13-Classical ASS

Wednesday, November 19, 2014 1:26 PM

$$\mathrm{Ext}^{s,t}_{E_*E}(E_*X, E_*Y) \Rightarrow [\Sigma^{t-s}X, Y^\wedge_E]$$

$X = S$

$E = H\mathbb{F}_p$

$H_* = H_*(-;\mathbb{F}_p)$

$$\mathrm{Ext}^{s,t}_{A_*}(\mathbb{F}_p, H_*(Y)) \Rightarrow \pi_{t-s}(Y^\wedge_p) \underset{\text{sometimes}}{\cong} \pi_{t-s}(Y)^\wedge_p$$

$\cong$ H_*Y finite type

$$\mathrm{Ext}^{s,t}_{A}(H^*(Y), \mathbb{F}_p)$$

(e.g. if $\forall k$ $\pi_k(Y)$ has p-torsion of bounded order)

Exercise

$$\pi_i\left(H(\mathbb{Q}/\mathbb{Z})^\wedge_p\right) = \begin{cases} \mathbb{Z}_p & i=1 \\ 0, & o/w \end{cases}$$

[Slideshow]

$\hookrightarrow$ Ext calculator...

$p=2$ Warmup: ku $\qquad ku = kU\langle 0\rangle$

Claim $H^*(ku) = A \otimes_{E[Q_0,Q_1]} \mathbb{F}_p$

$Q_0 = Sq^1$

$Q_1 = [Sq^2, Sq^1]$ $\qquad E[Q_0,Q_1] \hookrightarrow A$

Change of rings thm $\qquad A' \subset A$

$$\mathrm{Ext}_A(A \otimes_{A'} M, N) \cong \mathrm{Ext}_{A'}(M, N)$$

Also $\qquad H_*(ku \wedge X) \cong H_* ku \otimes H_* X$

$$\cong \left(A \otimes_{E[Q_0, Q_1]} \mathbb{F}_2\right) \otimes H_* X \qquad \text{"diagonal action"}$$

$$\underset{\text{Frown}}{\cong} A \otimes_{E[Q_0, Q_1]} H_* X \qquad \text{"left action"}$$

$$\left(\text{Cf.} \qquad G/H \times X \underset{G}{\cong} G \times_H X\right)$$

$$\mathrm{Ext}_A\left(H^*(ku \wedge X), \mathbb{F}_2\right) \Rightarrow (ku_* X)^\wedge_2$$

$$\cong$$

$$\mathrm{Ext}_{E[Q_0, Q_1]}\left(H_*(X), \mathbb{F}_2\right)$$
