15 - v_2-periodicity

Note Title 12/1/2008

$p \geq 5$ always

$$S(n) = \frac{\mathbb{F}_p[t_1, t_2, \ldots]}{(t_i^{p^n} = t_i)}$$

Begin by recalling v_1-periodic computations $\alpha_{i/j}$

Cohomology of $S(2)$

Modified May SS: $m = \left\lfloor \frac{pn}{p-1} \right\rfloor$

$$E_1^{**} = \Lambda_{\mathbb{F}_p}[h_{ij}]_{\substack{1 \leq i \leq m \\ j \in \mathbb{Z}/n}} \otimes \mathbb{F}_p[b_{ij}]_{\substack{1 \leq i \leq m-n \\ j \in \mathbb{Z}/n}} \Rightarrow \operatorname{Ext}_{S(2)}(\mathbb{F}_p, \mathbb{F}_p) \quad \underbrace{}_{s,t} \; \mathbb{Z}/(2(p^2-1))\text{-grad}$$

$$d_1 h_{ij} = \sum_{i_1 + i_2 = i} h_{i_1, j} h_{i_2, j + i_1}$$

$\underline{\underline{n = 0}}$ $\quad m = \left\lfloor \frac{2p}{p-1} \right\rfloor = 2$

$\left\lfloor \frac{10}{4} \right\rfloor = 2 \quad \left\lfloor \frac{14}{6} \right\rfloor = 2$

$\left\lfloor \frac{2 \cdot 3}{2} \right\rfloor = 3$

$$\Lambda_{\mathbb{F}_p}[h_{10}, h_{11}, h_{20}, h_{21}]$$

$d h_{20} = h_{10} h_{11}$ $\qquad \langle h_0, h_1, h_0 \rangle = g_0$

$d h_{21} = h_{11} h_{10}$ $\qquad \langle h_1, h_0, h_1 \rangle = g_1$

and $\quad h_1 g_0 = h_0 g_1$

$$d(h_{20} + h_{21}) = 0 \qquad \varsigma^2 = 0$$

$$|h_{20}|_t = 2(p^2-1) \qquad t_2$$

$$|h_{21}|_t = 2p(p^2-1) \qquad t_2^p$$

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get $\quad \mathrm{Ext}_{\Sigma(1)} = \Lambda_{\mathbb{F}_p}[\varsigma, h_0, h_1, g_0, g_1] \Big/ \begin{pmatrix} h_0 h_1,\ g_0 g_1 \\ h_1 g_0 = h_0 g_1 \end{pmatrix}$

