Combined Forecasts, Hidden Common Factors, and International Linkages

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Abstract

This paper suggests that, when a forecasting combination is formed with many macro series, forecasts made by simple dynamic factor models outperform both univariate models and the various forecasting combinations of bivariate models, such as equal weights, median, and trimmed mean. Furthermore, when the within-country model is expanded to consider international linkage, the forecasting improvement is possibly made on the best forecasts that are based on the domestic settings. The value of these linkages is usually stronger for output variables than for inflation, and especially holds for the United States.

Keywords: deep hidden common factor, forecasting combination, large macro data set International linkages

JEL Classification: C32, C52, C53

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1. Introduction

This paper considers forecast combinations from a large macro data set and some possible implications. We compare the performance of different forecasting models for price and output measures within country as well as across different countries. The paper builds on combined forecasts following Bates and Granger (1969) and forecasts constructed using principal components of the predictors following Stock and Watson (1998, 2002) and Forni, Hallin, Lippi and Reichlin (2000). Recently, Stock and Watson (2003a, 2003b) and Elliott and Timmermann (2002) consider the linear combination of many forecasts and find that using equal weights produces superior combinations compared to those using "optimal weights" based on regressions or portfolio theory weights using many estimated covariances. It is found that the estimation inefficiencies overcome the theoretical advantages when using a sophisticated rather than a simple approach.

The model considered here takes the form

$$Y_{t+h} = \mu + \alpha(L)Y_t + \beta(L)X_t + \varepsilon_{t+h}$$

to generate an h-step forecast. For example, Stock and Watson (2003a) consider Y_t being either price indices or real output, with two separate definitions of each, and for two distinct time periods. Data comes from seven countries. $\alpha(L)$ and $\beta(L)$ are lag polynomials, either of a fixed size, usually with four lags, or with lags chosen by the AIC or BIC model selection criteria. Stock and Watson consider about thirty different candidate explanatory variables as candidate predictors, each selected for the individual countries. They found that no individual X series provided consistently successful forecasts, compared to AR(4) forecasts, across time and country. However, an equal weighted combination of these thirty forecasts for each country was consistently superior to the AR(4) forecasts. Thus, individually the explanatory variables do not successfully forecast, but jointly they do so.

A possible explanation for this finding is that many of the explanatory variables contain a common factor, which contributes only a small amount to the variance of any such variable, but in the combination this factor shows up and helps forecast. The point can be illustrated in the following simplistic example. Denote the factor by W_t and, for a particular country, denote the explanatory variables by X_{jt}, j = 1, …, 30. Let

$$X_{jt} = c_j W_t + \widetilde{X}_{jt}$$

with

$$\text{cov}(W_t, \widetilde{X}_{jt}) = 0$$

so that

$$\text{var}(X_{jt}) = c_j^2 \text{ var}(W_t) + \text{var}(\widetilde{X}_{jt}).$$

Regressing X_{jt} on Y_t gives

$$Y = a_j X_{jt} + e_{jt}, \text{ with fit } R_j^2$$

and on the combination of forecasts gives

$$Y = \rho \sum a_j X_{jt} + \varepsilon_{jt}\ ,\ \rho = 1/30, \text{ with fit } R_C^2.$$

The Stock and Watson (2003a, 2003b) find that $R_C^2 > R_j^2$ for most j in this simplified example. However the combination is $\rho(\sum a_j c_j) W_t + \rho \sum a_t \widetilde{X}_{jt}$. If the $\widetilde{X}_{jt}$ terms are uncorrelated, the first term could have a variance comparable in size to the second. Take the simple case where all variables have zero mean, $a_j = 1$, $c_j = 1$ for all j and all $\widetilde{X}_{jt}$ have variance 1, then $\rho \sum_{j=1}^{30} \widetilde{X}_{jt}$ has variance $\frac{1}{30}$ as $\rho = \dfrac{1}{30}$, but the term $\rho(\sum a_j c_j) W_t$ has variance $\text{var}(W_t)$. Thus, there would be a factor which is useful in forecasting, but is not a large component in any of the individual explanatory variables, the X_{jt}. However, it can be brought into clearer sight via the combination.

