# On the End-to-End Delay Performance of Spatially Correlated Wireless Line Networks

Min Xie
University College London
Adastral Park Campus
Martlesham Heath, IP5 3RE, UK
Email: m.xie@adastral.ucl.ac.uk

Martin Haenggi
Department of Electrical Engineering
University of Notre Dame
Notre Dame, IN 46556, USA
Email: mhaenggi@nd.edu

Kai-Kit Wong
University College London
Adastral Park Campus
Martlesham Heath, IP5 3RE, UK
Email: k.wong@adastral.ucl.ac.uk

*Abstract* — The analytical end-to-end (e2e) performance of a wireless multihop network is largely unknown, because of the interconnections between several factors involved. Customarily, the nodes are often assumed to be spatially uncorrelated so that they can be analyzed in isolation, which is valid when all the traffic flows are independent. In practice, however, most traffic flows are correlated and cause spatial correlation among nodes. The results based on the assumption of spatially uncorrelated nodes may be far from the performance of real networks. In this paper, we aim to study the impact of the spatial correlation on the e2e delay in a wireless line network (WLN). In particular, we use queueing theory to reveal that the burstiness, the temporal correlation of the traffic flow and the underlying medium access control (MAC), together determine the spatial correlation, from which an analysis of the e2e delay of a WLN is accomplished.

## I. Introduction

The increasing demand for real-time applications over wireless networks necessitates the delay analysis of transmissions over error-prone channels. In multihop networks, the end-to-end (e2e) delay is determined by the joint distribution of the successive delays of a packet traversing multiple nodes. With network-wide traffic integration, all nodes could be assumed as independent and analyzed in isolation such that the joint distribution could be approximated in a product form [1]. However, if there exists a space-time correlation structure, it is difficult to derive the closed-form joint distribution. Here the *temporal correlation* is referred to as the correlation in two consecutive packet arrivals while the *spatial correlation* is the dependence between the activities of two nodes.

In general, the space-time correlation can be ignored under the conditions that i) the peak rate of each source does not exceed $5\%$ of the total link capacity; and ii) no more than $10\%$ of the departing sources go to the same immediate downstream link [2]. In other words, large-scale multiplexing and splitting are necessary to validate the assumption, which, however, may be impossible in networks with convergecasting (i.e., information gathering towards a central node). In an extreme case where all intermediate nodes are pure relays (Fig. 1, which is a representative of the area closer to the base station in random networks with convergecasting), the space-time correlation is too important to ignore.

The spatial correlation is mainly caused by the temporal correlation of the traffic flow, to which several factors contribute as well as the original traffic statistics. For instance,
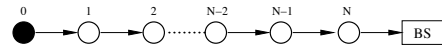
Fig. 1: Wireless Line Network.

channel errors cause distortions to the traffic flow, which, in turn, change the temporal correlation. Such distortions may be further accumulated with multihop transmission [5]. The other factor is multiple access control (MAC) that schedules the node transmission order and may incurs access delays, which certainly change the packet arrival pattern. Therefore, the study of the spatial correlation should take into account both the traffic statistics and the distortions caused by wireless channel errors and MAC.

The spatial correlation analysis is not scalable as it involves all the nodes in the network. Previous attempts usually focused on small networks, *e.g.*, two-node [3] or three-node networks [4], [5]. It was shown in [3] by simulations that a two-state Markov modulated Bernoulli (MMBP) traffic flow results in a positive spatial correlation. Earlier in [4], the correlation in a Jackson network with overtake-free paths was also proven to be positive. [5] looked at a general three-node network but assumed correlation exists only between neighboring nodes.

In this paper, we study the spatial correlation in a wireless line network (WLN) with one source (see Fig. 1). In particular, we consider three traffic models: i) constant bit rate (CBR), ii) on-off, and iii) Bernoulli, and two MAC schemes: i) $m$-phase time-division multiple-access (TDMA) and ii) persistent slotted ALOHA. Our contributions are two-fold. First, as an extension to [6] where the relayed flow convergence process was proven by entropy theory, we use queueing theory to analytically reveal that the direction of convergence is traffic- and MAC-dependent with consideration of both the queueing and access delays. Second, we explore the impact of the spatial correlation on the e2e delay. The correlation between two neighboring nodes is derived through the conditional probability of an upstream node being backlogged given a packet departure event to its downstream node. Based on the study of the direction of convergence, we further calculate the sign of the e2e correlation. The influence of the correlation on the e2e delay is examined and confirmed by the e2e delay *variance* via simulations.

