Hw3 Solutions

Note Title 2/23/2009

6.2(2)

Because there are ∞'ly many terms in the sequence, and only finitely many values, there must exist an i such that

$$a_i = x_n \quad \text{for } \infty\text{'ly many } n.$$

Let $\varepsilon > 0$

Then $|a_i - x_n| = 0 < \varepsilon$ for ∞'ly many n.

$\Longrightarrow$ a_i is a cluster point of $\{x_n\}$

6.3(1)

(a) Note that $-1 \leq \cos(a_n) \leq 1$

$\Rightarrow$ $0 \leq \cos^2(a_n) \leq 1$

$\Rightarrow$ $\{\cos^2(a_n)\}$ bounded

$\underset{\text{BW thm}}{\Longrightarrow}$ b_n has a convergent subsequence

(b) b_n does ~~NOT~~ always have to have a convergent subsequence

For example, take $a_n = -1 + 1/n$

then $\dfrac{a_n}{1+a_n} = \dfrac{-1+1/n}{1/n}$

$= -n + 1$

The sequence $\{-n+1\}$ has no convergent subsequences.

(c) Since $0 \le |a_n|$

$\Longrightarrow 1 \le 1 + |a_n|$

$\Longrightarrow \dfrac{1}{1+|a_n|} \le 1$ also $\dfrac{1}{1+|a_n|} \ge 0$

So $\{b_n\}$ is bounded

$\xRightarrow[\text{BW thm}]{}$ b_n has a convergent subsequence.

$\boxed{6.4(1)}$ Suppose $a_n \to L$.

We wish to show $\{a_n\}$ is Cauchy.

Let $\varepsilon > 0$.

Since $a_n \to L$,

$$a_n \approx_{\varepsilon/2} L \quad \text{for} \quad n \gg 0$$

in particular, for $n, m \gg 0$

$$a_n \approx_{\varepsilon/2} L$$
$$a_m \approx_{\varepsilon/2} L$$

$$\Longrightarrow \quad a_n \approx_{\varepsilon} a_m \quad \text{for} \quad n, m \gg 0.$$

$\boxed{6.5(1)}$

(a) sup $= 1$ max $= 1$

inf $= -1$ min $= -1$

(b) sup $= 1/2$ max $= 1/2$

inf $= -1$ min $= -1$

$-1, \frac{1}{2}, -1/3, \frac{1}{4}$

(c) $\sup = \frac{5}{4}$ $\max = \frac{5}{4}$

$\inf = -1$ $\min =$ d.n.e.

$1 + 0$

$\frac{1}{2} - 1$

$\frac{1}{3} + 0$

$\frac{1}{4} + 1$

$\frac{1}{5} + 0$

$\frac{1}{6} - 1$

$\vdots$

(d) $\sup = \frac{1}{2}$ $\max = \frac{1}{2}$

$\inf = 0$ $\min =$ d.n.e.

$\frac{1}{2}$

$\frac{2}{2^2} = \frac{1}{2}$

$\frac{3}{2^3} = \frac{3}{8}$

$\frac{4}{2^4} = \frac{1}{4}$

$\boxed{6.5(4)}$

Let $M = \sup S$

and $N = \inf T$

Claim: N is an upper bound for S.

Pf of claim: Let $s \in S$

$s \le t$ for all t

$\Rightarrow$ s is a lower bound

$\underset{\text{inf-2}}{\Rightarrow}$ $s \le N$

By sup-2, $M \le N$