In this paper we explore the Stock and Watson (2003a) data set directly to search for dynamic factors and compare the forecasts from models using one, two, and six of these factors with the univariate models, bivariate explanatory models, and three forms of combinations of the bivariate models, using equal-weighted means, medians, and trimmed means. Univariate and bivariate models have lags chosen either to be fixed or by AIC or BIC, and in dynamic factor models lags we selected by AIC and BIC. Overall results are reported by each of the seven countries and the four economic variables being forecast. This part of the paper covers similar ground to that covered in Stock and Watson (1998, 2002) and Stock and Watson (2003b).[1] However, the combinations considered are different, and we only consider the one-step (one-quarter) forecast horizon, and we get some clear-cut and suggestive results.

The study is expanded to consider variables from other countries. Thus, for example, the U.K. output series will use lags of outputs from the U.S., Canada, Germany, and France. Similar for prices, the U.K. price series uses lags of prices from the same set of prices. It was generally found that such linkages did produce improvements on the best forecasts from earlier sections of the paper based on the dynamic factors. Whilst it is common practice for European economists to use data from other countries, it is more unusual for US models to use this international data. We will show that there can be an advantage.

2. The Within-Country Empirical Study

The empirical results reported here follow, and expand, those of Stock and Watson (1998) and (2003a). Quarterly data for the period 1959 to 1999 is used, the period 1959 to 1970:IV is used to estimate initial forms of all models and from 1971:I onwards forecasts are made, models are updated with the arrival of each new quarter's data set and new sets of forecasts generated. Data is from seven developed countries: Canada, France, Germany, Italy, Japan, the U.K, and the U.S. The dependent variables are two output series:

[1] The first draft of this paper was written before Stock and Watson (2003b) was available to the authors.

RGDP: Real GDP and IP: Industrial Production

and two price series

CPI: Consumer Price Index and PGDP: Implicit GDP deflator.

The explanatory variables, or leading indicators, used include interest rates, term spreads, default spread, stock prices and dividend yields, and other financial indicators. The number of explanatory variables varies across countries from 30 to 63.[2] All forecast performances are judged in terms of relative mean squared error (RMSE) relative to that achieved by forecasts of an AR(4) model for the same series.[3]

Several alternative types of forecasts were formed: Univariate models of the form

$$Y_{t+1} = \alpha(L)Y_t + \varepsilon_{t+1}$$

were estimated where either $\alpha(L)$ was chosen to be of lag 4, denoted here the AR(4) or "fixed" case, or by using the AIC or BIC criteria, with an 8 maximum lag imposed.[4] All models have parameters that are progressively updated as new data is accumulated.

For the models involving leading indicators, used one at a time, take the form

$$Y_{t+1} = \mu_i + \alpha_i(L)Y_t + \beta_i(L)X_t^i + \varepsilon_{t+1}$$

for explanatory variable X_t^i. There will be between 30 to about 60 such equations, varying from country to country. The lag polynomials are either fixed at 4 lags or the lag length is chosen by AIC or BIC with a maximum length of 8. These are known here as the "bivariate models." Stock and Watson (2003a) found that the forecasts from these bivariate models were inconsistent in performance over two time periods and across countries for each of the four variables. However, Stock and Watson (2003a, 2003b) did find that combinations of the forecasts from the bivariate

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[2] Specific lists are given in Table 1(a)(b) of Stock and Watson (2003).

[3] Stock and Watson (2003a, page 808) provides simulation experiments on the statistical significance of RMSE, reporting that the 5 percent critical value ranges from 0.92 to 0.96.

[4] See Granger and Jeon (2002) for a discussion of the effect of imposing such lag restrictions.

models proved to be consistently superior.

In the empirical results reported later, three methods of combining are considered:

(i) *Simple Mean*; each forecast is given the same weight;

(ii) *The Median*; forecasts made at a particular date are ranked from the smallest to the largest. The median is the forecast at the middle of the ranking for an odd number of forecasts, or the simple average of the central two for an even number;

(iii) *Trimmed Average*; after ranking the largest and smallest forecasts are discarded and the remainder summed with equal weights. This procedure is thought to be useful when combining results from a panel of forecasts, a few of which may be of particularly poor quality.