$|\varsigma| = (1, 0)$

$|h_0| = (1, 2(p-1))$

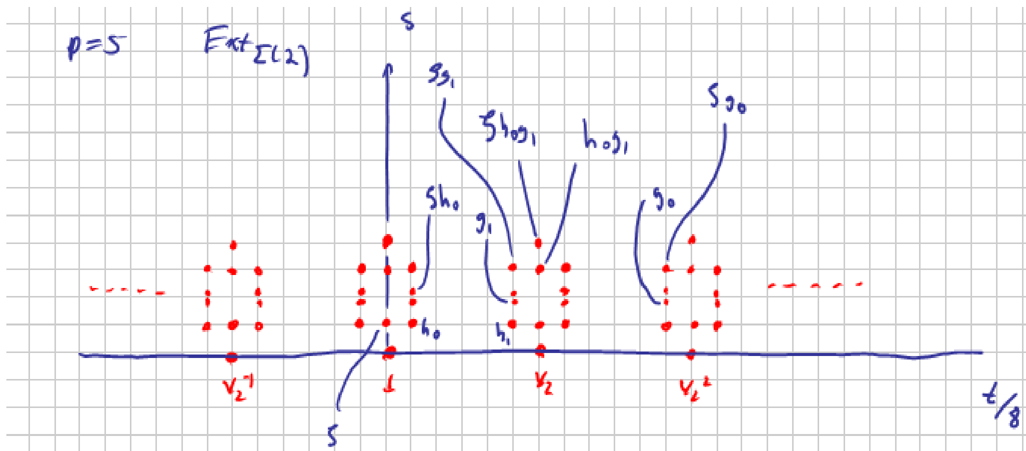
$|h_1| = (1, 2p(p-1))$

$|g_0| = (2, 2p(p-1))$

$|g_1| = (2, 2(p-1))$

$$\mathrm{Ext}_{\Sigma(2)}(\mathbb{F}_p, \mathbb{F}_p) = \mathbb{F}_p[v_2^{\pm 1}] \otimes \mathrm{Ext}_{S(n)}$$

Picture: $p = 5$



Computation of v_1 BSS

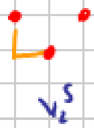
$$\mathrm{Ext}_{\Sigma(2)}(\mathbb{F}_p, \mathbb{F}_p) \otimes \frac{\mathbb{F}_p[v_1]}{v_1^\infty} \Rightarrow \mathrm{Ext}_{BP_*BP}\left(BP_*/p, v_1^\infty [v_2^{-1}]\right)$$

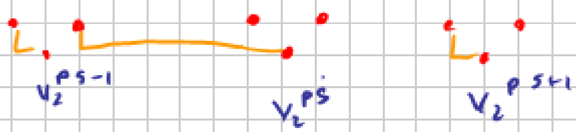
just compute 0-line (following Miller-Ravenel-Wilson)

Key: $\eta_R(v_2) \equiv v_2 + v_1 t_1^p - v_1^p t_1 \quad (p)$

Note: no t_2's $\Rightarrow$ S not involved.
just h_0 & h_1

$$d(v_2) = v_1 t_1^p - v_1^p t_1 \qquad (p)$$



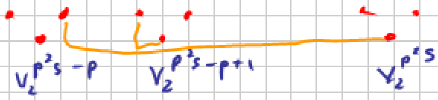


$$d(v_2^p) = v_1^p t_1^{p^2}$$
$$= v_1 v_2^{p-1} t_1$$

$$h_{12} = h_{10}$$

$$v_2 t_1^{p^2} = v_2^p t_1$$
$$t_1^{p^2} = v_2^{p-1} t_1$$

$$v_2^{ps-p+p-1} = v_2^{ps-1}$$

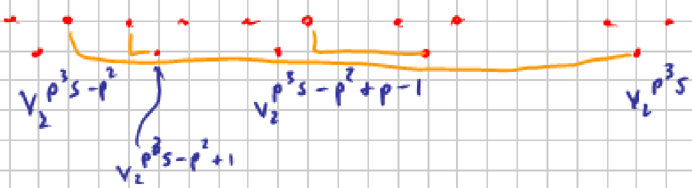


$$d(v_2^{p^2}) = v_1^{p^2} t_1^{p^3}$$
$$= v_1^{p^2} v_2^{p^2 - p} t_1^{p}$$

$$v_2^{p^2 s - p^2 + p^2 - p}$$

$$d\left(v_2^{p^2 - p + 1} v_1^{p^2 - 1}\right) = v_2^{p^2 - p} t_1^{p} v_1^{p^2} - v_1^{p^2 + p - 1} v_2^{p^2 - p} t_1$$

$$d\left(v_2^{p^3}\right) = v_1^{p^3} t_1^{p^4} + \cdots = v_1^{p^3} v_2^{p^3 - p^2} t_1^{p^2}$$
$$= v_1^{p^3} v_2^{p^3 - p^2 + p - 1} t_1$$



$$d\left(v_2^{p^3 - p^2 + p} v_1^{p^3 - p}\right) = v_2^{p^3 - p^2 + p - 1} v_1^{p^3} t_1$$
$$\pm\ v_2^{p^3 - p^2} v_1^{p^3 + p^2 - p} t_1^{p}$$

$$d\left(v_2^{p^3 - p^2 + 1} v_1^{p^3 + p^2 - p - 1}\right) = v_2^{p^3 - p^2} v_1^{p^3 + p^2 - p} t_1^{p} \pm v_2^{p^3 - p^2} v_1^{p^3 + p^2 - 1} t_1$$

