Regulations for Doctoral Students in Mathematics

A supplement to the regulations in the Graduate School Bulletin of Information

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New Students

A new graduate student should plan to arrive at Notre Dame several days before registration. The student will be assigned an initial adviser. The student should talk with the Director of Graduate Studies (DGS), with the initial adviser, and with other faculty members, to get acquainted in the department and to decide on appropriate first-year courses.

Course Requirements

In the first year, a student registers for four courses per semester. Four basic courses is a common load, but students are permitted and encouraged to "place out" of basic courses if they arrive with an adequate knowledge of the material (the procedure for doing this is described later). A first-year student not taking four basic courses is expected to take topics courses and/or readings courses to bring the course load to four. Occasionally, there is reason for a student to take a different number of courses, but this needs the approval of the adviser and the DGS.

Basic Courses

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The basic courses provide essential background for later work. Basic courses do not have prerequisites, except that for certain two-semester sequences such as Algebra, Algebra I is the prerequisite for Algebra II. Basic courses all have regular homework and comprehensive examinations.

At the end of these regulations is a syllabus for each of the basic courses. In addition, there are references covering this material.

The other graduate level courses offered by the Mathematics Department are directed readings courses (697, 698 and 699) and topics courses. Topics courses do not have fixed syllabi - the same course number is used for courses covering a broad selection of topics of current interest.

A first-year student may be able to place out of one or more basic courses. To do so, the student must convince the faculty member teaching the basic course that he or she has mastered the material on the syllabus. A student who places out of a basic course substitutes a readings course or a topics course. Thus, by placing out of one or more basic courses, the student may be able accelerate the process of choosing a permanent adviser and beginning research.

A student should select basic courses carefully, with the help of the adviser and other faculty members. A student who is undecided about a research area, or is trying to choose among several areas, must choose with special care, to keep open all of the options.

The selection of an area of specialization normally occurs by the beginning of the second year. Students should explore possible areas of research as soon as possible, by taking topics and readings courses, attending departmental colloquia and seminars, and talking with various faculty members.

In the second year, a student normally registers for three courses per semester. Sometimes there is reason for a different number, but this needs the approval of the adviser and the DGS.

A student who has not yet passed the oral candidacy examination may register for a 3-credit readings course with a possible adviser. After the second year, a student normally registers for one credit of research and dissertation with the adviser. This is sufficient to be considered full-time. However, a student should continue to register for interesting or important courses through the full 4-5 years at Notre Dame.

A student's schedule must always have the approval of the adviser, and anything unusual must also be approved by the DGS.

Course requirements are summarized as follows.

1. A student must, by the end of the second year, complete or place out of at least 6 basic courses, approved by the adviser. For many students, 8 or 10 basic courses may be the right number.

2. A student must accumulate at least 36 credits in basic and topics courses during the first three years at Notre Dame. There is no transfer of credits. (Fourth and fifth year students continue to register for topics courses, as appropriate.)

3. Grades in courses range from A to F. One condition for being in good academic standing is to maintain an average of at least B (3.0).

4. All students are expected to attend departmental colloquia. Students are also expected to participate in seminars related to their mathematical interests.

Advisers

A new student has an initial adviser assigned by the DGS. As soon as feasible, the student should be working with a permanent adviser/thesis director. Sometimes the initial adviser ends up as the thesis director, but often there is a change in adviser. Well before the oral candidacy examination, the student needs an adviser who has agreed at least to supervise the student's preparation for this examination.

To change advisers, the student needs the consent of the new adviser. In addition, the student should inform the DGS and, as a courtesy, the old adviser. After passing the oral candidacy examination, a student may wish to change area as well as adviser. There is no problem in doing this so long as the student has taken the appropriate courses and has the consent of the new adviser. The student need not re-take the candidacy examinations.

The student should always feel free to come to the adviser, or the DGS, to talk about various aspects of academic life.

Candidacy Examinations

The candidacy examination has two components: written and oral.

Written Candidacy Examinations

Each student must pass separate written examinations on two of the following three:

(a) Analysis (choice of Real or Complex)

(b) Algebra

(c) Numerical Analysis

Each examination covers the syllabus in one of the basic courses: Real Analysis I or Complex Analysis I for (a), Algebra I for (b), and Numerical Analysis I for (c).

The written candidacy examinations are offered shortly before the beginning of the fall and spring semesters, and at the beginning of the summer. A student who fails the candidacy examination on a given course may re-take it (more than once), or may take an examination on a different allowable course. The written candidacy examinations must be completed by the beginning of the second year.