## II. Queueing Theory-Based System Model

The WLN under consideration is composed of $N + 1$ transmitting nodes and a base station (BS) (Fig. 1). Denote

node $i$ by $n_i$ where a first-in, first-out (FIFO) discipline is used. A flow of fixed-length packets is generated from the source $n_0$ at rate $\lambda$ and all the remaining nodes act purely as relays. The time is slotted to the duration of one packet transmission. Assuming that all nodes are synchronized, the network can be modeled as a discrete-time queueing system.

Due to additive white Gaussian noise (AWGN) and channel fading, the wireless channels are assumed to have independent detection errors with a fixed capture rate $\mu \triangleq \Pr\{\text{SINR} \geq \Theta\}$ where SINR refers to the received signal-to-interference-plus-noise ratio and $\Theta$ is the target SINR. To guarantee 100% reliability, the failed packets will have to be retransmitted at each hop until successfully received. The number of transmission attempts to successfully send a packet follows a geometric distribution with parameter $\mu$, denoted by $\mathcal{G}_\mu$.

The traffic flow generated at $n_0$ is characterized by the interarrival time $A$ with the probability mass function (pmf) $a_k = \Pr\{A = k\}$. For the three traffic models considered, in i) CBR, the interarrival time is an integer constant $r = 1/\lambda$ with $a_k = 1$ for $k = r$ and $a_k = 0$ for $k \neq r$; ii) Bernoulli, a packet is generated with probability $\lambda$ in each time slot, i.e., $a_k = \lambda(1-\lambda)^{k-1}$; and iii) on-off, the arrival process is modulated by a two-state Markov chain that alternates between ON (1) and OFF (0) states, which is governed by the transition probabilities $a_{01}$ and $a_{10}$. The pmf is therefore given by

$$a_k = \begin{cases} 1 - a_{10} & k = 1, \\ a_{10}(1-a_{01})^{k-2}a_{01} & k > 1, \end{cases} \text{ and } \lambda = \frac{a_{01}}{a_{10} + a_{01}}.$$

The on-off source generates a stream of correlated and geometrically distributed bursty and idle periods. The mean burst size is $B = 1/a_{10}$. Note that Bernoulli is a degenerate on-off process with $a_{01} + a_{10} = 1$ and the resulting bursty and idle periods are independent. Denote the burst size of a Bernoulli source by $B_R = 1/(1-\lambda)$. Compared to this reference burst size $B_R$, an on-off source is said to be heavy or light if its burst size $B > B_R$ or $B < B_R$.

Denote the delay experienced by a packet at $n_i$ by $D_i$ with mean $\overline{D}_i$ and variance $\sigma_i^2$. The e2e delay $D = \sum_i D_i$ has the mean $\overline{D} = \sum_i \overline{D}_i$ and variance $\sigma^2$, given by

$$\sigma^2 = \sum_{i=0}^{N} \sigma_i^2 + 2\sum_{i=0}^{N}\sum_{j=i+1}^{N} \text{cov}(D_i, D_j), \quad (1)$$

where $\beta \triangleq \sum_{i=0}^{N} \sigma_i^2$, is the e2e delay variance if $D_i$'s were independent, *i.e.*, $\text{cov}(D_i, D_j) \equiv 0$ for any $i$ and $j$. However, with the spatial correlation, $\text{cov}(D_i, D_j) \neq 0$ and thus $\sigma^2 \neq \beta$. Henceforth, we regard the e2e correlation as positive if $\sigma^2 > \beta$ and negative if $\sigma^2 < \beta$.

The analysis of $D$ starts from $D_i$. Taking MAC into account, the delay $D_i$ consists of two parts, the queueing delay and the access delay. In $m$-phase TDMA, with full coordination, a node is scheduled to transmit once in $m$ time slots, and nodes, $m$ hops apart, can transmit simultaneously. Define $m$ slots as one frame. In the frame level, given the independence of channel errors, the service time is $S \sim \mathcal{G}_\mu$ and a TDMA node can be modeled as a GI/Geo/1 system. By contrast, in persistent slotted ALOHA, every node transmits independently, with a transmit probability $p_m$, *whenever it has packets*. Regarding both the access delay and the failed transmission attempts as unsuccessful transmissions, a packet

is successfully transmitted if and only if the node attempts to transmit *and* the transmission is successful, with probability $\mu_s \triangleq \mu p_m$. In other words, the service time is $S \sim \mathcal{G}_{\mu_s}$ at the slot level so that an ALOHA node can also be modeled as a GI/Geo/1 system.

