Lie Theory Seminar

University of Notre Dame
Department of Mathematics

Speaker: Jeb Willenbring
University of Wisconsin, Milwaukee

Thursday, November 12, 2009
1:00 PM
258 Hurley Hall

Title: Tensor powers of the oscillator representation and the spherical principal series of GL(n,R)

Abstract:

Let n_1, n_2,...,n_r be positive integers, and set n=n_1+...+n_r. A difficult problem in finite dimensional representation theory is to provide a combinatorial description of the branching rule from K=O(n) (the orthogonal group) to the block diagonally embedded subgroup M=O(n_1)x...xO(n_r). Among the known results is an expression for the branching multiplicities involving the Littlewood-Richardson coefficients. However, these formulas are valid only within a certain "stable range" requiring that n_1,...,n_r are large with respect to the data describing the K-representation.

We review how this multiplicity formula can be deduced, via dual pairs, from the decomposition of a tensor power of an irreducible, infinite dimensional, representation of the Lie algebra sp_2m(R). We emphasize the special case of finding the dimension of the M-invariant subspace of an irreducible K-representation. Although outside the stable range, one may consider the case when n_1=...=nr=1, which corresponds to the problem of decomposing a tensor power of the oscillator representation. This situation is of particular interest as it sheds light on the multiplicity of K-types in the spherical principle series of GL(n, R).