

MATH 80370, Topics in Complex Analysis

Fall 2009

Instructor: Mei-Chi Shaw

The purpose of this course is to study problems in real and complex analysis which are related to geometry. We begin the course by studying the Hodge theory in real and complex manifolds under the geometric setting, i.e., on manifolds with positive or negative curvature. We will make this course as self-contained as possible. The topics include

1. Bochner Techniques in Real and Complex Analysis.

The so-called Bochner techniques are Hilbert space methods to study many problems in geometry. We will give a systematic discussion of the Bochner techniques starting from the classical Bochner vanishing theorem in Riemannian manifolds. These methods are also useful to study the complex Hodge theorem, the Kodaira Vanishing and Embedding theorems and embedding CR manifolds in complex spaces.

2. Hermitian and Kähler manifolds.

Curvature of Kähler manifolds. Cartan-Chern formula for Holomorphic Vector Bundles. Invariant metrics on Kähler manifolds.

3. Function theory on manifolds with positive or negative curvature.

This is an area of active research. Elliptic partial differential equations and harmonic analysis play important roles here. We will discuss some recent results in this area, including the nonexistence of Levi-flat hypersurfaces in $\mathbb{C}P^n$ and bounded harmonic functions on negatively curved manifolds.

REFERENCES

- Bochner, S and Yano, K, *Curvature and Betti Numbers*, Princeton University Press, Princeton, New Jersey, 1953.
- Chen, S.-C. and Shaw, M.-C., *Partial Differential Equations in Several Complex Variables*, American Math. Society-International Press, Studies in Advanced Mathematics, Volume 19, Providence, R.I., 2001.
- Demailly, J.-P., *Complex Analytic and Differential Geometry*, to be published by American Math. Society.
- Krantz, S., *Function Theory of Several Complex Variables*, 2nd edition, Wadsworth, Belmont, California, 1992.
- Wu, H.-H., *The Bochner technique in differential geometry*, Hardwood Academic publishers publaddr New York, 1988.
- Zheng F., *Complex Differential Geometry*, American Math. Society-International Press, Studies in Advanced Mathematics, Volume 18, Providence R.I., 2000.

Prerequisite: Basic knowledge in one complex variable and some differential geometry.