Notably, in TDMA, the *arrival process* to the GI/Geo/1 system is an accumulated version of the original flow while in ALOHA, it is the *service rate* that is scaled by the ALOHA transmit probability. As such, TDMA acts like a deterministic traffic regulator that causes a distortion to the traffic flow while the influence of ALOHA lies in the service process and thus preserves the traffic statistics of the original flow. We shall show in the following sections that the distortion caused by MAC substantially affects the spatial correlation.

## III. CONVERGENCE OF RELAYED TRAFFIC FLOWS

To derive $D_i$ for $i \geq 2$, we first characterize the arrival processes at the relays. In [6], it has been shown that in a WLN of GI/Geo/1 nodes, the entropy of the relayed flows at $n_i$ increases with $i$. Since Bernoulli traffic has the maximum entropy, the relayed flows spatially converge to Bernoulli. The question remains whether all traffic flows converge in the same way regardless of their original statistics. For example, both CBR and on-off have smaller entropy than Bernoulli but they have completely different burstiness. Will they converge similarly or not?

To answer this question, we use queueing theory to discover the relationship between the direction of convergence and the traffic statistics. Characterize the departure process of $n_i$ (also the arrival process to $n_{i+1}$) by the interdeparture time $T_i$ that is composed of the packet service time $S$ and the node idle time $I$. The idle period $I$ can be derived by the delay model in [7], where the system state is the delay of the head-of-line (HOL) packet and negative states indicate the *number of idle slots*. Observe the system at the packet departure moment and let the steady-state probabilities be $\{\pi_k\}$. Then, $T_i$ is given by

$$T_i = \begin{cases} S & \text{with probability} \quad \widetilde{P}_B \triangleq \sum_{k\geq 0} \pi_k, \\ S + |k| & \text{with probability} \quad \pi_k, \ k < 0. \end{cases} \quad (2)$$

The details for the derivation of $\{\pi_k\}$ can be found in [8]. From (2), we calculate the probability generating functions (pgf) $G_{T_i}(z)$ of $T_i$ for different MAC and traffic models.

Denote $G_S(\mu, z)$ as the pgf of a geometric process $\mathcal{G}_\mu$. For *TDMA*, $T_i$ is measured in the number of frames. Due to the arrival accumulation, the average arrival rate becomes $m\lambda$, the service rate is $\mu$, and the traffic intensity is $\rho_T = m\lambda/\mu$.

- CBR—With a constant interarrival time $r < 2m$,

$$G_{T_0}(z) = \left[1 - \frac{(1-\rho_T)(1-z)}{m\lambda}\right] G_S(\mu, z), \quad (3)$$

corresponding to an on-off process with transition probabilities $a_{01}^{(1)} = \mu$ and $a_{10}^{(1)} = (r-m)\mu/m$. If $r > 2m$, the expression of $G_{T_i}(z)$ is too complex to be useful.

- On-off—With transition probabilities $(a_{01}, a_{10})$, let us denote $a_{00} = 1 - a_{01}$. The pgf can be expressed as

$$G_{T_0}(z) = \left[\widetilde{P}_B + \left(1 - \widetilde{P}_B\right)\frac{(1-a_{00}^m)z}{1-a_{00}^m z}\right] G_S(\mu, z), \quad (4)$$

$$\text{with } \widetilde{P}_B = 1 - \frac{(1-a_{00}^m)(1-\rho_T)}{m\lambda}. \quad (5)$$

For *ALOHA*, $T_i$ is measured in the number of time slots. For the GI/Geo/1 system established, the service rate is scaled to $\mu_s = \mu p_m$. Thus, the traffic intensity is $\rho_A = \lambda/\mu_s = \lambda/(\mu p_m)$. Proceeding as for TDMA, we have the pgf for

- CBR:

$$G_{T_0}(z) = \left[ \widetilde{P}_B + (1-\alpha)z \frac{\alpha^{r-1} - z^{r-1}}{\alpha - z} \right] G_S(\mu_s, z), \tag{6}$$

where $\alpha \in (0,1)$ is the unique root of the polynomial $f(x) = \mu_s x^r - x + 1 - \mu_s$ and

$$\widetilde{P}_B = \alpha^{r-1} = \rho_A - \frac{1 - r\mu\alpha^{r-1}}{r\mu}. \tag{7}$$

