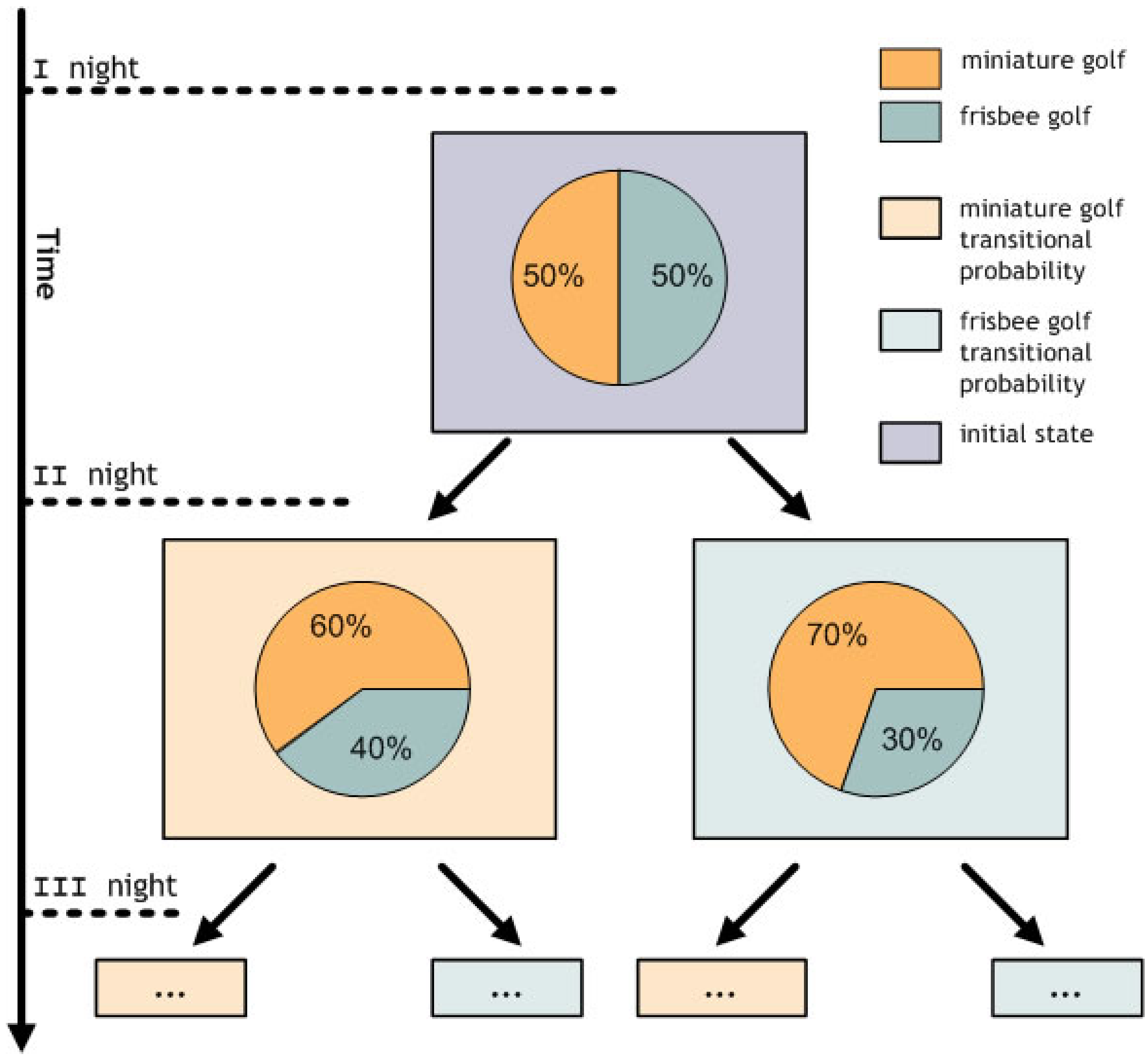
This section allows you – the student – to either print a copy of your work through this tutorial or email it to your instructor. Your teacher will inform you which format you should submit; either the filled out Word document or a printout of the pdf file with your comments written by hand. As for the Powerpoint form; this is available for teachers to use as an introduction to both the topic (such as this tutorial) or to the Notre Dame tutorial web site. Please feel free to download any of these to your own computer.