Inductively

$p \nmid s$

$$d\left(v_2^{p^n s}\right) = v_2^{p^n s - p^{n-1}} v_1^{p^n + p^{n-1} - 1} h_0$$

Get

$$\mathrm{Ext}^0_{BP_*BP}\left(BP_*/(p, v_1^\infty)[v_2^{-1}]\right)$$

$$= \mathbb{F}_p\left\{ \frac{v_2^{p^n s}}{v_1^j} \;\middle|\; \begin{array}{l} n \geq 0, \quad p \nmid s \\ 0 < j \leq p^n + p^{n-1} - 1 \end{array} \right\}$$

$$\oplus\ \mathbb{F}_p\left\{ \frac{1}{v_1^j} \;\middle|\; j > 0 \right\}$$

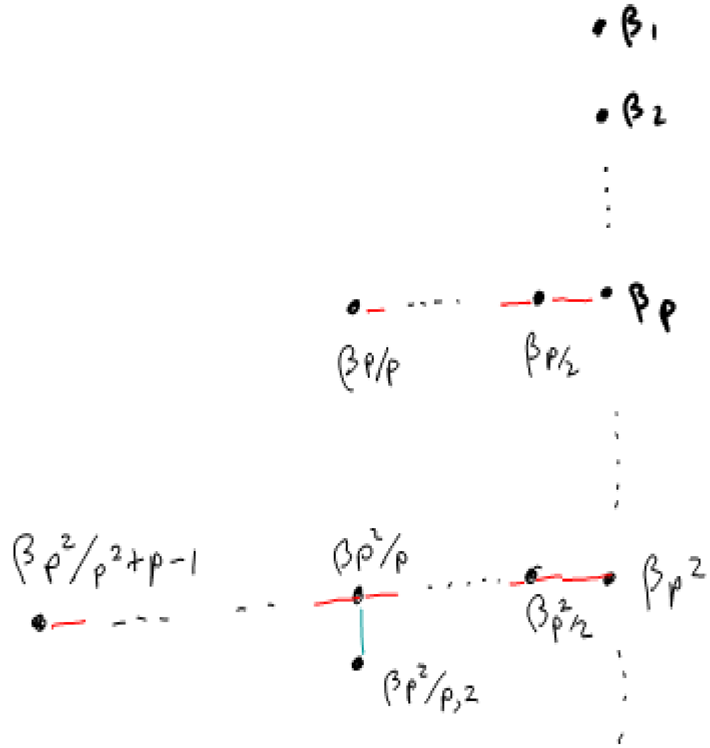
Thm: (MRW)

$$\mathrm{Ext}^{*}_{BP_*BP}\left(BP_*/(p^\infty, v_1^\infty)[v_2^{-1}]\right) =$$

