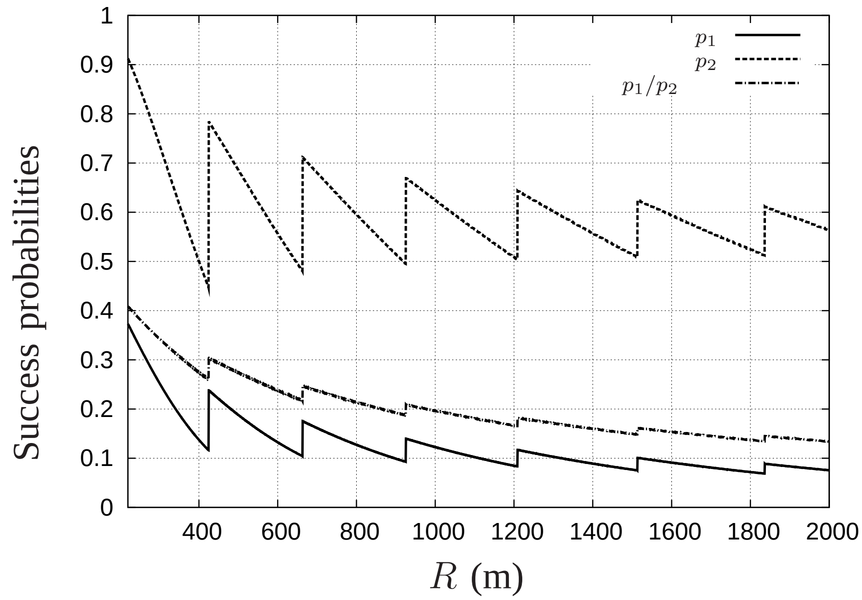

Fig. 10: Optimal success probabilities vs. R. ($\lambda = 10^{-4}$ sources/m^2, $b = 4$, $p = p_r = 0.05$)

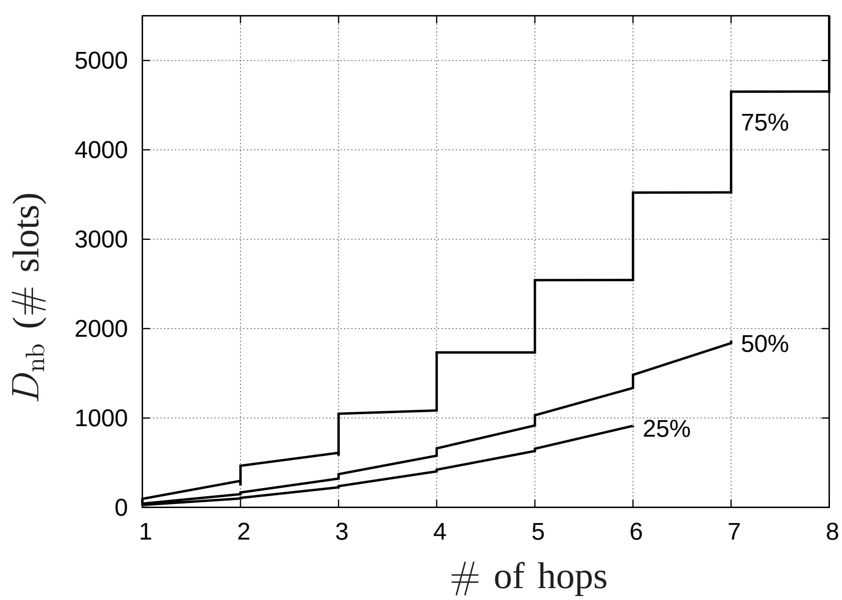

Fig. 11: $D_{\rm nb}$ vs. the optimal number of hops for different fractions of $\overline{\rm NT}_{\rm nb,max}$. ($R$ takes values from 50 m to 1200 m, $\lambda = 10^{-4}$ sources/m^2, $b = 4$, $p = p_r = 0.05$)