Continuation of an example given in Recitation 5 on 9/10

September 10, 2003

In recitation 5 I began a problem, but didn't finish it. Here it is.

The Problem Define points

$$\begin{aligned} Q_1 &= (1,0,0) \\ Q_2 &= (-1,1,0) \\ Q_3 &= (0,3,3) \end{aligned}$$

Let L be the line passing through Q_2 and Q_3. Let P be the plane containing Q_1 and L. Write an equation for the plane P.

Solution We first compute a normal vector $\vec{V}$ to the plane P. By subtracting the head of the vector from the tail of the vector, we compute

$$\begin{aligned} Q_2\vec{Q}_3 &= \langle 1,2,3 \rangle \\ Q_2\vec{Q}_1 &= \langle 2,-1,0 \rangle \end{aligned}$$

which gives a normal vector

$$\begin{aligned} \vec{V} &= Q_2\vec{Q}_3 \otimes Q_2\vec{Q}_1 \\ &= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 3 \\ 2 & -1 & 0 \end{vmatrix} \\ &= 3\hat{\imath} + 6\hat{\jmath} - 5\hat{k} \end{aligned}$$

The plane P is the set of all points $Q = (x,y,z)$ with the property that

$$Q_2\vec{Q} \cdot \vec{v} = 0$$

We may write the vector

$$Q_2\vec{Q} = \langle x,y,z \rangle - \langle -1,1,0 \rangle = \langle x+1, y-1, 0 \rangle$$

thus we have

$$0 = \langle x+1, y-1, z\rangle \cdot \langle 3, 6, -5\rangle = (3x+3) + (6y-6) - 5z.$$

Bring the constants to the other side, we have

$$3x + 6y - 5z = 3.$$

This is the equation we wanted.