GSS Talk for November 10, 2008
Speaker: Steven Broad

Umbilics, Lines of curvature and a conjecture of Carathéodory

We will introduce umbilics and discuss the structure of the lines of curvature around generic isolated umbilics. We will also discuss a conjecture attributed to Carathéodory which states that a smooth, convex embedding of the 2-sphere into $\mathbb{R}^3$ has at least two umbilics. Finally, we will discuss the deep connection between the Carathéodory conjecture and Loewner's conjecture about the index of the vector field $\partial_{\bar{z}}^2 f$ for C^2 functions $f : \{|z| \leq 1\} \to \mathbb{R}$.