

Algebra Review

Microeconomic Analysis

Exponents or powers

Basics

$$a^0 = 1$$

$$a^1 = a$$

$$a^2 = aa$$

Fractional Powers

Fractional powers represent "roots" of the number raised to that power. For example:

$$a^{\frac{1}{2}} = \sqrt{a} \quad \text{square root of } a$$

$$a^{\frac{1}{3}} = \sqrt[3]{a} \quad \text{cubic root of } a$$

$$a^{\frac{2}{3}} = \sqrt[3]{a^2} \quad \text{cubic root of a squared}$$

Note that there are two square roots of any positive number - a negative root and a positive root of the same absolute value. Only roots of positive numbers or real roots are used in this course.

Negative Powers

Negative powers represent the inverse or reciprocal of the number raised to that power. For example:

$$a^{-1} = \frac{1}{a}$$

$$a^{-2} = \frac{1}{a^2}$$

$$a^{-\frac{2}{3}} = \frac{1}{a^{\frac{2}{3}}} = \frac{1}{\sqrt[3]{a^2}}$$

Rules of Operations Involving Power Functions

$$a^m a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$(ab)^m = a^m b^m$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \text{ if } b \neq 0$$

LogarithmsBasics

In this course we only use base e logarithms - so called "natural logarithms". The natural logarithm of a number a can be represented as $\ln a$. $e \approx 2.718$. We find the antilogarithm of a number b by computing e^b . $e^{\ln a} = a$.

Rules of Operations Involving Logarithms

$$\ln ab = \ln a + \ln b$$

$$\ln \frac{a}{b} = \ln a - \ln b$$

$\ln a^m = m \ln a$ This will be a very useful trick in the course for converting power functions to linear equations and vice versa. Simultaneous linear equations can be solved for unknown variables.

$$\ln e = 1$$

$$\ln 1 = 0$$

Logarithms of negative numbers are undefined.

Exponential Functions

$$e^m = \exp(m)$$

The rules applying to exponential functions are the same as those that apply to power functions. Exponential functions are just power functions involving the number e.

Simultaneous Equations

If we have n unknown variables we need to have at least n linearly independent linear equations involving those variables in order to find the values for the n variables. Each equation must also involve a non-zero constant term. The most elegant solutions use matrix algebra, which is not required for this course. The brute force technique is to multiply equations by numbers that when the equations are subtracted or added to each other result in a variable being eliminated. Example:

$$2x + 3y = 10$$

$$5x + 2y = 15$$

If we multiply the first equation by 2.5:

$$5x + 7.5y = 25$$

and then subtract the second equation from the first we get:

$$5x - 5x + 7.5y - 2y = 25 - 15 \rightarrow 5.5y = 10 \rightarrow y = 10 / 5.5$$

We then **substitute** this value of y into either the first or the second equation to find the value of x .

The Quadratic Equation Formula

A quadratic equation has the form:

$$ax^2 + bx + c = 0$$

If you have an equation involving both the square of x and x itself, first organize it into this form. A quadratic function has a parabolic graph. If $c \neq 0$ there are two values of x for which the equation holds true. To find those two values we use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The sign \pm means that we compute the formula twice - once using $+$ and once using $-$. In some economic examples only one of the values will be relevant - for example the positive value or the larger value. In other examples both values will be relevant.