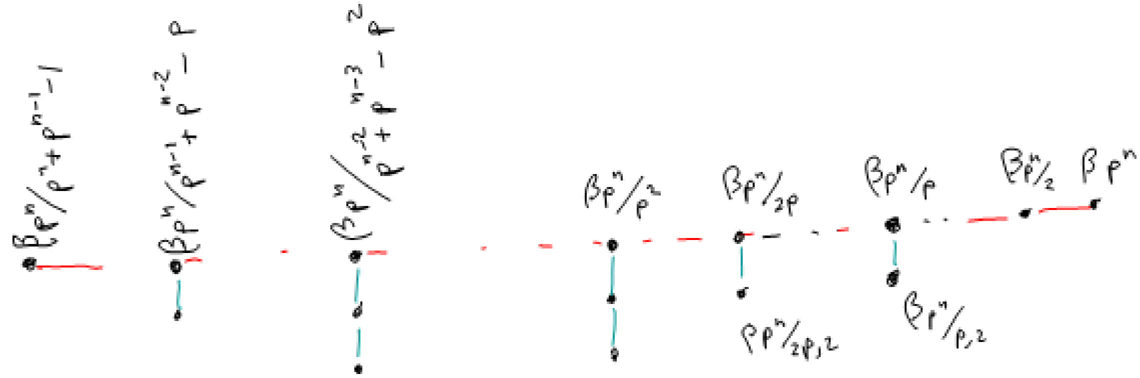
$$\left\langle \frac{v_2^{p^n s}}{p^k v_1^j} \;\middle|\; \begin{array}{l} n \geq 0,\quad p \nmid s \neq 0 \\ 1 \leq j \leq p^n + p^{n-1} - 1 \\ \text{and if } i \text{ is chosen such that} \\ p^{n-i-1} + p^{n-i-2} - 1 < j \leq p^{n-i} + p^{n-i-1} - 1 \\ \qquad 1 \leq k \leq \min(i+1, \nu_p(j)+1) \end{array} \right\rangle$$

$$\oplus \left\langle \frac{1}{p^k v_1^j} \;\middle|\; \begin{array}{l} j \geq 1 \\ 1 \leq k \leq \nu_p(j)+1 \end{array} \right\rangle$$

Draw pictures of

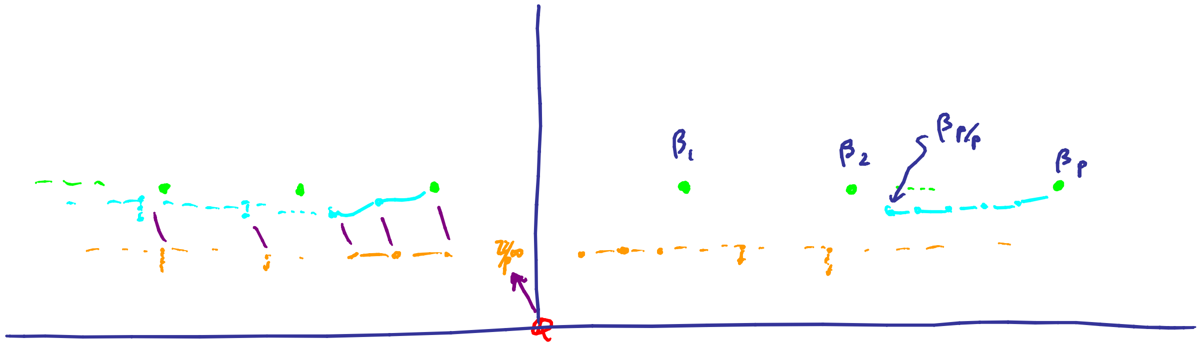


β_1
β_2
β_p
$\beta_{p/p}$
$\beta_{p/2}$
β_{p^2/p^2+p-1}
$\beta_{p^2/p}$
β_{p^2}
$\beta_{p^2/2}$
$\beta_{p^2/p,2}$



$\beta_{p^n/p^n+p^{n-1}-1}$
$\beta_{p^n/p^{n-1}+p^{n-2}-p}$
$\beta_{p^n/p^{n-2}+p^{n-3}-p^2}$
β_{p^n/p^2}
$\beta_{p^n/2p}$
$\beta_{p^n/p}$
$\beta_{p^n/2}$
β_{p^n}
$\beta_{p^n/2p,2}$
$\beta_{p^n/p,2}$

Picture of ACSS



Constructing $\beta_{i/j,k} \in \pi_*^S$ (no v_2-J-hom)

$$
\begin{tikzcd}
S^{2i(p^2-1)-2j(p-1)-2} \arrow[r, "\text{bottom cell}"'] \arrow[drr, bend right, "\text{detected by } \beta_{i/j,k} \in \mathrm{Ext}^2"'] & \Sigma^{2i(p^2-1)-2j(p-1)-2} M(p^k, v_1^j) \arrow[d, "v_2^i"] & \\
& \Sigma^{-2j(p-1)-2} M(p^k, v_1^j) \arrow[r, "\text{top cell}"'] & S
\end{tikzcd}
$$

