10 - Low dim'l ANSS computations

Note Title 10/20/2008

$$\mathrm{Ext}^{s,t}_{BP_*BP}(BP_*, BP_*) \Rightarrow \pi_{t-s} S_{(p)}$$

we will compute this for

$$t-s < 2(p^2+p)(p-1)$$

In this range: only see

v_1, v_2, t_1, t_2 $\qquad |v_3| = |t_3| = 2(p^3-1)$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad ||$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad 2(p^2+p+1)(p-1)$

For convenience: do this at $p=5$

but in this range "same" calculation holds for $p \geq 3$

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v_n - BSS

Filter BP_*/I_n by (v_n^k)

$$\Rightarrow \quad \mathrm{Ext}^{s,t}_{BP_*BP}(BP_*, BP_*/I_{n+1})[v_n] \Rightarrow \mathrm{Ext}^{s,t}(BP_*, BP_*/I_n)$$

diff'ls $\quad (s,t) \to (s+1,t)$

$$\mathrm{Ext}_{BP_*BP}(BP_*, BP_*/I_3) \xrightarrow{v_2\text{-BSS}} \mathrm{Ext}(BP_*/I_2) \xrightarrow{v_1\text{-BSS}} \mathrm{Ext}(BP_*/I_1) \xrightarrow{v_0\text{-BSS}} \mathrm{Ext}(BP_*)$$

$\cong$ in our range

$$\mathrm{Ext}_{BP_*BP/I}(BP_*/I, BP_*/I)$$

$$\cong$$

$$\mathrm{Ext}_{P_*}(\mathbb{F}_p, \mathbb{F}_p)$$

$$\Uparrow \text{ MSS}$$

$$\mathrm{Ext}_{E^0P_*}(\mathbb{F}_p, \mathbb{F}_p) = E_2^{MSS}$$

$$\Uparrow \text{ KSS}$$

$$\mathrm{Ext}_{E^0E^0P_*}(\mathbb{F}_p, \mathbb{F}_p) = E_2^{KSS} = E_1^{MSS}$$

$$E^0E^0P \cong \mathrm{Tr}_{\mathbb{F}_p}(P_i^j)_{\substack{i \geq 0 \\ j \geq 0}}$$

$$E_2^{KSS} = E_1^{MSS} = \Lambda_{\mathbb{F}_p}[h_{1,0}, h_{1,1}, h_{1,2}, h_{20}, h_{21}] \otimes \mathbb{F}_p[b_{1,0}, b_{1,1}, b_{20}]$$

