

Barwick - Curves in K-thy, the return

Note Title

4/17/2009

Recap:

Idea: $K(-): \mathcal{B}/s \rightarrow \mathrm{Sp}_{\geq 0}$

is a Green functor

Green functors $/s \rightsquigarrow$ Grand Witt Complexes
 ΩC_*

Remark:

$$\begin{array}{c} X \\ \downarrow G \\ s \end{array} \text{ étale Galois } \Rightarrow \text{ set germs } G\text{-spectra}$$

$K(x)$

$\mathcal{B}^{\mathbb{Z}} := \mathcal{B}_{\mathbb{Z}}^{\text{op}}$

↑ spectral cat

↓ finite \mathbb{Z} -sets

spectral category

↑

\mathcal{B}^{Φ}

↑ finite directed \mathbb{Z} -set

$\Phi =$ poset of positive integers by divisibility

$$M: \mathcal{B}^{\Phi} \rightarrow \text{Ab}$$

$$W_n \in \text{Ab} \quad n \geq 1$$

$$m|n \quad F_{m|n}: W_n \rightarrow W_m$$

$$m|n \quad V_{m|n}: W_m \rightarrow W_n$$

Sit. -----

$$F_{e|n} \circ V_{m|n} = \binom{n}{\text{lcm}(e,m)} V_{\text{gcd}(e,m)} | e \circ F_{\text{gcd}(e,m)|m}$$

$$e|m|n$$

If M is a green functor

- $F_{m|n}(xy) = F_{m|n}(x) F_{m|n}(y)$

- $V_{m|n}(F_{m|n}(x) \cdot z) = x V_{m|n}(z)$

Def: $R = m_3$

① Grand Witt coo / R is

a comm Green functor
(E_{oo})

$$A: \mathcal{B}^{\pi} \rightarrow Sp_{\geq 0} \quad \text{spectral functor}$$

via connection, and an isomorphism

$$\pi_0 A \rightarrow W(R)$$

of Green functors $\pi_0 \mathcal{B}^{\pi} \rightarrow Ab$

② The left initial object

of coo of Grand Witt coo 's

is called the Grand de Rham-Witt
 coo_e

$$W\Omega_R$$

$$\pi_0 W\Omega_R = \text{big de Rham Witt } coo_e$$

Concept: = bunch of functions $A_{\langle n \rangle}$
together w/ restriction
maps =

$\{W_{\langle n \rangle}(R)\}_n$ or Restriction
gives $W(R)$

Claim: \mathcal{I} core algorithmically
from the fact that
 $W\Omega_R$ is spectral
valued.

Thm: (Bloch - Hesselholt) $R = \text{regular}$
 \mathbb{Z}/s

$$\Omega_{GK}(R) \cong W\Omega_R$$

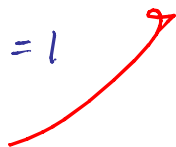
If $R = \mathbb{Z}_p$ - sly both sides given

$$\prod_{(p,k)=1} p\text{-typical}(\Omega_{C_*K(R)}) \simeq \prod_{(p,k)=1} p\text{-typical}(W^{(p)}\Omega_R)$$

$(p,k)=1$



$(p,k)=1$



equivalent factors

tree ab for $X = \text{reg}/s$

Make sense of $C_*K(R)$

Rank

$$A: B/s \longrightarrow Sp_{30}$$

given X/s

get $C_*A(X): B^{\mathbb{Z}} \longrightarrow Sp_{30}$

In fact

$$B/s \longrightarrow \text{Mack}(B^{\mathbb{Z}}, Sp_{30})$$

Def: A concrete σ_n a
 Mackey functor

$$M: \mathbb{B}^{\mathbb{Z}} \longrightarrow S_{p \geq 0}$$

is a compatible sequence
 of equivalences of Mackey
 functors

$$\rho: \beta_{n_1}! M \xrightarrow{\cong} M$$

$$\beta_n: \mathbb{B}^{\mathbb{Z}} \longrightarrow \mathbb{B}^{\mathbb{Z}}$$

$$X \longmapsto n_1 X$$

$$\mathbb{Z} \xrightarrow{\cong} \mathbb{Z}$$

$$\begin{array}{ccc} X & & n_1 X \\ \downarrow & \swarrow & \\ \text{Sets} & & \end{array}$$

$$\mathbb{Z}/m\mathbb{Z} \longmapsto \mathbb{Z}/n_1 m \mathbb{Z}$$

left Kan
 extn

"Think induction"

$$\beta_{n,!} : \text{Mack}^{\mathbb{Z}} \longrightarrow \text{Mack}^{\mathbb{Z}}$$