- On-off:

$$G_{T_0}(z) = \left[ \widetilde{P}_B + \left(1 - \widetilde{P}_B\right) \frac{(1-a_{00})z}{1 - a_{00}z} \right] G_S(\mu_s, z), \tag{8}$$

$$\text{where } \widetilde{P}_B = 1 - \frac{a_{01}(1-\rho_A)}{\lambda}. \tag{9}$$

For Bernoulli traffic, $G_{T_0}(z)$ can be obtained by plugging $a_{01} + a_{10} = 1$ into (4) and (8), respectively. From (3)–(9), we observe that the flow departs $n_0$ as bursty and correlated, regardless of the burstiness and temporal correlation of the original flow. The only exception is Bernoulli in ALOHA as it becomes a Geo/Geo/1 system. Since the departure of $n_i$ is the arrival to $n_{i+1}$, plugging the above departure characterization into $n_{i+1}$, the relayed flows of all nodes can be analyzed in the same way. However, the characterizations would be too complex and analytically intractable for large-scale networks. In order to improve the tractability, simplifications have to be done. On-off, as a special MMBP case, is fairly general and able to capture both the burstiness and the strong correlation in time, and yet being analytically tractable [9]. Therefore, we approximate the departure process as on-off. Using (3)–(8), we obtain the transition probabilities $\{a_{01}^{(i)}, a_{10}^{(i)}\}$ [where the superscript $(i)$ represents the arrival to $n_i$] of the approximated on-off process based on $a_{10}^{(i)} = 1 - \Pr\{T_i = 1\}$. For TDMA, $a_{01}^{(i)} = m\lambda a_{10}^{(i)}/(1 - m\lambda)$, and

$$a_{10}^{(i)} = \begin{cases} \dfrac{(r-m)\mu}{m}, & \text{for CBR,} \\ 1 - \mu + (1 - (a_{00}^{(i-1)})^m)\dfrac{1-\rho_T}{\rho_T} & \text{for on-off,} \end{cases} \tag{10}$$

For ALOHA, $a_{01}^{(i)} = \lambda a_{10}^{(i)}/(1 - \lambda)$ and

$$a_{10}^{(i)} = \begin{cases} \dfrac{1-\mu_s}{\alpha} & \text{for CBR,} \\ 1 - \mu_s + a_{01}^{(i-1)}\dfrac{1-\rho_A}{\rho_A} & \text{for on-off.} \end{cases} \tag{11}$$

In so doing, $n_i$ can be analyzed as a GI/Geo/1 system with on-off arrivals. The corresponding departure process can also be approximated as on-off with (10) and (11).

Fig. 2 shows the analytical results of $a_{01}^{(i)}$. In TDMA, $a_{01}^{(i)} \to m\lambda$ while in ALOHA, $a_{01}^{(i)} \to \lambda$, corresponding to the average arrival rates of the established GI/Geo/1 systems for TDMA and ALOHA, respectively. Since an on-off process with $a_{01} = \lambda$ reduces to a Bernoulli process, the relayed flows converge to a Bernoulli process, sometimes referred to as the limiting Bernoulli as in [6] by entropy theory. It is worth noting

that due to the arrival accumulation, the Bernoulli source in TDMA is not the limiting Bernoulli process while in ALOHA, the Bernoulli source itself is the limiting Bernoulli. Moreover, our analysis has revealed the following facts:

- The relayed flows converge to Bernoulli in a direction in accordance with the relative burstiness of the limiting Bernoulli. For instance, CBR and heavy on-off have different burstiness and therefore converge to Bernoulli from opposite directions.
- MAC plays an important role in determining the direction of convergence. In TDMA, the accumulated versions of both on-off and Bernoulli sources are more bursty than the limiting Bernoulli process and hence converge from the same direction. On the other hand, in ALOHA, without arrival accumulation, heavy and light on-off converge from opposite directions because of their different burstiness compared to the limiting Bernoulli.
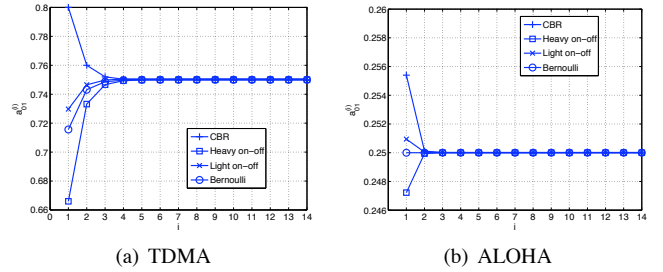


(a) TDMA      (b) ALOHA

Fig. 2: The convergence of the analytical $a_{01}^{(i)}$ to $m\lambda$ and $\lambda$ in TDMA and ALOHA networks with $N = 14$.