$$d h_{20} = h_{1,0} h_{1,1}$$

$$d h_{21} = h_{1,1} h_{1,2}$$

$h_{1,0} = h_0$

$h_{1,1} = h_1$

$h_{1,2} = h_2$

$b_{1,0} = b_0$

$b_{1,1} = b_1$

get $\langle \overbrace{h_0, \ldots, h_0}^{p} \rangle = b_0$

$\langle \overbrace{h_1, \ldots, h_1}^{p} \rangle = b_1$

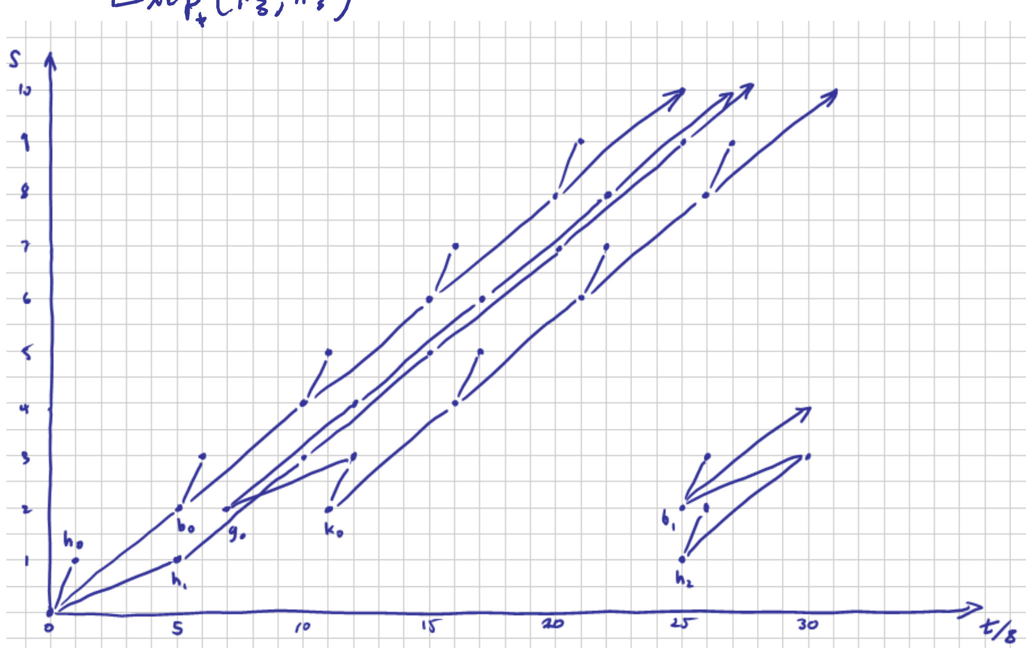
$\langle h_0, h_1, h_0 \rangle = g_0$

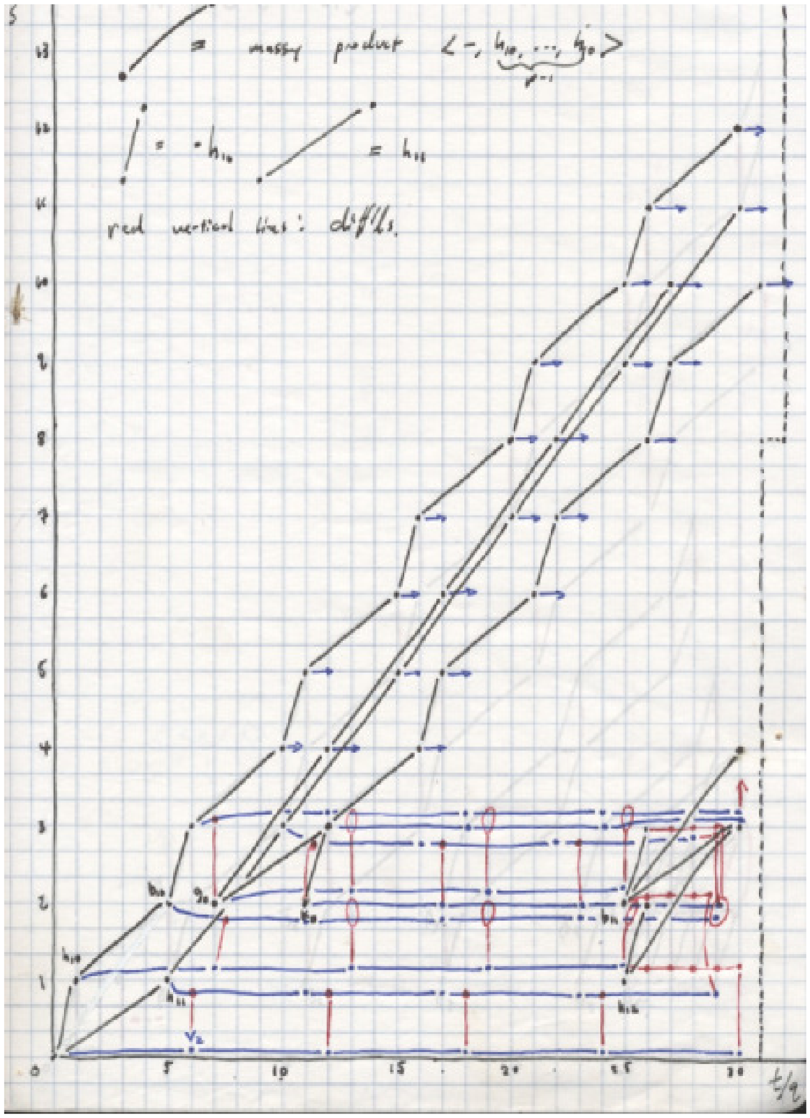
$\langle h_1, h_0, h_1 \rangle = k_0$

any involving h_2 are out of our range.