The dynamic factor analysis or diffusion index is adopted with time-varying factor loadings that are weighted averages of the predictors, following Stock and Watson (1998). The approximate dynamic factor model places strict constraints on the joint behavior of many leading indicators, allowing one to substitute many predictors with a small number of dynamic factors.[5] This model is generated as follows. First, at time t estimate a $t \times 1$ eigenvector $\hat{\Lambda}^k$ corresponding to the kth largest eigenvalue of $X_t X_t'$ where X_t is a $t \times g$ matrix with g leading indicators, iteratively for $t = 1, \cdots, T-1$.[6] Second, calculate a $t \times 1$ matrix W_t^k, the kth factor at time t as $\hat{W}_t^k = \sqrt{t}\hat{\Lambda}^k$.[7] Then, stack the lagged dynamic factors and the lagged dependent variables with a maximum lag restriction. Finally, iterate over $t = 1, \cdots, T-1$, to produce the forecasting values:

$$f_{t+1,1}^{df} = \mu + \alpha(L) y_t + \sum_{k=1}^{K} \beta_k (L) W_t^k$$

Three different number of dynamic factor models are considered: just one dynamic factor (K=1),

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[5] Stock and Watson (1998, p.2) discuss more details.

[6] Leading Indicators are individually adjusted for outliers (that are replaced with missing values after detecting that the difference between a value and median is 20 times larger than inter-quartile range) and then standardized before either variable models or estimating approximate dynamic factors.

[7] This calculation is computationally efficient when t<g. See more on Bai and Ng (2002, page 198).

two different factors (K=1,2), and six dynamic factors (K=1,2,…,6). With the given maximum number of dynamic factors, the lag polynomials and the number of dynamic factors are simultaneously determined by either AIC or BIC while the same maximum lags of 8 are imposed on $\alpha(L)$ and $\beta_k(L)$.

If the dynamic factor model possesses smaller MSFE than both the univariate models and the combinations of bivariate models, then hidden common factors (having lags of dependent variable as well as those of dynamic factors) improve the quality of forecast. If not, idiosyncratic factors from each individual bivariate forecast provide a major component in improving the combined forecasts.

3. Results

Relative mean squared errors from one-step forecasts for these various individual and combining methods are presented in the tables. The first table shows all of the RMSE results for the four variables and seven countries, and thus has 28 rows and 18 columns corresponding to the various forecasts considered. As all are relative MSE compared to the AR(4) model, the first column is all 1.00 as the fixed model is the AR(4). Any number under 1 corresponds to a forecast that has numerically beaten the AR(4), a number over 1 is a forecast that has performed less well.

The last two rows of the table act as summaries. The first shows the mean of the numbers above it, and the last shows the number of the terms in the column that are greater than one; that is, do not perform better than a simple AR(4) forecast.

Looking just at the means, it seems that

a) For the univariate models, BIC does somewhat better than AIC and both are slightly superior to fixed;

b) In the bivariate models:

(b1) All the mean combinations beat the fixed and the univariate models. AIC and BIC are virtually identical

(b2) Using trimmed means produces slightly lower RMSE values compared to the untrimmed;

(b3) The median combinations are all superior than any other bivariate combination, with improvements in RMSE of between 8% or 9% over AR(4). AIC is better than BIC. Median forecasts produce few values over 1.0.

c) Turning to the dynamic factor models:

(c1) Using six factors leads to poorly performing forecasts; by far the worst of those investigated. Not only are the number of RMSE values over 1 high but some particularly large values occur such as 12.00, for RGDP (Italy) and 4.56 for PGDP (Germany).

(c2) Adding factors does not appear to improve forecasts as AIC2 is worse than AIC1, and similarly for BIC.

(c3) However, the one dynamic factor forecasts are spectacularly successful compared to others, particularly AIC1 with a mean of 0.77. This is giving a score of 23% over the AR(4) in terms of MSE. The dynamic factor part of the table contains eight examples of RMSE of 0.51 or less, roughly half having values below 0.70 and a highest value of 1.15, suggesting that in most cases this is a successful approach. AIC does seem to be superior to BIC in this context.

