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A Reexamination of Economies of Scale in Banking Using a Generalized Functional Form

*A Note by Bernard J. Kilbride, Bill McDonald,
and Robert E. Miller*

Using a sample of 1,205 unit banks for the years 1972–77, Clark (1984) addresses a number of critical issues in ascertaining the nature of the cost function underlying the banking industry. The generalized functional form methodology used by Clark is ideal for empirically analyzing these questions. This paper addresses three issues evolving from Clark's study. First, a number of econometric problems that were not addressed by Clark can arise in the application of the functional form method. Since the findings of Clark have major implications for the question of scale economies in banking, it is important to verify that these findings are not an econometric artifact. Second, and more importantly, the major modifications that have occurred in the structure of the banking industry during the past few years suggest that the corresponding operating parameters may have changed substantially from those in Clark's 1972–77 sample. Finally, previous literature has debated the relative scale economies of independent banks versus multi-bank holding company (MBHC) affiliates. The methodology of Clark provides a unique opportunity to extend previous research on this issue.

1. Methodology and Estimation

The specification of the basic cost function as developed by Clark is given by

$$C_i^{(\lambda_0)} = \beta_0 + \beta_1 Q_i^{(\lambda_1)} + \beta_2 W_i^{(\lambda_2)} + \beta_3 r_i^{(\lambda_3)} + \beta_4 p_i^{(\lambda_4)} + \mu_i, \quad (1)$$

where C , w , r and p are, respectively, the average of total operating expenses, the price of labor, the price of loanable funds, and the price of real capital inputs. The measure of output used corresponds to Clark's Q_3 , which is the unweighted sum of the bank's earning assets. Clark shows that the results are not sensitive to alternative means of measuring output.

Data for the study were provided by the Jesse H. Jones Research Data Base. The manuscript has benefitted from substantive comments by the editor and two anonymous reviewers.

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The transformation parameter is defined as in Box and Cox (1964), with the log-likelihood function given by

$$\ln L = -\frac{N}{2} \ln \sigma^2 - \frac{1}{2\sigma^2} \sum_{i=1}^N \mu_i^2 + (\lambda_0 - 1) \sum_{i=1}^N \ln C_i. \quad (2)$$

The interpretation of the estimated transformation parameters in the context of the current model is critical. If the parameters are significantly different from zero, the logarithmic form of the traditional Cobb-Douglas specification is not appropriate. In addition, the implied elasticity of substitution and scale economies under these conditions need not be constant for all levels of output. The latter point is especially important, in that even if economies of scale do exist in banking, there is little reason to believe that they are the same for all levels of output. The empirical results of Clark suggest that for the time period he analyzed, the assumption of constant elasticities and a Cobb-Douglas functional form was generally acceptable (the logarithmic model could only be marginally rejected and the elasticities exhibited very little variation). Clark's estimate of the output elasticity is less than one, and indicates statistically significant scale economies.

In estimating the transformation parameters, Clark assumes each variable is transformed by the same λ . (He reports difficulties in estimating the more general specification.) The generalized model of equation (1) does not restrict the elasticities of each cost component to be the same. In addition, if the measure of one cost component is poorly approximated, its impact on the estimated transformation of other components is minimized. The importance of extending the model to multiple transformations can be tested by comparing the likelihood value of equation (1) with a restricted model where $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4$.¹

The first sample examined parallels that of Clark, and is based on all unit banks with complete data for the 1979–83 time interval.² This sample consists of 1,737 unit banks with total assets in nominal dollars ranging from \$1.2 million to \$10.1 billion (versus Clark's sample, which ranged from \$7 million to \$425 million). The other two samples considered are independent banks and MBHC affiliates with complete data for 1979–83. The sample of independents contains 1,858 banks with total assets in nominal dollars ranging from \$1.2 million to \$1.6 billion. The sample of MBHC affiliates includes 1,036 affiliates with total assets in nominal dollars of \$3.2 million to \$54 billion.