← enriched left Kan extension.

$$\Phi \longrightarrow \left\{ \mathcal{S}_{p \geq 0} \text{ - categories} \right\}$$

$$m \longmapsto \text{Mack}^{\mathbb{Z}}$$

$$m/n \longmapsto \beta_{n/m,!}$$

Formal construction of cat of modly functors w/ correction

Compatible

$$\text{Ob } \Phi \longrightarrow \Phi$$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ * & \longrightarrow & C\Phi \end{array}$$

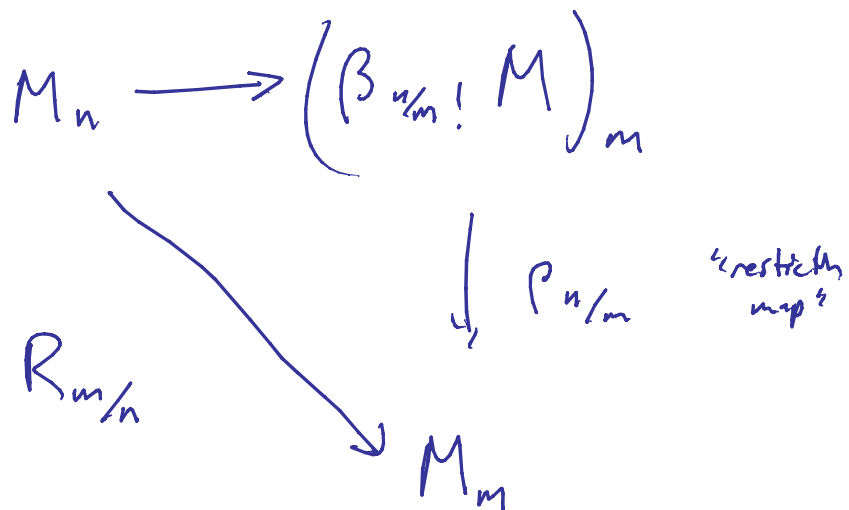
$$\beta : C\Phi \longrightarrow \left\{ \text{Spectral cats} \right\}$$

$$\text{when } \beta =: \text{Mack}_{\text{com}}^{\mathbb{Z}}$$

Rank: can find a semi-nodal cut of Mackey matrix of connections, can do no better.

[Conj: this is same as cyclic spectral?]

m/n



Claim: These are restriction maps

A Green Subalgebra w/ Connection,

$$\rho: \mathcal{B}_{n,1} M \longrightarrow M$$

is an evaluation
of Green subalgebras

Rank cannot extend this

to \mathcal{B}_s

due to finiteness
issues,

Curves Functor:

B/s :

$$\mathbb{H}_n : B/s \longrightarrow B/s$$

$$X \longmapsto X[t]/t^n$$

$$P_X : X[t]/t^n \longrightarrow X$$

$$A : B/s \longrightarrow Sp_{\geq 0}$$

$$T^n A := \mathbb{H}_n^* A$$

$$T^n A \xrightarrow{p} A$$

p gives such a map
of Green functors

$$C_n A \longrightarrow T^n A \xrightarrow{p} A$$

m/n:

$$\phi_{m/n, X} : X[u] / u^n \longrightarrow X[t] / t^m$$

$$u^{n/m} \longleftarrow t$$

$$\phi_{m/n} : \textcircled{+}^n \longrightarrow \textcircled{+}^m$$

get:

