Topology Seminar

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Title: A survey of some smooth concordance invariants

Abstract: With an appropriate notion of equivalence called concordance, knots in the three-dimensional sphere can be given the structure of an abelian group under the operation of connected sum. The equivalence relation is defined in terms of embedded surfaces in the four-dimensional ball. A surprising aspect of this group, called the knot concordance group, is that its structure varies greatly depending on whether the surfaces in its definition are smoothly or topologically embedded. The existence of this discrepancy is parallel to the distinction between smooth and topological closed four-manifolds, first observed using the powerful theories of Donaldson and Freedman, respectively.

In the past few years, several new knot concordance invariants have been discovered in the smooth category. Perhaps most notable are the invariants $\tau(K)$ and $s(K)$, both of whose values for the (p, q) torus knots provide new proofs of Milnor's famous conjecture on the unknotting number of these knots. $\tau(K)$ was discovered by Ozsváth-Szabó, and independently by Rasmussen, and its definition relies on the analytically defined knot Floer homology theory developed by these authors. $s(K)$, on the other hand, was discovered by Rasmussen and its definition is in terms of the combinatorial knot homology theory of Khovanov. Though quite different in their definition, the two invariants share several formal properties, and agree for many knots. Indeed, it was conjectured by Rasmussen that the two invariants are equal, up to normalization.

In this talk I will survey what is known about the two invariants, and discuss some of my recent results regarding the invariant τ. I will present the first known examples where the invariants disagree, discovered jointly with Philip Ording of Columbia University, and discuss what implications these examples have for the structure of the smooth knot concordance group.