A student may take a written candidacy examination without taking the corresponding course. Placing out of the course does not count as passing the written candidacy examination.

Oral Candidacy Examination

The oral candidacy examination, taken after the written candidacy examinations are completed, focuses on an "advanced" topic. This may be related to a topics course or seminar, or it may come from advanced research texts or articles. In any case, the student should begin working on the advanced topic, with an adviser, well in advance of the examination. The material to be counted as the advanced topic must have the approval of the adviser and the DGS.

The oral candidacy examination begins with a presentation by the student, lasting approximately 30 minutes. This is followed by questions on material from a syllabus related to the presentation. The topic should be chosen months before the examination. The syllabus must be made available to all members of the examining board at the time they agree to serve.

The board of examiners for the oral candidacy examination consists of five people: an outside chair (not in the Mathematics Department) and four examiners from the Mathematics Department. The chair is selected by the Graduate School. The other members of the examining board are selected by the DGS. Normally, the adviser is one of the examiners.

All examiners should restrict their questions to the advanced topic or other material on the given syllabus. The examination lasts from one and a half to three hours.

After the completion of the examination, the four examiners vote "pass" or "fail." A vote of "pass" means that, in the eyes of the particular examiner, the student has passed all parts of the examination. The student is considered to have passed the oral candidacy examination only if at least three of the four examiners vote "pass." In case the student does not pass, the examiners vote whether to recommend the student for a master's degree.

The student is informed of the outcome of the examination immediately.

The oral candidacy examination is usually held in early September, late November, late January and/or April. All students must take the oral candidacy examination by the end of January in the second year. Students who fail the first time may take the examination again, but must do so no later than April of the second year. The DGS arranges the oral candidacy examination in consultation with the student.

In the Mathematics Department, there is no requirement for a "doctoral dissertation proposal." The material for the advanced topic might include specific research problems and partial results but, for most students, the candidacy examinations come when they can only propose to work in a certain area.

Foreign Language Requirement

In addition to knowing English, a student must demonstrate the ability to read technical material in another language, said language to be chosen after consultation with the adviser and with the approval of the DGS. The usual choices are French, German, and Russian, but other languages are allowed. The choice should take into account the body of literature in the student's field of study, and the specific books and papers which the adviser has found to be important and might conceivably ask the student to read.

Certification should be done by a faculty member other than the adviser, but the adviser may be asked to suggest references from which a test passage may be drawn. In case of languages for which there is no member of the faculty competent to administer an examination, the student may be asked to translate a passage for which a translation already exists. In such a case, it is up to the adviser to provide the examiner with a choice of appropriate articles. In obvious cases, the DGS may deem the candidate to have passed.

The language and literature departments of the University offer language courses for graduate students during the summer—contingent on sufficient enrollment.

The language examination must be taken by the end of the third year. The examination is given in the Mathematics Department, normally in April and in November. Arrangements are made by the DGS, in consultation with the student.

The Thesis

Thesis research, under the supervision of the thesis director, normally begins after the successful completion of the candidacy examinations. The thesis director is expected to be concerned with the interest and significance of a thesis topic, with the originality of the research, and with the accuracy and the style of the manuscript. The final draft of the thesis should provide enough background and detail to make for easy reading by a semi-expert in the area, but should also be in a form that can easily be edited and shortened for publication, so that it would be suitable for publication in a good mathematics journal.

After the thesis has been approved by the thesis director, it is submitted to three official readers. Normally, these are professors in the Mathematics Department at Notre Dame (exceptions require special documentation). The official readers are appointed by the Graduate School, on the recommendation of the department chair. After the thesis has been approved by all three official readers, the thesis defense is scheduled. In approving the thesis, the official readers certify that it is worthy of defense. They may continue to require changes.

The thesis defense is an oral examination on the contents of the thesis and its relation to other work in the same area. The board of examiners for the thesis defense consists of five people: a chair (from outside the Mathematics Department), and four examiners. The chair is appointed by the Graduate School, the other four by the DGS. Normally they will be the thesis director and the three official readers

The examination begins with a 30-50 minute presentation by the Ph.D. candidate, prepared in consultation with the thesis director (who also sets the length). This is followed by a round of questions by the examiners. There may be questions about specific points in the thesis, and also about the importance of the research and what further work it suggests. A thesis defense is public, in the sense that people other than the candidate and the members of the board of examiners may be present for the lecture. Such people leave the room prior to the vote. Voting is as for the oral candidacy examination. As for the oral candidacy examination, the candidate is informed of the outcome immediately.