By using the same data and similar forecasting models, Stock and Watson (2003b) reach the opposite conclusions that an equally weighted combination of a large number of forecasts, obtained from different single predictors, performs well. The differences come possibly from the following reasons; (1) this paper focuses on one-quarter ahead forecasts while Stock and Watson consider 2, 4, 8-quarters ahead forecasts, (2) we construct out-of-sample forecasts over the post-1970 period, while Stock and Watson consider the post-1983 period, (3) there are minor differences in the model specifications, and (4) Stock and Watson does not report the results using only one-dynamic factor. Using multifactors in the estimation may bring multicollinearity and thus worse forecasting performance. A single dynamic factor with ample dynamics chosen by the AIC provides a better forecasting performance.

The remaining two tables summarize the information in Table 1 by considering in Table 2 the variables aggregated over the countries, and in Table 3 the results for countries by aggregation over variables.

For the bivariate models for the two price variables, the median combination (either AIC or BIC) seemed superior to either other bivariate combinations or the univariate models, but, for the two output series, the AIC trimmed mean combination did well. However, the AIC single factor model (AIC1) was clearly superior to all alternatives, beating the AR(4) RMSE by 20% for CPI and IP and over 25% for PGDP and RGDP.

In Table 3, the results for each country are displayed. The results for the combinations of bivariate models do not display many clear patterns. It is perhaps noteworthy that the median combinations have RMSE values, all below 1.0 with the single exception of Germany, the AIC1 dynamics factor model is the best for each row. For Germany, it is beaten by every BIC and AIC bivariate model.

4. Forecasts With Country Links

The models are now linked with each output series of a country having lagged outputs from some other countries, and these outputs are added to the model. Similarly for prices.[8] The following country inputs were used for each dependent variable:

[8] The more complete and systematic investigation remains in a future research agenda.

	Input Countries
Canada	US, UK, France, Japan
France	UK, US, Germany
Germany	US, UK, France, Italy
Italy	US, Germany, France
Japan	US, UK, France, Germany
UK	US, Canada, Germany, France
US(a)	UK, Germany, France, Canada
US(b)	UK, Germany, France, Japan
US(c)	Canada

In most cases what might be considered as the natural trading partners were considered amongst the seven countries in the study. The US, as the dominant economy, might be expected to be little influenced by outputs or prices in other countries, so three alternative sets of countries were considered as an experiment.

The results are shown in Table 4. As the single common factor model (AIC1) had previously been found to be generally superior, results are given relative to it. Regressions were run of the form

$$Y_t = \alpha(\text{AIC1}) + \sum_{k=1}^{q} \beta_{jk} X_{j,t-k-1} + e_t$$

where Y_t is output or price for some country, where the *X*'s are the other country's corresponding variable (i.e., output or price) with lags 1 to q. In the study, only lags 1 or 2 were used. Column 3 shows the MSE (using lag 1)/MSE (AIC1) and Column 4 shows MSE (using q =2)/MSE (AIC1). A number less than 1 suggests that an improved forecast was achieved over the previous best method. Columns 5, 6, and 7 show MSE (AIC1)/MSE (AR(4)), MSE (new using lag 1)/MSE (AR(4)), and MSE (new using lags 1 and 2)/MSE (AR(4)).

The main conclusions seem to be

i. For the countries, other than US, the output forecasts were generally improved, two lags are not clearly better than one;

ii. For these same countries, there is little evidence of improvement for the inflation forecasts;

iii. For the US, there appears to be a substantial improvement for output forecasts, but this seems to come just from the inclusion of Canada. The CPI forecasts are improved by about 9% using one lag for all sets of inputs. The results suggest that the US economy may be forecast better using external information.

iv. Columns 3 and 4 illustrate how much better most of the forecasts are than the standard AR(4). The major exceptions are Germany, Canadian prices, and Japanese PGDP. 40% of the values in these columns are 0.7 or below, suggesting that a worthwhile amount of forecastability appears to have been found.

5. Conclusions

In this study involving data from seven countries for both outputs and prices, many alternative forecasting models can be constructed using alternative explanatory variables one at a time. Combinations of these forecasts work well, particularly median combinations. Using a single common factor to represent the explanatory variables, with lags chosen by AIC, produces a generally superior model. However, introducing similar lagged dependent variables from other countries often further improves forecasts, particularly for output variables and even for the US. A systematic investigation for international linkage, including foreign factors that are found to be highly informative domestically, amongst natural trading partners still remains as a future work.