During the period of Clark's study, commercial banking was on the threshold of a vast number of changes which would permanently alter the banking environment. The changes which took place from 1972 to 1977 were not only relatively few in number but also, by comparison with those occurring after 1977, quite modest. Perhaps the most notable change in the earlier period was the introduction of NOW accounts by Massachusetts mutual savings banks in 1972. Regulatory, competitive, and financial product changes increased not only in number after 1977 but also in

¹A more general specification that allows for heteroskedasticity (as in Lahari and Eby 1981) was also tested, since the presence of heteroskedasticity can bias the estimate of the transformation parameter. This extension was not found to be statistically significant; therefore, its derivation and results are not reported.

their impact on the banking environment. The rate of evolution increased substantially in 1977 when brokerage firms offered the public cash management accounts, followed in 1978 by the availability of money market CDs at depository institutions, and the enactment of the Depository Institutions Deregulation and Monetary Control Act of 1980. In the latter period, the vast number of regulatory and competitive developments, combined with higher rates of inflation, more volatile money markets, and higher interest rates, sharply narrowed net interest margins of commercial banks.

The parameters for the various models are estimated using maximum likelihood techniques. Clark reports using iterative grid search techniques to estimate the model, which implies that the ordinary-least-squares (OLS) covariance matrix was used to test the significance of the estimated coefficients. As shown in Spitzer (1984) and detailed in Blackley, Follian, and Ondrich (1984), these estimates seriously understate the standard error. To avoid this problem in the current study, asymptotic standard errors for the regression parameters are estimated from the Cramer-Rao lower bound.³

2. Unit Bank Results

Parameter estimates for the fully generalized model of equation (1) and various restricted forms of that model using the unit bank sample appear in Table 1. In general, the magnitude and sign of the coefficients are very similar to those of Clark. The *t*-statistics of the GFF2 model reported by Clark appear to be overstated which, based on the findings of Blackley et al. (1984), would be expected if OLS estimates were in fact substituted. As in Clark, the coefficient for real capital inputs (*p*) is not significant and the coefficient for labor (*w*) is significantly negative. The R^2 for the logarithmic model was 0.983 which again is comparable to Clark's results.

The comparative significance of each specification is examined using the likelihood ratio test.⁴ The relevant likelihood ratio tests for each of the sample groups considered in this study are reported in Table 2. The series of comparisons for the sample paralleling Clark's (all unit banks) provides the following conclusions: The comparison of the logarithmic (LOG) and two-transformation functional form (GFF2) models indicates that the extension of the Cobb-Douglas (LOG) model to the simplified functional form is statistically significant. In addition, comparison of the two-transformation model (GFF2) to the more general model where each variable is

²The data are taken from the Report of Income and Report of Condition provided by the FDIC. To average the 1979-83 data, all dollar-based units were expressed in thousands and denominated in 1967 dollars.

³The Cramer-Rao lower bound is given by $-E(\partial^2 L / \partial \theta \partial \theta')^{-1}$, where $\theta' = \{\beta_0 \dots \beta_4, \lambda_0 \dots \lambda_4\}$. A quasi-Newton method is used to estimate the likelihood function in order to minimize the tendency of first-derivative-based routines to overestimate the standard error. In Spitzer's (1984) tests, this effect appears to be negligible. Spitzer also notes that although the standard errors of equation (3) are correct, they are not generally scale-invariant. To assure that the significance tests in this study are not simply an artifact of scaling, the scale-invariant form of all functional form models tested was also estimated. (Although the modified model has the advantage of scale invariance, the form of the elasticities is restricted.) The *t*-statistics for the scaled models were in general within plus or minus 10 percent of the unscaled version, with the exception being a few cases where the scaled *t*-statistic was substantially larger.

⁴Although Clark compares the linear and logarithmic models, a direct comparison is not appropriate since neither specification corresponds to a restricted form in relation to the other.

