

To isometrically immerse a surface in \mathbf{R}^3 one needs to find solutions of the equation $\det(\nabla^2\theta) = (1 - |\nabla\theta|^2)K$ (of Monge-Ampere type) with $|\nabla\theta| < 1$; solutions of the same equation with $|\nabla\theta| > 1$ give isometric immersions in the Lorentz space $\mathbf{R}^2 \times i\mathbf{R}$ and cyclic systems as follows: intersect the tangent planes of the surface with a light cone, thus highlighting a circle in each tangent plane. When the surface will be immersed in \mathbf{R}^3 , the circles rigidly attached to the tangent planes will form a cyclic system, and all cyclic systems appear this way. This equation is not amenable to the classical method of integration of Monge-Ampere by finding intermediary integrals of first order, except when $K = 0$. Knowledge of such a cyclic system allowed Weingarten to reduce the linear element of a surface to a special form, and using this special form Weingarten proposed another equation, still of the Monge-Ampere type, for the isometric immersion problem. This new equation is amenable in some particular cases to the classical method of integration of Monge-Ampere by finding intermediary integrals of first order. Julius Weingarten received the prize of the French Academy of Sciences for this work.