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Table 1: Relative MSE of Various Forecasting Models over AR(4)

		UNIVARIATE			COMBINATIONS OF BIVARIATE MODELS									DYNAMIC FACTOR MODELS					
					MEAN			MEDIAN			TRIMMED MEAN			AIC			BIC		
		FIXED	AIC	BIC	FIXED	AIC	BIC	FIXED	AIC	BIC	FIXED	AIC	BIC	AIC1	AIC2	AIC6	BIC1	BIC2	BIC6
cn	pgdp	1.00	1.01	0.99	1.00	1.07	0.98	0.92	0.97	0.96	0.98	1.03	0.98	0.92	0.95	1.02	1.03	1.18	1.46
cn	cpi	1.00	1.00	0.98	0.91	0.92	0.94	0.93	0.95	0.96	0.92	0.92	0.95	0.88	0.95	1.08	0.95	1.14	1.37
cn	rgdp	1.00	0.97	1.00	0.96	0.91	0.90	0.90	0.89	0.91	0.94	0.89	0.89	0.83	0.95	0.95	1.00	1.01	1.02
cn	ip	1.00	1.06	1.12	0.98	1.00	1.05	0.94	0.98	1.07	0.97	0.99	1.04	1.01	1.11	1.11	1.16	1.00	0.84
fr	pgdp	1.00	1.02	1.06	1.04	1.09	1.05	0.96	0.97	1.00	1.02	1.04	1.01	0.36	0.40	0.66	0.39	0.44	0.83
fr	cpi	1.00	0.90	0.88	1.57	1.46	1.46	0.95	0.83	0.85	1.51	1.39	1.40	0.69	0.74	0.71	0.75	0.72	1.47
fr	rgdp	1.00	1.00	0.96	0.65	0.69	0.70	0.80	0.80	0.86	0.65	0.69	0.70	0.52	0.51	0.51	0.62	0.71	0.89
fr	ip	1.00	1.06	1.03	0.86	0.90	0.91	0.85	0.91	0.94	0.86	0.89	0.91	0.68	0.72	0.74	0.71	0.69	0.78
gy	pgdp	1.00	0.86	0.84	1.09	0.86	0.85	1.06	0.84	0.84	1.08	0.86	0.85	0.75	1.12	2.10	0.76	1.12	4.56
gy	cpi	1.00	1.06	1.04	1.10	1.20	1.18	1.07	1.04	1.03	1.08	1.17	1.15	1.15	1.26	1.26	1.15	1.14	1.06
gy	rgdp	1.00	0.90	0.91	0.88	0.77	0.78	0.89	0.84	0.86	0.88	0.77	0.79	0.89	1.20	1.42	1.04	1.25	3.11
gy	ip	1.00	1.04	1.02	0.95	0.91	0.94	0.95	0.97	0.99	0.95	0.92	0.94	1.10	1.07	1.07	1.01	1.13	1.42
it	pgdp	1.00	0.91	0.91	0.81	0.78	0.80	0.88	0.81	0.84	0.82	0.78	0.81	0.66	1.25	3.94	0.66	1.61	7.76
it	cpi	1.00	1.03	1.03	0.91	1.04	0.94	0.93	0.95	0.95	0.91	1.02	0.93	0.61	0.63	0.64	0.62	0.54	1.06