TABLE I
ESTIMATED COST FUNCTIONS: Unit Banks ($N = 1737$)

Restrictions (from Equation 1)	Specification			
	Linear	Logarithmic	GFF2	GFF5
	$\lambda_i = 1 \forall i$	$\lambda_i = 0 \forall i$	$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4$	—
Parameter Estimates				
Intercept	-929.123 (-3.69)	-3.471 (-51.50)	-3.793 (-15.72)	-2.986 (-12.64)
$Q(\lambda_1)$	0.113 (430.69)	0.958 (275.69)	0.623 (23.11)	0.632 (10.99)
$w(\lambda_2)$	-12.222 (-0.56)	-0.103 (-5.91)	-0.173 (-5.52)	-0.002 (-0.67)
$r(\lambda_3)$	106.443 (3.53)	0.820 (28.29)	1.364 (14.62)	0.465 (7.13)
$p(\lambda_4)$	1.089 (1.09)	-0.003 (-0.56)	-0.002 (-0.30)	-0.000 (-0.07)
λ_0	—	—	0.124 (9.48)	0.127 (6.29)
$\lambda_1(Q)$	—	—	0.139 (10.09)	0.140 (5.78)
$\lambda_2(w)$	—	—	0.139 (10.09)	2.528 (3.11)
$\lambda_3(r)$	—	—	0.139 (10.09)	0.673 (4.65)
$\lambda_4(p)$	—	—	0.139 (10.09)	3.032 (1.16)

Note: t -statistics are in parentheses.

transformed separately (GFF5) indicates that the two-transformation model is not sufficient. Given these results, the subsequent discussion will concentrate on the comparison between the traditional Cobb-Douglas form and the GFF5 model.

Insight into the question concerning economies of scale in banking is provided by examining the unit bank output elasticities for the LOG and GFF5 specifications. The output elasticity at the mean for the GFF5 model is 0.958, which is equivalent to the estimated elasticity for the log model. The average elasticities for both models are significantly less than one and similar to those reported by Clark for the 1972–77 sample. If the elasticity of output is not constant across all levels of output, then the industry could be characterized by large economies of scale for smaller firms and

TABLE 2
LIKELIHOOD RATIO TESTS

Test	df	Critical Value ^a ($\alpha = 0.05$)	Observed Value				
			MBHC			Independent	
			Unit	Unit	Branch	Unit	Branch
GFF2:LIN	2	3.00	4075.56	762.88	2187.73	490.87	1029.32
GFF2:LOG	2	3.00	43.60	11.74	3.65	27.61	41.23
GFF5:GFF2	3	3.91	9.53	4.89	13.18	9.81	11.77

^aBased on a likelihood ratio test, where the difference between the log-likelihood of one model versus its restricted counterpart is distributed as $1/2\chi^2(r)$, where r is the number of restrictions.

diseconomies for large firms, and still have an average elasticity of one. The significance of the GFF5 model is the implication that the elasticities are not constant. The average cost per unit of output is plotted in Figure 1. The results indicate that a constant elasticity of substitution model is not appropriate and that the nature of the scale economies is a function of the level of output. The most notable difference from Clark's study revealed in Figure 1 is the finding that potential economies of scale are decreasing with size. (Clark's Table 6 indicates they are increasing.) The finding that scale economies are favorable for smaller banks is consistent with the results of Gilligan, Smirlock, and Marshall (1984) and Benston, Hanweck, and Humphrey (1982).

3. Cost Functions and Bank Structure

A number of studies have examined the economies of scale for MBHCs versus independent banks (see Frieder and Apilado [1982] for a synthesis of this literature.) The functional form methodology provides a unique means to examine this question. The MBHC and independent bank samples are subdivided into unit and branch banks, thus providing four groups for comparison.⁵ The likelihood ratio tests for each of these sample groups (reported in Table 2) indicate that in each case the GFF5 model dominates the restricted counterpart. For this reason, only the GFF5 model will be considered in subsequent tests.