The dependence of the direction of convergence on traffic burstiness can be explained from the viewpoint of traffic regulation. Regard the geometric server as a Bernoulli regulator that regulates the traffic flows by randomly inserting "holes" into the arrival flows [10]. The insertion limits the maximum burstiness that the traffic flow can sustain as it traverses through the network. In the WLN, after hop-by-hop regulation, the flow is turned into Bernoulli that possesses the "natural" level of burstiness favored by the network under a given traffic load. As such, a heavy bursty flow will converge with the burstiness decreasing while a smooth flow will converge with the burstiness increasing. Notice that here the traffic burstiness is the one after MAC regulation. In other words, the direction of convergence is burstiness- *and* MAC-dependent.

## IV. DERIVATION OF THE SPATIAL CORRELATION

As shown in (1), the e2e delay variance $\sigma^2$ is determined by $\text{cov}(D_i, D_j)$, which depends on the spatial correlation between $n_i$ and $n_j$. In this section, we first study the correlation between neighboring nodes and then proceed to the e2e correlation based on the direction of convergence.

In a WLN (Fig. 1), the correlation between $n_i$ and $n_{i+1}$ can be reflected through the queueing activities of $n_i$ and $n_{i+1}$ when a packet departs from $n_i$ and arrives at $n_{i+1}$. As shown in (2), the interdeparture time $T_i$ depends on the node backlog state and the idle period. Denote the backlogged probability of $n_i$ *upon a packet departure* by $\widetilde{P}_B$ [defined in (2)] and the backlogged probability *at any moment* by $P_B(= \rho)$. It is well known that $\widetilde{P}_B \neq P_B$ if the traffic flow is temporally correlated [11]. Denote $\theta = \widetilde{P}_B - P_B$. Previous work on

queueing theory showed that for memoryless Bernoulli traffic, not only $\theta = 0$, but also there is no spatial correlation. On the other hand, for temporally correlated traffic like on-off and CBR, the spatial correlation exists and $\theta \neq 0$. Naturally, $\theta$ could be used to evaluate the spatial correlation. Between $n_i$ and $n_{i+1}$, if $\theta > 0$, upon the departure moment, $n_i$ is more backlogged than usual that will lead to increasing queueing delays at $n_i$. Meanwhile, because of the non-zero idle period, the packets depart a backlogged $n_i$ in a more bursty manner than departing an idle $n_i$. Based on queueing theory, a bursty flow results in a longer delay in $n_{i+1}$ than a smooth flow. Therefore, $\theta > 0$ indicates an increase in both $D_i$ and $D_{i+1}$, i.e., $D_i$ and $D_{i+1}$ are positively correlated. Similarly, if $\theta < 0$, $n_i$ is less backlogged at the packet departure moment than usual and $D_i$ and $D_{i+1}$ are negatively correlated. To start with, we calculate $\theta$ for $n_0$ and $n_1$ with $\tilde{P}_B$ obtained from (5), (7) and (9). For TDMA,

$$\theta = \begin{cases} -\dfrac{(r-m)(1-\rho)}{m} < 0, & \text{for CBR,} \\ (1-\rho)\dfrac{m\lambda - (1-a_{00}^m)}{m\lambda} > 0, & \text{for on-off,} \\ (1-\rho)\dfrac{m\lambda - (1-\lambda^m)}{m\lambda} > 0, & \text{for Bernoulli.} \end{cases} \quad (12)$$

In contrast to conventional queueing theory, even if the original flow is Bernoulli, spatial correlation exists and $\theta \neq 0$ since TDMA, as a deterministic regulator, destroys the memoryless property of the Bernoulli source. For ALOHA,

$$\theta = \begin{cases} -\dfrac{1 - r\mu\alpha^{r-1}}{r\mu} < 0, & \text{for CBR,} \\ (1-\rho)(1 - a_{01} - a_{10}) < 0, & \text{for light on-off,} \\ (1-\rho)(1 - a_{01} - a_{10}) > 0, & \text{for heavy on-off,} \\ 0, & \text{for Bernoulli,} \end{cases} \quad (13)$$

where $\alpha \in (0,1)$ is the root of $f(x) = \mu_s x^r - x + 1 - \mu_s$. Because the local minimum $x_{\min}$ of $f(x)$ is between 1 and $\alpha$, $f'(\alpha) = \mu r \alpha^{r-1} - 1 < 0$, leading to $\theta < 0$. As a Bernoulli regulator, ALOHA does not change the temporal correlation property and hence $\theta = 0$ for Bernoulli.