it	rgdp	1.00	1.09	1.04	0.87	0.89	0.90	0.90	0.99	0.99	0.86	0.90	0.90	0.94	0.96	4.97	0.92	1.23	12.00
it	ip	1.00	0.85	0.88	0.82	0.70	0.76	0.86	0.75	0.78	0.82	0.71	0.77	0.52	0.57	1.64	0.54	0.78	2.88
jp	pgdp	1.00	1.14	0.97	0.93	0.99	0.99	0.93	1.03	0.93	0.91	0.97	0.97	1.00	1.16	1.16	1.19	1.74	1.53
jp	cpi	1.00	1.11	1.02	0.83	0.91	0.86	0.83	0.89	0.89	0.82	0.89	0.86	0.77	0.90	0.93	0.99	1.09	1.30
jp	rgdp	1.00	0.97	1.06	0.87	0.83	0.84	0.92	0.88	0.93	0.87	0.82	0.85	0.78	1.04	1.08	1.10	1.05	0.96
jp	ip	1.00	0.99	0.96	0.85	0.88	0.90	0.92	0.92	0.93	0.85	0.87	0.90	0.85	0.92	0.92	1.10	1.10	1.11
uk	pgdp	1.00	1.06	0.95	1.05	1.03	1.01	0.92	0.93	0.85	1.03	1.02	1.00	0.74	0.78	0.78	0.77	0.77	0.81
uk	cpi	1.00	0.82	0.78	1.05	0.95	0.92	0.89	0.75	0.74	1.03	0.93	0.89	0.58	0.63	0.62	0.58	0.59	0.51
uk	rgdp	1.00	0.99	0.89	0.90	0.90	0.88	0.91	0.87	0.88	0.90	0.90	0.88	0.49	0.56	0.56	0.64	0.64	0.72
uk	ip	1.00	1.06	1.02	0.87	0.87	0.94	0.92	0.92	0.99	0.88	0.88	0.95	0.78	0.80	0.80	0.86	0.98	0.98
us	pgdp	1.00	0.95	1.01	0.99	1.01	1.01	0.98	1.03	1.01	1.03	1.03	1.01	0.69	0.86	1.42	0.84	1.18	2.10
us	cpi	1.00	0.95	0.99	0.90	0.88	0.91	0.91	0.91	0.91	0.92	0.88	0.90	0.97	0.99	1.07	0.80	0.89	0.59
us	rgdp	1.00	0.91	0.97	0.92	0.85	0.85	0.91	0.83	0.85	0.91	0.83	0.84	0.76	0.78	0.78	0.83	0.73	0.96
us	ip	1.00	1.07	0.99	0.93	0.93	0.86	0.91	0.91	0.93	0.92	0.90	0.87	0.71	0.87	0.87	0.82	0.82	0.71
mean		**1.00**	**0.991**	**0.974**	**0.946**	**0.935**	**0.933**	**0.919**	**0.906**	**0.917**	**0.938**	**0.924**	**0.926**	**0.772**	**0.881**	**1.243**	**0.851**	**0.975**	**1.957**
number>1			**13**	**11**	**6**	**7**	**6**	**2**	**3**	**3**	**7**	**7**	**5**	**3**	**8**	**14**	**8**	**14**	**16**