Parameter estimates for the four sample subsets appear in Table 3. As in the unit bank tests, the parameters and significance for w , r and p are not consistent across specifications. Aggregate output elasticities and elasticities disaggregated by level of output are presented in Table 4. The results suggest notable differences among the

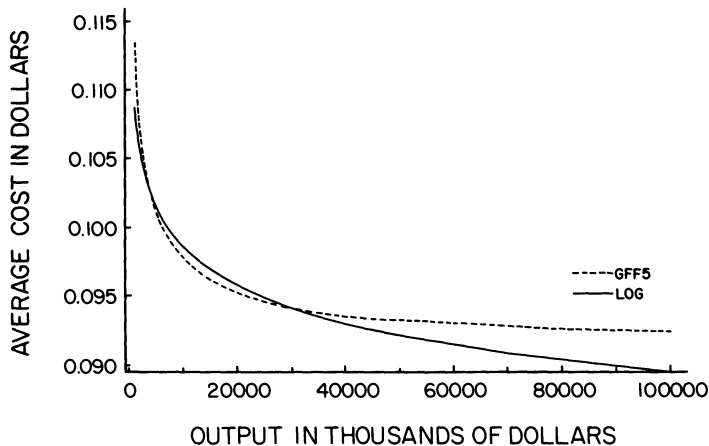


FIG. 1. Average Cost per Unit for the GFF5 and LOG Models (all unit banks).

⁵For both the MBHC and independent bank samples, the hypothesis that the cost functions were equivalent for unit and branch banks was rejected at the 0.05 level. The MBHC subsamples contain 297 unit banks and 739 branch banks. The independent bank subsamples contain 803 unit banks and 1,055 branch banks.

TABLE 3
ESTIMATED COST FUNCTIONS FOR THE INDEPENDENT BANK AND MBHC SAMPLES (GFF5 SPECIFICATION)

Type	Int.	Q	w	r	P	A ₀	A ₁	A ₂	A ₃	A ₄
MBHC										
Unit	-1.612 (-2.95)	0.629 (7.68)	-0.000 (-0.49)	0.102 (1.23)	0.000 (0.26)	0.103 (2.47)	0.122 (2.77)	7.233 (6.77)	1.175 (2.51)	2.694 (3.51)
Brch	-3.848 (-28.13)	0.936 (33.44)	0.000 (0.01)	1.509 (46.92)	0.089 (3.88)	-0.005 (-0.94)	0.002 (0.31)	-17.250 (-42.39)	-0.600 (-22.59)	-0.355 (-3.54)
Ind										
Unit	-1.703 (-3.67)	0.456 (8.49)	-0.003 (-0.54)	0.105 (4.55)	-0.005 (-0.59)	0.205 (7.45)	0.232 (7.66)	2.378 (2.78)	1.700 (11.96)	0.653 (1.62)
Brch	0.771 (1.60)	0.489 (15.92)	-2.419 (-3.15)	0.038 (1.44)	0.000 (0.18)	0.093 (4.51)	0.139 (7.00)	-2.419 (-6.20)	1.717 (5.24)	2.972 (2.63)

Note: t-statistics are in parentheses.

TABLE 4
AVERAGE OUTPUT ELASTICITIES AT THE MEAN BY LEVEL OF OUTPUT: MBHC'S AND INDEPENDENT BANKS (GFF5 SPECIFICATION)

