Calculus Review Sheet

Microeconomic Analysis

Concept of Derivatives

Marginal value of Y= f(X) is ΔY/ΔX. Derivative is the limit:

$$\frac{dY}{dX} = \lim_{\Delta X \to 0} \frac{\Delta Y}{\Delta X}$$

Formulae for Derivatives

Power Rule	$Y = aX^b$	$\frac{dY}{dX} = baX^{b-1}$
Constant	$Y = aX^0$	$\frac{dY}{dX} = 0$
Linear	$Y = aX^1$	$\frac{dY}{dX} = a$
Natural Logarithm	$Y = \ln X$	$\frac{dY}{dX} = \frac{1}{X}$
Exponential	$Y = \exp X$	$\frac{dY}{dX} = \exp X$
Sums	$Y = U(X) + W(X)$	$\frac{dY}{dX} = \frac{dU}{dX} + \frac{dW}{dX}$
Products	$Y = UW$	$\frac{dY}{dX} = U\frac{dW}{dX} + W\frac{dU}{dX}$
Quotients	$Y = \frac{U}{W}$	$\frac{dY}{dX} = \frac{W\frac{dU}{dX} - U\frac{dW}{dX}}{W^2}$
Chain Rule	$Y = f(W(X))$	$\frac{dY}{dX} = \frac{dY}{dW}\frac{dW}{dX}$
Implicit Function Theorem	$0 = f(X,Y)$	$\frac{dY}{dX} = -\frac{df/dX}{df/dY}$

Maximization and Minimization for Single Variable Functions

Marginal value = 0 for maximum in dependent variable so $\frac{dY}{dX} = 0$

Maximum vs. minimum - second derivative:

$\frac{d^2Y}{dX^2} < 0$ for maximum $\frac{d^2Y}{dX^2} > 0$ for minimum

Partial Differentiation and Maximization of Multivariable Functions

Partial derivative: marginal effect of one variable holding all other independent variables constant ***- Ceteris paribus***

$Y = f(X_1, X_2)$, $\frac{\partial Y}{\partial X_1}$, $\frac{\partial Y}{\partial X_2}$

Extreme value: $\frac{\partial Y}{\partial X_i} = 0, \forall i$

Discriminating between a maximum and a minimum. For a two variable function:

If $\frac{\partial^2 Y}{\partial X_1^2}\frac{\partial^2 Y}{\partial X_2^2} - \left(\frac{\partial^2 Y}{\partial X_1 \partial X_2}\right)^2 > 0$ then this is a **maximum or minimum**. And additionally:

$\frac{\partial^2 Y}{\partial X_1^2} < 0, \frac{\partial^2 Y}{\partial X_2^2} < 0$ for a **maximum**, and

$\frac{\partial^2 Y}{\partial X_1^2} > 0, \frac{\partial^2 Y}{\partial X_2^2} > 0$ for a **minimum**, and

if $\frac{\partial^2 Y}{\partial X_1^2}\frac{\partial^2 Y}{\partial X_2^2} - \left(\frac{\partial^2 Y}{\partial X_1 \partial X_2}\right)^2 < 0$ then this is a **saddle point**.

When the expression is zero the test fails.

Notice the notation used for second partial derivatives. The third derivative in the inequality is known as the **cross-derivative**. It is found by first differentiating the function with respect to one variable and then differentiating the resulting derivative with respect to the other variable. The order of differentiation doesn't matter as the two cross-derivatives are equal.

Total Differential
The total differential shows the total change in a function of several variables:

$$Y = f(X_1, X_2), \qquad dY = \frac{\partial Y}{\partial X_1} dX_1 + \frac{\partial Y}{\partial X_2} dX_2$$

Constrained Optimization

Form of problem:

Maximize $Y = f(X_1, X_2)$

Subject to: $g(X_1, X_2) = 0$

Form **Lagrangian Function**:

$$L = f(X_1, X_2) + \lambda g(X_1, X_2)$$

Maximize this function

Lagrange Multiplier, λ, is marginal effect on Y of relaxing the restriction

We can determine whether the optimum reached is a maximum or minimum using the discrimination rule above. Usually it will suffice to check the derivatives of the two variables of the objective function f(). Formally, we need to generalize the rule for checking for a maximum. This involves checking the sign of the determinant of the so-called Hessian matrix of the Lagrangian function. This course does not require matrix algebra, so this will be ignored.