Answer Key 1

MATH 10560: Calculus II **Practice Exam 3.2** *November 27, 2007*

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this page. There are 8 multiple choice questions worth 6 points each and 4 partial credit problems worth 10 points each. You start with 12 points. On the partial credit problems try to simplify your answer and indicate your final answer clearly. *You must show your work and all important steps to receive credit.*

You may not use a calculator.

1. [•] b c d e
2. a b c [•] e
3. [•] b c d e
4. a b [•] d e
5. a b c d [•]
6. [•] b c d e
7. a b c [•] e
8. a [•] c d e

MATH 10560: Calculus II **Practice Exam 3.2** *November 27, 2007*

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this page. There are 8 multiple choice questions worth 6 points each and 4 partial credit problems worth 10 points each. You start with 12 points. On the partial credit problems try to simplify your answer and indicate your final answer clearly. *You must show your work and all important steps to receive credit.*

You may not use a calculator.

1. [a] [b] [c] [d] [e]
2. [a] [b] [c] [d] [e]
3. [a] [b] [c] [d] [e]
4. [a] [b] [c] [d] [e]
5. [a] [b] [c] [d] [e]
6. [a] [b] [c] [d] [e]
7. [a] [b] [c] [d] [e]
8. [a] [b] [c] [d] [e]

1. Use the Binomial Series to expand $\sqrt{1+x^2}$ as a power series.

   (a) $1 + x^2/2 - x^4/8 + x^6/16 - 5x^8/128 + \cdots$

   (b) $1 - x^2/2 + 3x^4/8 - 5x^6/16 + 35x^8/128 + \cdots$

   (c) $1 + x/2 - x^2/8 + x^3/16 - x^4/128 + \cdots$

   (d) $1 - x/2 + 3x^2/8 - 5x^3/16 + x^4/128 + \cdots$

   (e) $1 + x^2/2 + x^4/8 + 3x^6/16 + 5x^8/128 + \cdots$

2. Deteremine which of the following series are convergent.

   (I) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{n^8+3}$ (II) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^3-1}}$ (III) $\displaystyle\sum_{n=2}^{\infty} \frac{5}{n\ln(n)}$

   (a) *none* (b) (I), (II), & (III) (c) (II) & (III)

   (d) (I) & (II) (e) (I) & (III)

3. Determine the third-degree Taylor polynomial, $T_3(x)$, for $f(x) = x + \dfrac{1}{x}$ at $x = 1$.

   (a) $2 + (x-1)^2 - (x-1)^3$ (b) $2 - (x-1) + (x-1)^2/2 - (x-1)^3/3$

   (c) $2 - 2(x-1) + 2(x-1)^2 - 2(x-1)^3$ (d) $2 + 2(x-1)^2 - 6(x-1)^3$

   (e) $2 + (x-1)^2/2 + (x-1)^3/3$

4. For which values of k is the series $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2n}{kn+7}\right)^n$ convergent?

   (a) $|k| < 7$ (b) *none* (c) $|k| > 2$ (d) $|k| < 2$ (e) $|k| > 7$

5. Evaluate $\sum_{n=1}^{\infty} \left(\frac{n}{n+1} - \frac{n+1}{n+2} \right)$.

(a) $-1/6$ (b) *diverges* (c) $1/3$ (d) 0 (e) $-1/2$

6. Estimate the maximum error of approximating $\sin(x)$ by its fourth-degree Taylor polynomial, $T_4(x)$, on the interval $[-\pi/2, \pi/2]$.

(a) $\frac{\pi^5}{2^5 \cdot 120}$ (b) $\frac{\pi^4}{2^4 \cdot 24}$ (c) $\frac{\pi^3}{48}$ (d) $\frac{\pi}{2}$ (e) $\frac{\pi^2}{24}$

7. Find the limit of the sequence defined recursively by $a_1 = 1$ and $a_{n+1} = 1 + \frac{1}{a_n}$.

(a) ∞ (b) 1 (c) 0 (d) $(1+\sqrt{5})/2$ (e) $(1-\sqrt{5})/2$

8. Use power series to calculate the limit $\lim_{x \to 0} \frac{e^{-x^2/2} - \cos(x)}{x^4}$.

(a) *does not exist* (b) $1/12$ (c) $1/8$

(d) $1/24$ (e) $1/3$

9. Estimate the series $\sum_{n=1}^{\infty} \frac{1}{n^4}$ to within $\frac{1}{300}$. You may leave you answer in the form of a (finite) sum, but be sure to explain your reasoning and state any theorems you apply.

10. Determine whether the series $\sum_{n=0}^{\infty} \frac{(-1)^{n+1} n}{n^2+1}$ is absolutely convergent, conditionally convergent, or divergent.

11. Find the radius of convergence and the interval of convergence for the series $\sum_{n=1}^{\infty} (-1)^n \frac{(x+8)^n}{n6^n}$.

12. Evaluate the indefinite integral $\int \frac{x^3}{1+x^{10}}\, dx$ as a power series.