Like the direction of convergence, Bernoulli and light on-off traffic flows cause different $\theta$ in TDMA and ALOHA. In TDMA, the accumulated versions of both on-off and Bernoulli become more bursty than the limiting Bernoulli process and have $\theta > 0$. Their burstiness remain the same in ALOHA, which is consistent with $\theta < 0$ for light on-off and $\theta = 0$ for Bernoulli. In both TDMA and ALOHA, heavy on-off (resp. CBR) is always more (resp. less) bursty than the limiting Bernoulli process and thus has consistent $\theta > 0$ (resp. $\theta < 0$). Therefore, $\theta$ and the corresponding spatial correlation depend on the MAC-regulated traffic burstiness, which is a function of both MAC and the original traffic burstiness.

Similar correlations exist in $(n_{i+1}, n_{i+2})$, $(n_{i-1}, n_i)$, and so on. As a result, $n_i$ is correlated with all $n_j$'s. To determine the e2e correlation, recall that in Section III, we reveal that if the source flow is more bursty than Bernoulli, then the relayed flows will converge with the burstiness decreasing, i.e., all the relayed flows are more or equally bursty than the limiting Bernoulli. Then, all the neighboring nodes are positively correlated with $\theta > 0$. This correlation will extend to nodes more than one hop away, say $n_i$ and $n_{i+2}$, and so on so forth. Overall, the e2e correlation is positive as well. Similarly, if the source flow is smoother than Bernoulli, then the e2e correlation is negative. As a result, the sign of the correlation between $n_i$ and $n_{i+1}$ can be used as the sign of the e2e correlation.

Though we have derived the sign of the correlation, it is still difficult to explicitly derive $\text{cov}(D_i, D_j)$, especially if $|j - i| > 1$. Even in a simple tandem system of two D/M/1 nodes, the calculation involves of partitioning the state space into four parts and solving them individually [12]. Instead, we use simulation to explore the degree of the e2e correlation.

## V. SIMULATION RESULTS

In the simulations, all traffic flows have the same rate $\lambda = 0.25$ and all channels have the same probability of success $\mu = 0.8$. In TDMA, $m = 3$. In ALOHA, we let the transmit probability be $p_m = 1/m$ so that the average number of transmission opportunities and the traffic intensity $\rho$ are equal for TDMA and ALOHA. The transition probabilities $(a_{01}, a_{10})$ for heavy and light on-off are $(0.125, 0.375)$ and $(0.292, 0.875)$, respectively. Delays are measured in the number of time slots the packet stays.

### A. Convergence

We first justify the impact of traffic burstiness and MAC on the direction of convergence. Fig. 3 provides the simulated mean $\overline{D}_i$ of per-node delays in TDMA and ALOHA. The average per-node delays converge as the node index $i$ increases. The asymptotic delay means are those for a Geo/Geo/1 system with Bernoulli arrivals. Moreover, a traffic flow with a longer burst size causes a longer delay and thus converges from above while a flow with a shorter burst size converges from below, consistent with our analysis on $a_{01}^{(i)}$ in Section III.
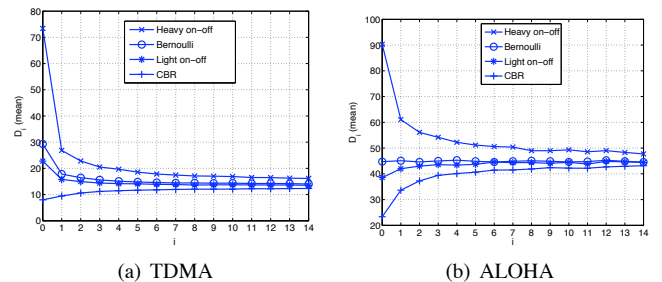


(a) TDMA        (b) ALOHA

Fig. 3: The mean $\overline{D}_i$ of single node delays at $n_i$ in TDMA and ALOHA WLN with $N = 14$.