After a successful defense, the candidate may still need to make some minor changes in the thesis. Then the final version of the thesis, signed by the thesis director, is submitted to the Graduate School.

Getting the thesis read and approved, scheduling the thesis defense, making corrections, and having the thesis accepted by the Graduate School is a time-consuming process that requires strict adherence to the timetables set by the Department of Mathematics and the Graduate School. The thesis must be submitted to the readers well before the Graduate School deadline for submission of theses. The latter is roughly two months before the graduation date. August graduation entails special difficulties, since there are fewer faculty members available during the summer to serve as official readers.

There are strict rules about formatting, margins, etc., which must be observed if the thesis is to be accepted by the Graduate School. The Ph.D. candidate should be sure to consult the Graduate School's *Guide for Formatting and Submitting Doctoral Dissertations and Master's Theses.* This document changes from year to year, so it is important to consult the current version.

Summary of Official Requirements for the Ph.D. Degree

1. Courses and credits—appropriate basic courses, 36 credits in basic and topics courses

2. Residency—4 consecutive semesters of full-time study (as required by the Graduate School)

3. Written and oral candidacy examinations

4. Foreign language requirement

5. Admission to Degree Candidacy (see the Bulletin of Information of the Graduate School)

6. Thesis carried through the following steps:

   a. approved by all three readers
   b. typed
   c. signed by adviser
   d. accepted by the Graduate School

7. Thesis Defense (this cannot be officially scheduled until steps 1-5 and 6a are completed).

Student Status

A student is considered to be full-time if he or she is registered for appropriate courses and the adviser indicates that the student is working full-time. After a certain point, the student may be registered for as little as one credit. The student must give the adviser reason to believe that he or she is spending sufficient time and effort in individual study and research. The Mathematics Department does not admit students planning to study on a part-time basis.

To be in good academic standing, a student must maintain a B average and be on schedule in terms of course work and examinations. In addition, once the student is not registered for basic courses, the student's adviser must indicate that the student is making satisfactory progress. Thus, it is essential for the student to keep the adviser informed about his or her progress. Normally, a student who is not registered for basic courses is registered for a readings or research and dissertation course with an adviser, and the adviser indicates that the student is making satisfactory progress by giving a grade of at least B in this course.

A student who does not succeed in passing the first year courses is not permitted to continue in the second year. A student who does not pass the candidacy examinations by the end of the second year is not permitted to continue in the third year. A student who, for two consecutive semesters, fails to maintain good academic standing (as described above) is not permitted to continue further.

A student must fulfill all doctoral requirements, including the dissertation and its defense, within eight years from the time of matriculation. Failure to complete any of the Graduate School or departmental requirements within the prescribed period results in forfeiture of degree eligibility. A student is normally expected to finish in five years or less.

The M.S. Degree

The graduate program in the Mathematics Department is almost entirely a Ph.D. program. Students are not normally admitted directly to a Master's program. There is an Master of Science in Applied Mathematics, for students who do not need funding and wish to pursue an interdisciplinary project, or to carry out serious mathematical work while pursuing a Ph.D. in another department. (The requirements for the MSAM are described elsewhere.)

A student who is working toward a Ph.D. may qualify for a Master of Science degree along the way, if he or she has accumulated 30 credit hours, has passed the written candidacy examination, and has either passed the oral candidacy examination or (without passing) exhibited sufficient knowledge to obtain a positive recommendation from the examiners. Having met the requirements, a student must also ask to have his or her name put on the graduation list. The degree is not given automatically.

Financial Support

Continued financial support requires the student to maintain good academic standing, and to carry out any teaching duties associated with the support. The Mathematics Department does not provide financial support beyond the fifth year. Neither teaching assistants nor fellowship holders are allowed to take outside employment without special approval. (Occasionally, students are permitted to tutor up to four hours per week, with the permission of the adviser and the Department Chair.)

Decisions about financial support are made at the end of the spring semester for the next academic year. Students are informed about support at that time.

Teaching Opportunities

First year students have no teaching duties. Second and third year students are generally assigned to conduct tutorial sessions or to grade papers. After gaining some experience with the tutorial sessions, students are assigned to teach a course—usually one section of a multi-section introductory course, under the supervision of a senior faculty member.