$$\phi_{m/n}^\# : T^m A \longrightarrow T^n A$$

morph of
green functors

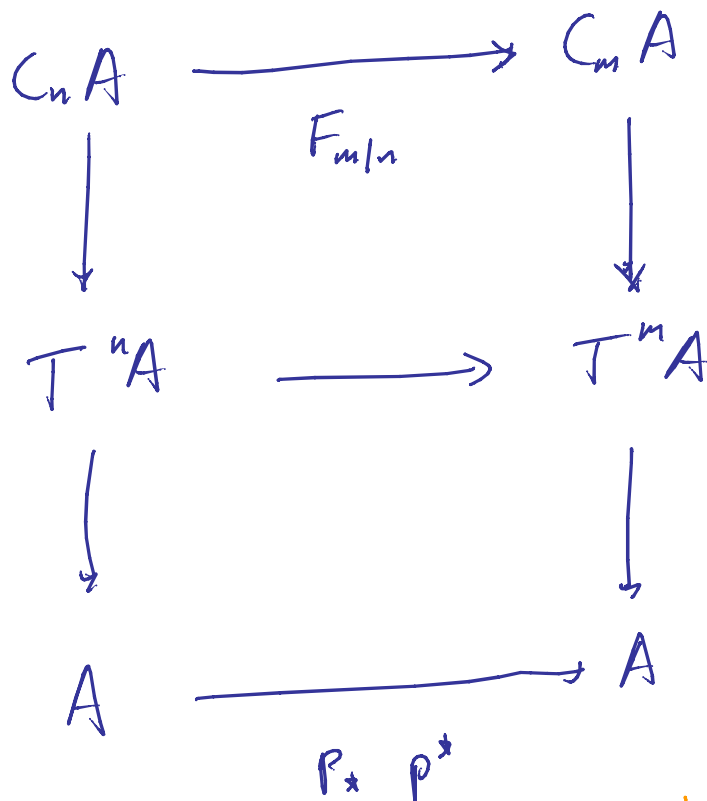
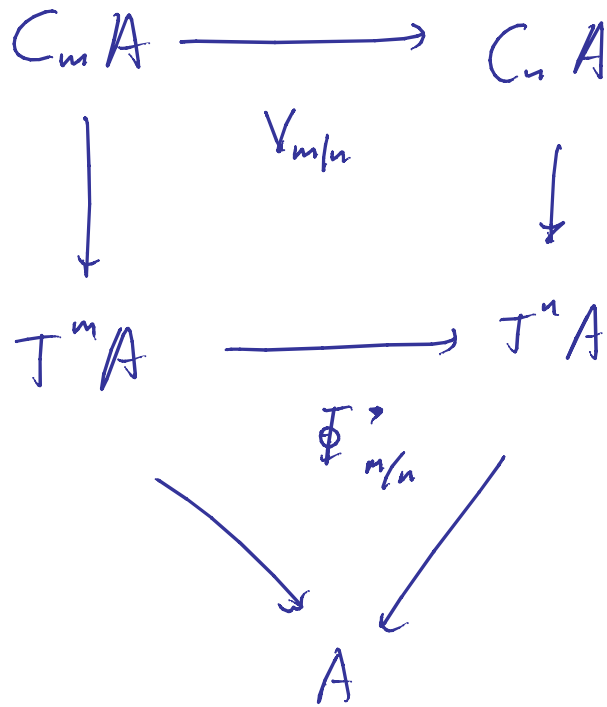
$$(\phi_{m/n})_* : T^n A \longrightarrow T^m A$$

morph of
 $T^n A$ -modules

$$\begin{array}{ccc}
 T^m A & \xrightarrow{\phi_{m/n}^*} & T^n A \\
 \downarrow & & \downarrow \\
 A & \xlongequal{\quad} & A
 \end{array}$$

$$\begin{array}{ccc}
 & (\phi_{m/n})_* & \\
 & T^n A \longrightarrow & T^m A \\
 \downarrow p_x & & \searrow p_* \\
 A & \xrightarrow{p^*} & T^{n/m} A \xrightarrow{p_*} A
 \end{array}$$

result: $p : X[t]_{/t^n} \longrightarrow X$



Important: For $A=K$
 + ws is mult by n/m

$$\underline{\text{Rank}} \quad \Omega C_+ A \quad \longleftrightarrow \quad C_+ A$$

$$\text{THH} \quad \longleftrightarrow \quad \text{Der}$$

$$\pi_0 \Omega C_m K(R) \longrightarrow W_m(R)$$

← truncate set

1 2 3 4 ... m

$$\text{but } W_m(R) \neq W_{\langle m \rangle}(R)$$

← truncate set
d(m)

Point

m/n

$$C_{\langle n \rangle} A(R) \xrightarrow{R_{m/n}} C_{\langle m \rangle} A(R)$$

Cores from:

C_m are compatibly
by restriction of jet
maps

$$C_{\langle m \rangle} \supseteq \lim_{d/m}^{\text{Rest maps}} C_m$$

C_m : easily use functorially
to make $\Omega G \rightarrow A$
a green functor.