The influence of MAC is also confirmed. In TDMA, the asymptotic value lies between the smooth CBR and the three more bursty flows [see Fig. 2(a) and Fig. 3(a)]. In contrast, in ALOHA, the original Bernoulli process itself is the limiting Bernoulli. So the asymptotic value lies between the heavy on-off and the light on-off [see Fig. 2(b) and Fig. 3(b)]. In short, as a single flow traverses multiple nodes, the relayed flows converge in a direction that highly depends on the original traffic burstiness and the MAC scheme although they will be shaped into the same Bernoulli process regardless of the original burstiness.

## B. The e2e Delay Variance

The e2e correlation is evaluated by the difference between $\sigma^2$ and $\beta$. In Fig. 4, the solid lines are for the simulated delay variance $\sigma^2$ and the dash-dotted lines represent $\beta$, the variance as if the nodes were spatially uncorrelated as assumed in the previous works. Obviously, $\sigma^2 = \beta$ occurs only when the arrival process is Bernoulli in the established GI/Geo/1 model, e.g., Bernoulli in ALOHA, Fig. 4(b). Otherwise, a gap exists between $\sigma^2$ and $\beta$. Sometimes, this gap is too large to ignore the spatial correlation, *e.g.*, for heavy on-off.
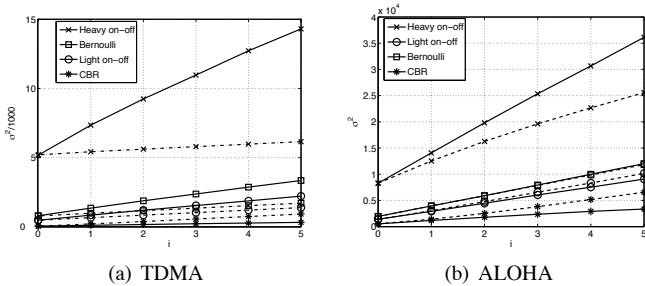


(a) TDMA          (b) ALOHA

Fig. 4: The e2e delay variance in TDMA and ALOHA WLN with $N = 5$.

In Section IV we have proven that if the source flow is more bursty (smooth) than the limiting Bernoulli, then $\theta > 0 (< 0)$ and the correlation should be positive (negative). More specifically, our analysis concludes that in TDMA, CBR results in $\theta < 0$ and negative correlation, which is confirmed by $\sigma^2 < \beta$ in Fig. 4(a) and 4(b). Similarly, all other three flows cause $\theta > 0$, meaning a positive correlation supported by $\sigma^2 > \beta$. In ALOHA, both CBR and light on-off are less bursty than Bernoulli giving $\theta < 0$ and hence $\sigma^2 < \beta$ as expected. The only flow with a heavier burstiness is heavy on-off that has $\sigma^2 > \beta$ to give $\theta > 0$. Therefore, the sign of $\theta$ is sufficient to indicate the sign of the e2e correlation. Remarkably, unlike [3], our results have revealed that with MAC, on-off, as a special MMBP flow, could give rise to both positive and negative correlations.

Smooth traffic causes not only a small per-node delay, but also a negative correlation and a decreased e2e delay variance compared to the uncorrelated case. In contrast, bursty traffic incurs both large per-node delays and a positive e2e correlation. That explains the huge gap between the e2e delay variances of CBR and heavy on-off traffic in Fig. 4, e.g., in TDMA, $\sigma^2_{\text{heavy on-off}} \approx 14\sigma^2_{\text{CBR}}$ and in ALOHA, $\sigma^2_{\text{heavy on-off}} \approx 11\sigma^2_{\text{CBR}}$. In order to guarantee the delay performance of delay-sensitive applications, the heavy bursty flow should be shaped before entering the network by traffic regulation.

Though $\theta$ itself is not sufficient to determine the degree of the e2e correlation, it still provides an insight. To show this, we define $\eta = \sigma^2/\beta$. If $\eta \to 1$, then the correlation coefficient decreases to zero. Simulation results show that for bursty traffic, $\eta$ is non-increasing while for smooth traffic, $\eta$ is non-decreasing. A similar relationship can be found in the analytical quantity $\theta' = \frac{\partial \theta}{\partial \mu}$ where $\theta'$ is decreasing and increasing with $\mu$ for bursty and smooth traffic, respectively. More importantly, the separation between $\theta'$ for different traffic models is consistent to that between $\eta$, showing a great potential of analyzing the e2e delay correlation degree by $\theta'$.