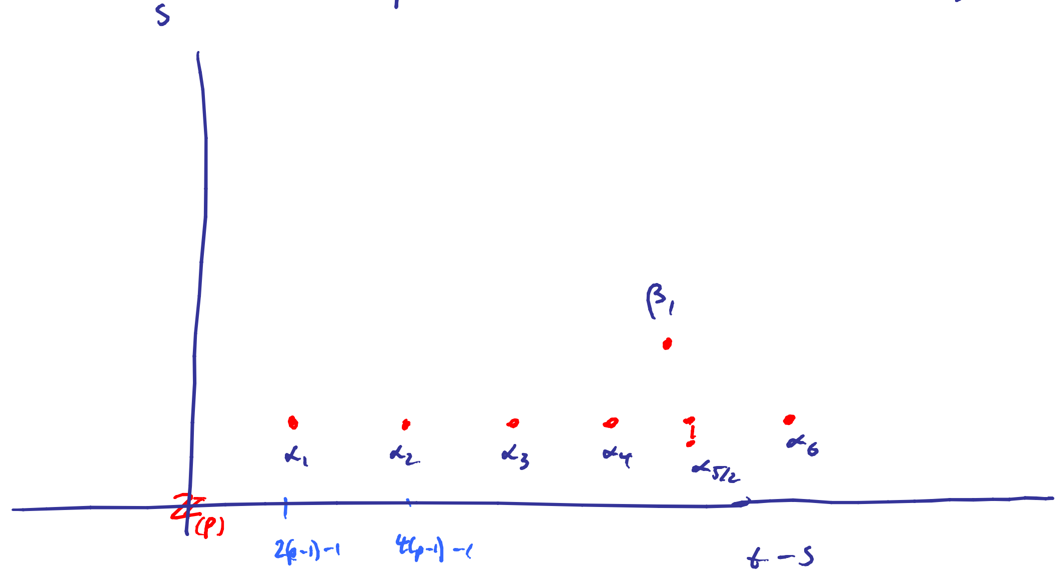
Process (1) Construct v_2-self map v_2^n on $M(p^k, v_1^j)$

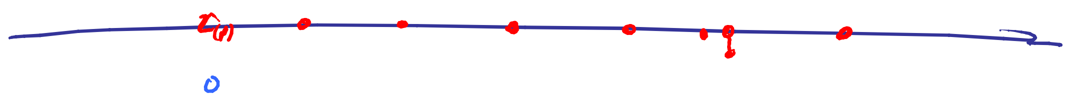
(2) iterate to get infinite family $t > 0$

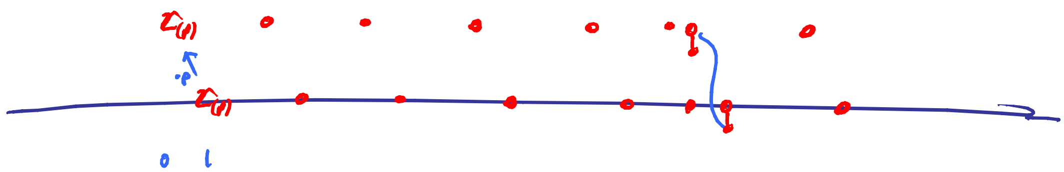
$\beta_{nt/j,k}$

$p = 5$ low dim'l π_*^S

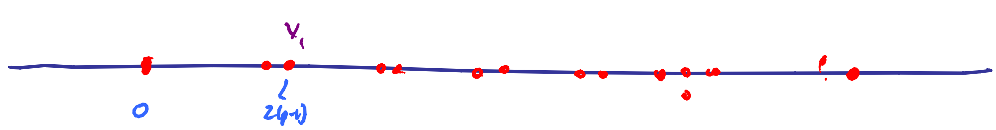
e.g.