Table 2: Relative MSE of Various Forecasting Models by Variables

	UNIVARIATE			COMBINATIONS OF BIVARIATE MODELS									DYNAMIC FACTOR MODELS					
				MEAN			MEDIAN			TRIMMED MEAN			AIC			BIC		
	FIXED	AIC	BIC	FIXED	AIC	BIC	FIXED	AIC	BIC	FIXED	AIC	BIC	AIC1	AIC2	AIC6	BIC1	BIC2	BIC6
pgdp	1.00	0.993	0.962	0.987	0.976	0.955	0.950	0.938	0.919	0.981	0.962	0.947	0.731	0.931	1.580	0.806	1.150	2.720
cpi	1.00	0.980	0.960	1.040	1.050	1.030	0.931	0.904	0.905	1.030	1.030	1.010	0.806	0.872	0.904	0.835	0.874	1.050
rgdp	1.00	0.973	0.974	0.865	0.833	0.836	0.890	0.870	0.896	0.858	0.828	0.836	0.743	0.856	1.470	0.878	0.945	2.800
ip	1.00	1.020	1.000	0.893	0.881	0.909	0.905	0.911	0.947	0.890	0.879	0.911	0.808	0.863	1.020	0.885	0.929	1.250

Table 3: Relative MSE of Various Forecasting Models by Countries

	UNIVARIATE			COMBINATIONS OF BIVARIATE MODELS									DYNAMIC FACTOR MODELS					
				MEAN			MEDIAN			TRIMMED MEAN			AIC			BIC		
	FIXED	AIC	BIC	FIXED	AIC	BIC	FIXED	AIC	BIC	FIXED	AIC	BIC	AIC1	AIC2	AIC6	BIC1	BIC2	BIC6
cn	1.00	1.010	1.020	0.963	0.973	0.967	0.924	0.948	0.975	0.952	0.959	0.964	0.910	0.993	1.040	1.040	1.080	1.170
fr	1.00	0.994	0.982	1.030	1.030	1.030	0.889	0.880	0.913	1.010	1.000	1.010	0.563	0.591	0.654	0.619	0.640	0.994
gy	1.00	0.964	0.953	1.000	0.932	0.938	0.992	0.921	0.929	0.996	0.926	0.933	0.972	1.160	1.460	0.989	1.160	2.540
it	1.00	0.967	0.967	0.851	0.853	0.850	0.891	0.874	0.890	0.849	0.850	0.850	0.683	0.850	2.800	0.682	1.040	5.910
jp	1.00	1.050	1.000	0.869	0.899	0.897	0.898	0.930	0.920	0.861	0.888	0.895	0.848	1.000	1.020	1.100	1.250	1.230
uk	1.00	0.981	0.910	0.966	0.938	0.939	0.909	0.868	0.865	0.961	0.930	0.931	0.646	0.691	0.688	0.713	0.747	0.756
us	1.00	0.970	0.987	0.934	0.916	0.907	0.930	0.919	0.927	0.945	0.911	0.905	0.781	0.874	1.040	0.822	0.905	1.090

Table 4: Forecasts With Country Links

		RMSE over AIC1		RMSE over AR(4)			Input Countries
		lag 1	lags 1&2	AIC1	lag 1	lags 1&2	
cn	pgdp	0.93	0.96	0.92	0.86	0.89	US,UK,FR,JP
cn	cpi	1.10	1.11	0.88	0.97	0.98	
cn	rgdp	0.71	0.61	0.83	0.59	0.51	
cn	ip	0.80	0.80	1.01	0.81	0.81	
fr	pgdp	1.02	1.16	0.36	0.37	0.42	UK, US, GY
fr	cpi	1.04	1.12	0.69	0.72	0.77	
fr	rgdp	0.80	0.82	0.52	0.41	0.43	
fr	ip	0.93	0.86	0.68	0.63	0.59	
gy	pgdp	0.93	1.09	0.75	0.70	0.82	US,UK,FR,IT
gy	cpi	1.10	1.16	1.15	1.26	1.33	
gy	rgdp	1.11	1.17	0.89	0.98	1.04	
gy	ip	0.88	0.88	1.10	0.96	0.97	
it	pgdp	1.12	1.51	0.66	0.74	0.99	US, GY,FR
it	cpi	1.12	1.19	0.61	0.68	0.73	
it	rgdp	0.79	0.92	0.94	0.74	0.87	
it	ip	0.92	0.87	0.52	0.48	0.45	
jp	pgdp	1.00	1.14	1.00	1.00	1.14	US,UK,FR,GY
jp	cpi	1.16	1.15	0.77	0.89	0.89	
jp	rgdp	1.03	1.00	0.78	0.81	0.78	
jp	ip	0.96	0.92	0.85	0.82	0.78	
uk	pgdp	0.92	0.72	0.74	0.68	0.53	US,CN,GY,FR
uk	cpi	1.12	1.07	0.58	0.65	0.62	
uk	rgdp	0.89	0.98	0.49	0.43	0.48	
uk	ip	0.92	0.98	0.78	0.72	0.77	
us	pgdp	0.96	0.92	0.69	0.67	0.63	UK,GY,FR,CN
us	cpi	0.88	0.97	0.97	0.85	0.94	
us	rgdp	0.57	0.58	0.76	0.43	0.44	
us	ip	0.66	0.69	0.71	0.47	0.49	
us	pgdp	0.90	1.04	0.69	0.62	0.72	UK,GY,FR,JP
us	cpi	0.92	0.98	0.97	0.89	0.95	
us	rgdp	0.99	1.06	0.76	0.75	0.81	
us	ip	0.92	0.97	0.71	0.65	0.69	
us	pgdp	1.02	1.01	0.69	0.70	0.70	CN
us	cpi	0.93	0.93	0.97	0.90	0.90	
us	rgdp	0.67	0.65	0.76	0.51	0.50	
us	ip	0.63	0.55	0.71	0.44	0.39	