It is interesting to observe that even with the correlations, the e2e delay variance is almost linear with the number of nodes (Fig. 4). Then it is reasonable to assume that the impact of the correlations is uniform in a line network and a product-form joint distribution of all $D_i$'s could be possible. Moreover, a huge $\eta$, say $\eta > 2$, implies that strong correlations exist not only between neighboring nodes, but also between nodes that are more than one hop away [Fig. 4(a) for heavy on-off]. In this case, the assumption used in previous work that the correlation mainly exists between neighboring nodes does not hold.

## VI. CONCLUSIONS

This paper has adopted queueing theory to analyze the e2e delay variance of a WLN with spatial correlation among nodes. Using a correlated bursty on-off model to approximate the relayed flows, we have confirmed the convergence behavior in [6]. More importantly, we have revealed that although all relayed flows converge to the same Bernoulli process, they converge from different directions depending on the original traffic burstiness and the underlying MAC. The other contribution is to derive the sign of the correlation between two neighboring nodes through the parameter $\theta$, which, confirmed by simulation, is sufficient to indicate the sign of the e2e delay correlation. Furthermore, the derivative of $\theta$ can well indicate the degree of the e2e delay correlation.

Since it is the traffic burstiness that affects the e2e correlation, our work could be extended to non-linear networks with inter-flow coupling and other MAC schemes, whose influence could be regarded as "traffic regulator". For instance, multiplexing could be considered as a burstiness booster while splitting reduces traffic burstiness. The combined burstiness determines how the nodes are correlated.

## REFERENCES

[1] F. P. Kelly, "Blocking probabilities in large circuit-switched networks," *Adv. App. Prob.*, vol. 18, pp. 473–505, 1986.

[2] W. C. Lau, and S. Q. Li, "Traffic analysis in large-scale high-speed integrated networks: Validation of nodal decomposition approach," in Proc. *IEEE Info. Commun. Conf.*, pp. 1320–1329, 28 Mar.-1 Apr. 1993, San Francisco, USA.

[3] N. Ryoki, K. Kawahara, T. Ikenaga, and Y. Oie, "Performance analysis of queue length distribution of tandem routers for QoS measurement," in Proc. *IEEE Sym. App. and the Internet*, pp. 82–87, 28 Jan.-1 Feb. 2002, Nara city, Nara, Japan.

[4] H. Daduna, and R. Szekli, "On the correlation of sojourn times in open networks of exponential multiserver queues," *Queueing Systems*, vol. 34, pp. 169–181, 2000.

[5] M. Conti, E. Gregori, and I. Stavrakakis, "Large impact of temporal/spatial correlations on per-session performance measures: Single and multiple node cases," *Perf. Eval.*, vol. 41, no. 2–3, pp. 83–116, Jul. 2000.

[6] B. Prabhakar, and R. Gallager, "Entropy and the timing capacity of discrete queues," *IEEE Trans. Info. Theory*, vol. 49, no. 2, pp. 357–370, Feb. 2003.

[7] Kelvin K. Lee, and Samuel T. Chanson, "Packet loss probability for bursty wireless real-time traffic through delay model," *IEEE Trans. Veh. Tech.*, vol. 53, no. 3, pp. 929–938, May 2004.

[8] M. Xie, and M. Haenggi, "Towards an end-to-end delay analysis of wireless multihop networks," submitted to *Ad Hoc Networks*, available at http://www.nd.edu/~mhaenggi/pubs/adhoc07.pdf.

[9] T. H. Cheng, C. H. Ng, L. Zhang, and C. H. Tan, "Cell loss probability of a finite ATM buffer queue," *IEE Proc. – Commun.*, vol. 146, no. 1, pp. 9–14, Feb. 1999.

[10] N. T. Plotkin, and C. Roche, "The entropy of cell streams as a traffic descriptor in ATM networks," in Proc. *IFIP Perf. Commun. Net.*, Oct. 1995.

[11] J. J. Hunter, *Mathematical Techniques of Applied Probability*, Academic Press, Oct. 1983.

[12] C. Knessl, "An explicit solution to a tandem queueing model," *Queueing Systems*, vol. 30, pp. 261–272, 1998.