Sam and Joan are two teenagers living in a small town, Nowhere, Indiana. In this town there are only two outdoor evening entertainment centers, a Putt-Putt and a Frisbee Golf. Not having much to do in the evenings, Sam and Joan regularly hang out at one of these places. They begin to notice that there are regular customers, but also that some customers switch from one place to the other. Intrigue by this, they decide to do an analysis. At first they try to follow particular people and record their habits, but this is too hard even in the little town. So they decide to keep track of how many people repeat and how many change in each of the two places. They enlist several of their (also bored) friends and after many nights of observations, they arrive at the following: 60% of those attending Putt-Putt on any given night also go there the next night. At Frisbee Golf, 70% of those playing on a given night also play golf the next night.



Sam and Joan decide that this information might be useful to the owners of the two entertainment centers. On approaching them, Sam and Joan are offered a job if they could predict how many people will attend on future nights. With the hope of making money, they take the challenge.

Sam and Joan begin with assuming that there are 100 people total who attend both entertainment centers on any given evening, and they start with one night when they are 50 in each.

1. How many people are at Putt-Putt the first night?

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2. How many people are at Frisbee Golf the first night?

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3. How many people who were at Putt-Putt the first night are also at Putt-Putt the second night?

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4. How many people who were at Frisbee the first night are at Putt-Putt the second night?

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5. How many people are at Putt-Putt the second night?

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6. How Many people who were at Frisbee the first night are at Frisbee the second night?

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7. How many people who were at Putt_putt the first night are at Frisbee the second night?

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8. How many people are at Frisbee the second night?

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9. Your answers to questions 5 and 8 add up to

They decide to predict for night number three. Now they have new numbers, those from night number 2.

10. How many people are at Putt-Putt on night 3 who were there on night 2?

11. How many people are at Putt-Putt on night 3 who were at Frisbee on night 2?

12. How many people are at Putt-Putt on night 3?

13. How many people are at Frisbee on night 3 who were there on night 2?

14. How many people are at Frisbee on night 3 who were at Putt-Putt on night 2?

15. How many people are at Frisbee on night 3?

16. Your answers to 12 and 15 add up to

One of the answers this time is 38.5 people, which doesn't make sense. However, this can be interpreted as a probability of .385. In addition, the 60% in the story can be interpreted as the conditional probability of a person being at Putt-Putt on a night, given that they were there the previous night.

Let us analyze this example. First of all, our **model** involved a **mathematical process**. We did something. Second, our information was **probabilistic**. The information we obtained did not tell where a specific individual will eat on any given day. It did tell us the probability of someone changing restaurants (or not). A process based on probability is called a **stochastic** process. Third, it was convenient to represent this information in a square matrix, called a **transition matrix**. We could then use matrix multiplication to get from one day to the next. We used the word **state** to describe the situation on a given day. This too was a matrix, a column matrix. **Time stepping** occurs because we asked the question in **discrete** time intervals (days) rather than continuously. Fourth, in this example we eventually reach a state in which subsequent moves do not change the state matrix. When this occurs, we say that we have a **steady state** (or an equilibrium state). This is actually a dynamic equilibrium: the probabilities are constant although individual people could still be changing restaurants. Fifth, there is another property in this example which is important but is easily overlooked. To predict what will happen on the next day, we only need the transition matrix and the current situation – no previous days' information is necessary. The process "has no memory". This property is what makes the process **Markov**. This is most evident in Sam's recursive approach. We summarize by giving the following definition.

Definition: A mathematical process that is stochastic, discrete, and has the property that the probability that a particular outcome occurs depends only on the previous outcome is called a Markov chain.

Notes.

It is important to keep a record of your work (and not just for evaluation by your instructor). Good record-keeping will allow you to refer back to your notes and make it easier to cement the basic concepts in your mind . . . where you will likely draw them out at the appropriate time.

Finally, submit one copy of the document to your teacher and keep a personal copy in your laboratory notebook/logbook.