Level of Output	MBHC		Independent Banks	
	Unit	N	Unit	N
\$0 < Q ≤ \$5M	0.925	38	0.981	12
\$5M < Q ≤ \$10M	0.942	62	0.985	78
\$10M < Q ≤ \$25M	0.962	111	0.991	183
\$25M < Q ≤ \$50M	0.977	42	0.996	194
\$50M < Q ≤ \$100M	0.987	22	1.000	118
\$100M < Q ≤ \$500M	0.998	18	1.009	107
\$500M < Q ≤ \$17B	1.036	4	1.024	47
All	0.975	297	1.007	739
			Unit	N
			0.905	352
			0.936	253
			0.955	167
			0.974	25
			0.990	5
			0.995	1
			—	—
			0.938	803
			Branch	N
			0.874	90
			0.910	219
			0.947	437
			0.981	193
			1.011	83
			1.052	32
			1.114	1
			0.966	1055

four organizational types. Consistent with the findings of Drum (1978), the aggregate output elasticities indicate greater scale economies for independent banks. The disaggregated data, however, indicate that the advantage of independent banks dominates in output levels less than \$25 million. In the range of \$25 million to \$50 million this advantage is less clear, and for banks with output levels greater than \$50 million, MBHCs appear to minimize diseconomies of scale. The general pattern for both samples is consistent with Benston et al. (1982), indicating that economies of scale are predominant only for banks with output levels of less than \$50 million. In addition, for both groups, unit banks consistently experience higher scale economies than branch banks.

Figure 2 presents the average cost function for each sample subgroup. In addition to highlighting the previous results, the graph reveals that MBHC branch banks experience virtually no scale economies, and their overall cost per unit of output is higher.

4. Conclusions

This paper provides an extension of the functional form analysis used by Clark to examine economies of scale in banking. The empirical results suggest that:

- a. The econometric limitations of Clark's analysis do not appear to materially affect the results. Differences reported in this paper are apparently attributable to the more recent sample and the inclusion of larger banks.
- b. Unlike the earlier sample of Clark's, the functional form transformation is significant for the 1979-83 samples of unit banks, independent banks, and holding company affiliates. The use of separate transformations on the dependent variable and each of the independent variables was also found to be significant in comparison to a two-transformation model.

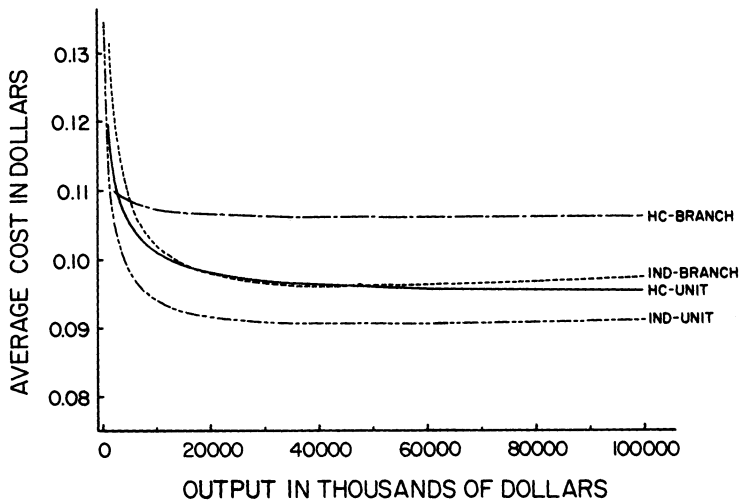


FIG. 2. Average Cost per Unit for Sample Subgroups.

- c. In contrast to Clark, the level of scale economies was found to be a decreasing function of the level of output for each of the three samples. The MBHC organizational form appears to be advantageous relative to scale economies only for banks with output levels greater than \$50 million. Scale economies are notably different for unit versus branch banks.
- d. The significance of the functional form models and the parameter estimates in each of the samples indicate that the scale economies are not constant. The sample estimates suggest a broad range of elasticities including both substantial economies and diseconomies.

The banking environment has experienced substantive changes in the past few years. The results of this study vis-à-vis those of Clark suggest that these changes have influenced the underlying cost structure. Future research should examine the dynamics of this process in more detail and determine the sources of the observed economies and diseconomies.

Data for this paper are available from the JMCB editorial office.

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