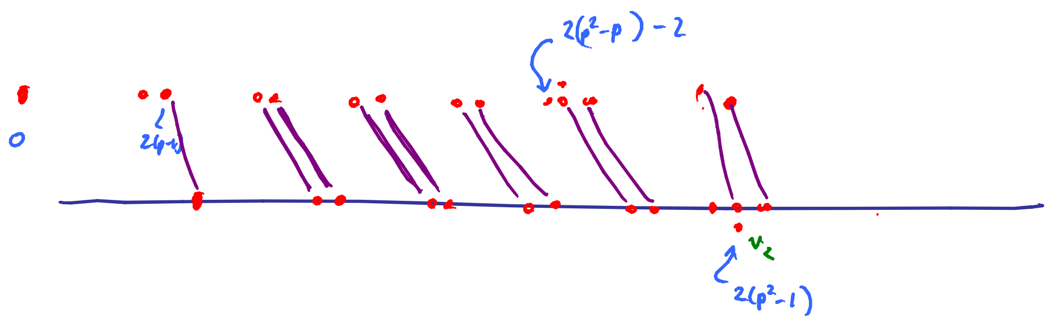






$\pi_*(M(p))$





$M(p,v_1)$ is a ring spectrum.

$$\begin{tikzcd}
S \wedge M(p,v_1) \arrow[r, "\text{bottom cell} \wedge 1"] \arrow[dr, equal] & M(p,v_1) \wedge M(p,v_1) \arrow[d, dashed] \\
 & M(p,v_1)
\end{tikzcd}$$

$$\begin{tikzcd}
S \wedge M(p) \arrow[r, "p"] & S \wedge M(p) \arrow[r] \arrow[dr, equal] & M(p) \wedge M(p) \arrow[d, dashed] \\
 & & M(p)
\end{tikzcd}$$

$$\mathbb{Z}/p = [M(p), M(p)] \leftsquigarrow \pi_0(M(p)) = \mathbb{Z}/p$$

$$\leftsquigarrow \pi_1(M(p)) = 0$$

$$\left[p=2 \quad \pi_1(M(p)) = \mathbb{Z}/2\{\eta\} \right]$$

$$\left[\Sigma^{2(p-1)} M(p,v_1), M(p,v_1)\right] \leftsquigarrow \pi_{2(p-1)}(M(p,v_1)) = 0 \qquad \pi_{2(p-1)+1}(M(p,v_1)) = 0$$

$$\leftsquigarrow \pi_{2(p-1)+1}(M(p,v_1)) = 0 \qquad \pi_{2(p-1)+2}(M(p,v_1)) = 0$$

[e.g. $p=3$ $\quad \mathbb{Z}/3\{\beta_1\} \neq 0$]

$v_2 \in \pi_{2(p^2-1)} M(p, v_1)$

$$\begin{tikzcd} \Sigma^{2(p^2-1)} M(p,v_1) \arrow[r, "v_2 \wedge 1"'] \arrow[rr, bend right, "v_2"'] & M(p,v_1) \wedge M(p,v_1) \arrow[r, "\mu"'] & M(p,v_1) \end{tikzcd}$$

$\Longrightarrow \beta_i \in \pi^s_{2i(p^2-1)-2(p-1)-2} \quad \forall \; i > 0$

<u>Contrast</u>: $p=3$ [B-Pemmaraju]

β_i exists for $i = 0, 1, 2, .5, 6$

$$\Sigma^{144} M(p,v_1) \xrightarrow{v_2^9} M(p,v_1)$$

<u>Shimomura</u> β_{9t+7} β_{9t+8} d.n.e.

<u>Conj</u> β_{9t+3} exists (soln recently proposed by Shimomura)