Hw 10

Note Title 4/27/2009

18.3 (4) [5 pts]

Let $P = \{0 = x_0 < x_1 < \cdots < x_k = 1\}$

be a partition w/ mesh $< 1/N$

since $[x_{i-1}, x_i]$ contains irrational numbers

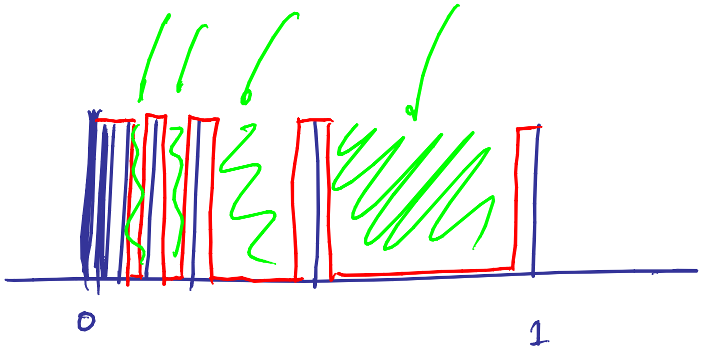
$$m_i = 0$$

$$\Rightarrow L_f(P) = 0$$

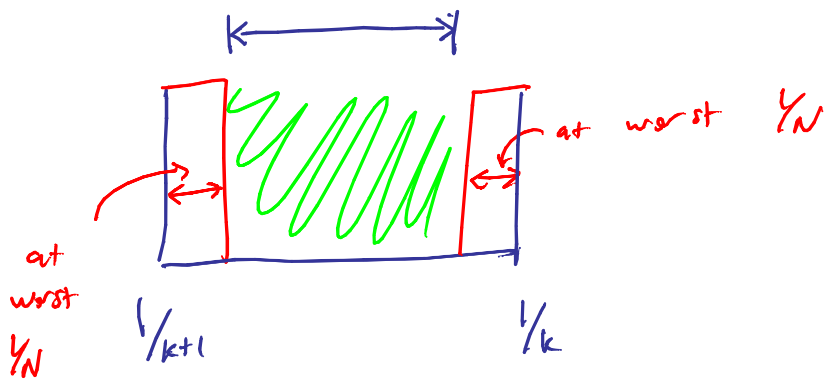
For $U_f(P)$, analyze the

"negative space"

[c.f. Donald Judd
american minimalist sculptor]



get at least $\frac{1}{k} - \frac{1}{k+1} - \frac{2}{N}$



get such negative space

whenever $\frac{1}{k} - \frac{1}{k+1} - \frac{2}{N} > 0$

i.e., $\frac{1}{k^2+k} > \frac{2}{N}$

or when $k^2+k < \frac{N}{2}$

In particular when $k^2 < \frac{N}{2}$

or $k < \sqrt{\frac{N}{2}}$

Let k_0 be the biggest integer less than $\sqrt{\frac{N}{2}}$

$$k_0 + 1 > \sqrt{\frac{N}{2}}$$

$$\Rightarrow \frac{1}{k_0+1} < \sqrt{\frac{2}{N}}$$

$$\text{Negative space} > \sum_{k=1}^{k_0} \left(\frac{1}{k} - \frac{1}{k+1} - \frac{2}{N} \right)$$

$$\|$$

telescoping sum

$$\left(\frac{1}{1} - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right) + \cdots + \left(\cancel{\frac{1}{k_0}} - \frac{1}{k_0+1}\right) - \frac{2k_0}{N}$$

$$= 1 - \frac{1}{k_0+1} - \frac{2k_0}{N}$$

$$> 1 - \sqrt{\frac{2}{N}} - \frac{2}{N} \cdot \sqrt{\frac{N}{2}}$$

$$= 1 - \sqrt{\frac{2}{N}} - \sqrt{\frac{2}{N}} = 1 - 2\sqrt{\frac{2}{N}}$$

So $U_f(P) = 1 - (\text{negative space})$

$$< 2\sqrt{\frac{2}{N}}$$

Let $\varepsilon > 0$

choose N such that $2\sqrt{\frac{2}{N}} < \varepsilon$

For $|P| < \frac{1}{N}$

$$U_f(P) - L_f(P) < 2\sqrt{\frac{2}{N}} < \varepsilon$$

$\Longrightarrow$ $f(x)$ is integrable on $[0,1]$

19.3 (1) [5 pts]

Choose a sequence of partitions

$\{P_n\}$ such that

$$|P_n| \xrightarrow[n\to\infty]{} 0$$

For each partition

$$P_n = \{a = x_0 < x_1 < \cdots < x_k = b\}$$

choose $x_i^* \in [x_{i-1}, x_i]$ to be rational

Then

$$S_f(P_n) = \sum_i f(x_i^*) \Delta x_i = 0$$

$$\Big\downarrow n \to \infty$$

$$0$$

The Riemann sum theorem implies that $\int_a^b f(x)\,dx = 0$

19-1(a) [5 points]

Since f is bounded, then is M so that

$$|f(x)| < M \quad , \quad x \in I$$

Let $\varepsilon > 0$.

Choose $\delta = \frac{\varepsilon}{M}$

Suppose $|x - x_0| < \delta$

Then (assume $x > x_0$, $x < x_0$ similar)

interval addition thm

$$\left| \int_a^x f(t)\,dt - \int_a^{x_0} f(t)\,dt \right| = \left| \int_{x_0}^{x} f(t)\,dt \right|$$

$$\leq \int_{x_0}^{x} |f(t)|\,dt \leq \int_{x_0}^{x} M\,dt$$

comparison thm

$$= (x - x_0) M < \frac{\varepsilon}{M} \cdot M = \varepsilon$$

So

$$F(x) \underset{\varepsilon}{\approx} F(x_0) \quad \text{for} \quad x \approx x_0.$$

$\Longrightarrow F(x)$ continuous. $\square$