$h_1 b_0 = 0$ $h_1 g_0 = h_0 k_0$ $h_0 g_0 = 0$	Nakamura $b_{2,0} = \beta P^0(h_{2,0})$ $d\, b_{2,0} = \beta P^0(d\, h_{2,0})$ $= \beta P^0(h_{1,0} h_{1,1}) = h_{1,1} b_{1,1} + b_{1,0} h_{1,2}$
$h_1 b_1 = b_0 h_2$	

$\mathrm{Ext}_{P_*}(\mathbb{F}_3, \mathbb{F}_3)$





v_1-BSS

e.g. $dv_2 = \eta_R(v_2) - \eta_L(v_2)$

$= v_1 t_1^p - v_1^p t_1 \equiv v_1 t_1^p \quad (v_1^p)$

h_{11}

So $dv_2 = v_1 h_{11}$ $\quad dh_{20} = h_0 h_1$

$d\, t_2 = t_1 | t_1^p + \frac{v_1}{(p^{p-1}-1)} b_{10}$ $\quad$ mod v_1^2

$\Rightarrow$ So $h_1 h_0 \doteq v_1 b_{1,0}$

$\doteq$ means 'up to a unit'

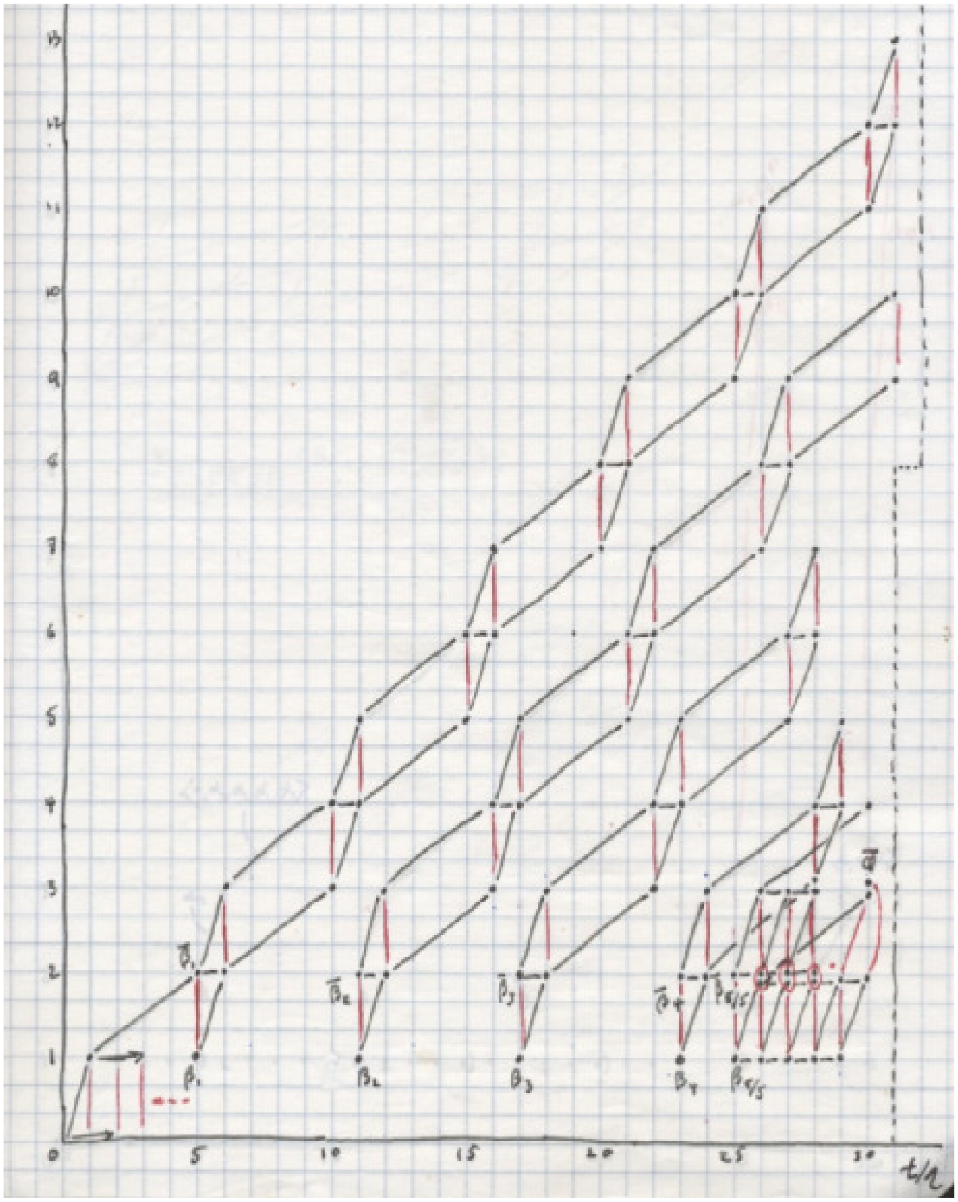
So $d(v_2 h_0) = v_1 h_0 h_1 = v_1^2 b_0$

$$d\left(v_2^k\right) = k\, v_1\, v_2^{k-1} h_1 \qquad \bmod \left(v_1^p\right)$$

$d\left(v_2^p\right) = v_1^p t_1^{p^2} \quad \bmod \quad v_1^{p^2}$

$\Rightarrow \boxed{d v_2^p = v_1^p h_2}$

etc ....



$$dh_1 = dt_1^p$$

$$\Delta(t_1^p) = \sum_i{}' \binom{p}{i} t_1^i \otimes t_1^{p-i}$$

$$= p\, b_0 \quad \text{mod } p^2$$

etc.....

Most Importantly

$$\eta_R(v_1) = v_1 + p\, t_1 \mod p^2$$

$$\Rightarrow \eta_R(v_1^i) = v_1^i + i\, v_1^{i-1} p\, t_1 \mod p^2$$

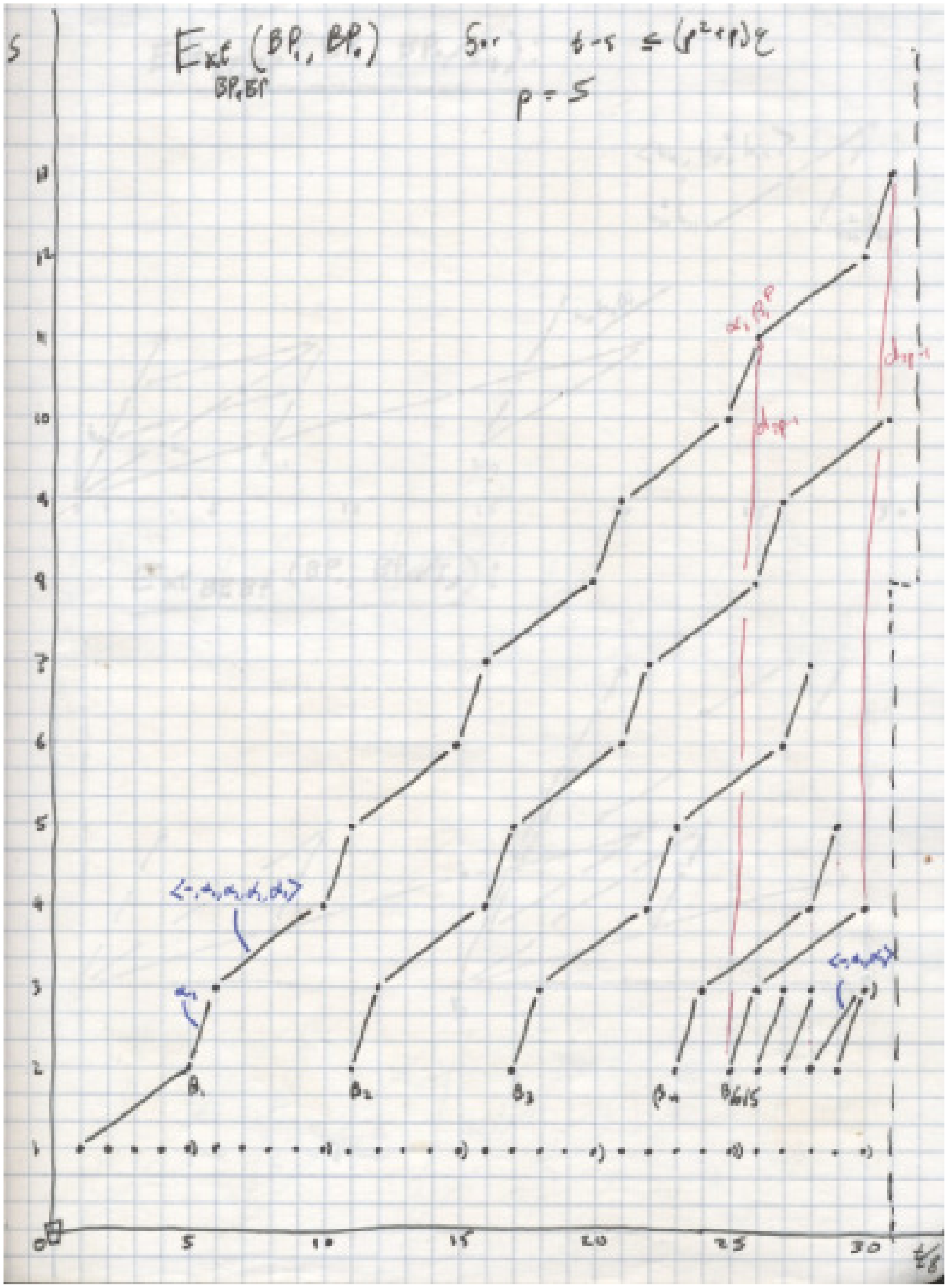
so $p \nmid i$ $\quad d(v_1^i) = p\, v_1^{i-1} h_0$

$$\eta_R(v_1^p) = v_1^p + \underbrace{p\, v_1^{i-1} p\, t_1}_{\substack{\text{lowest } p \\ \text{much}}} + \frac{p(p-1)}{2} v_1^{i-2} p^2 t_1 + \cdots$$

So $d\, v_1^p = p^2\, v_1^{p-1} h_0$

In general, if $\nu_p(i) = j$

$$\Rightarrow d(v_1^i) \doteq p^{j+1} h_0\, v_1^{i-1}$$



Toda diff'l

$$\beta_{5/5} = b_1 = P^0(b_0)$$

$$|b_0| = 2p(p-1) - 2$$

$B\Sigma_p$

$$\begin{array}{lll} & \circ \quad 2p(p-1)^2 & P^0 \\ \alpha_1 & \circ \quad 2p(p-1)^2-1 & \beta P^0 \\ & \circ \quad 2p(p-1)^2-2(p-1) & P^1 \end{array}$$

$$d\left(P^0(b_0)\right) = p\ \cancel{\beta P^0(b_0)} + \underbrace{\alpha_1 P^1(b_0)}_{\alpha_1 b_0^p}$$

<u>Note:</u> $b_0 h_0 \doteq \langle \alpha_1, \alpha_1, \beta_1^5 \rangle$

Final remark

$\alpha_{i/j}$ on 1-line

generates image of J

$$\pi_* SO \longrightarrow \pi_*^S$$

$$4k-1 \quad \mathbb{Z} \twoheadrightarrow \mathbb{Z}/p^j$$

note $2(p-1) \equiv 0 \quad (4)$

$j = 0$ unless $4k = 2(p-1)i$

then $j = \nu_p(i) + 1$

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generator is $\alpha_{i/j}$

Q: is β_1^k always nontrivial

Nishida nilpotence thm

$\Longrightarrow \exists\, k$ s.t. β_1^k trivial

e.g. $p = 5$, $\beta_1^{18} = 0$