Grievance Procedure

If a graduate student has a grievance, the procedure is to go first to the DGS, unless the grievance involves the DGS, in which case, the student should approach the Department Chair. The DGS, or the Department Chair, will try work with the parties involved to reach a solution. If the student feels that this is not enough, then he or she may appeal to the Graduate School.

The Director of Graduate Studies

The DGS has direct responsibility in the following areas:

1. Providing information about the program to prospective and current graduate students

2. Assigning initial advisers, and overseeing course placement

3. Selecting examiners for oral candidacy examinations (for approval by the Graduate School)

4. Overseeing language examinations, and the scheduling of thesis defenses.

Teaching assignments are made by the department's Associate Chair.

Recommendations for fellowships and stipends are made by the Department Chair and the DGS, in consultation with the adviser, when appropriate.

Syllabi for Basic Courses

The syllabi for the basic courses are given below, with references for each subject.

Algebra I, II

1. Groups
Subgroups, quotient groups, direct products, the homomorphism theorems. Automorphisms, conjugacy, commutators, solvability. Theorems of Lagrange, Sylow. EXAMPLES: Cyclic groups, groups of permutations, matrix groups

2. Rings
Ideals, quotient rings, the homomorphism theorems. Prime and maximal ideals, localization. Local rings, polynomial rings, Noetherian rings. Principal ideal domains, Euclidean rings, unique factorization domains. EXAMPLES: Rings of functions, matrix rings

3. Modules
Theory of modules, projective, finitely generated free modules. The language of categories and functors. Localization of modules. Structure theory of modules over a principal ideal domain (structure of finitely generated Abelian groups). Tensor products. Algebras, Wedderburn's theory of simple algebras, Brauer groups. EXAMPLES: Finite dimensional vector spaces over fields: Basis, dimension, duality, linear transformations and matrices, rank and nullity. Canonical forms for the matrix of a linear transformation, invariant factors and elementary divisors of a matrix.

4. Fields
Algebraic and transcendental field extensions, degree, transcendence base, algebraic closure, structure of finite fields. Separability, normal extensions, Galois groups, fundamental theorem of Galois theory. Examples of Galois groups over the rationals. Cyclotomic extensions, cyclic extensions, theory of equations.

References

T. Hungerford, *Algebra, Graduate Texts in Mathematics 73.*

N. Jacobson, *Basic algebra I, II*.

S. Lang, *Algebra*.

Algebra I covers groups and rings, and introduces modules (omitting material on the language of categories and functors, and localization of modules, but including the structure theory for modules over a principal ideal domain. The material is roughly Chapters 1-4 of Hungerford. Algebra II covers the remaining material.

Real Analysis I, II

1. Calculus
Calculus of one and several variables, including line integrals, surface integrals, Stokes' theorem, the Implicit and Inverse Function Theorems, pointwise and uniform convergence of sequences of functions, integration and differentiation of sequences, the Weierstrass Approximation Theorem, the existence and uniqueness of solutions of ordinary differential equations.

2. Lebesgue measure and integration on the real line
Measurable sets, Lebesgue measure, measurable functions, the Lebesgue integral and its relation to the Riemann integral, convergence theorems, functions of bounded variation, absolute continuity and differentiation of integrals.

3. General measure and integration theory
Measure spaces, measurable functions, integration convergence theorems, signed measures, the Radon-Nikodym Theorem, product measures, Fubini's Theorem, Tonelli's Theorem.

4. Families of functions
Equicontinuous families and the Arzela-Ascoli Theorem, the Stone-Weierstrass Theorem.

5. Banach spaces
L^p- spaces and their conjugates, the Riesz-Fisher Theorem, the Riesz Representation Theorem for bounded linear functionals on L^p, C(X), the Riesz Representation Theorem for C(X), the Hahn-Banach Theorem, the Closed Graph and Open Mapping Theorems, the Principle of Uniform Boundedness, Alaoglu's Theorem, Hilbert spaces, orthogonal systems, Fourier series, Bessel's inequality, Parseval's formula, convolutions, Fourier transform, distributions, Sobolev spaces. (In regard to the last three topics consult Folland's book—see the references—for an indication of what is expected).

References

Apostol, *Mathematical Analysis.*

Riesz-Nagy, *Functional Analysis.*

Royden, *Real Analysis.*

Rudin, *Principles of Mathematical Analysis.*

Rudin, *Real and Complex Analysis.*

Rudin, *Functional Analysis.*

Simmons, *Introduction to Topology and Modern Analysis.*

Wheeden-Zygmund, *Measure and Integration.*

Folland, *Real Analysis.*

Real Analysis I covers the material on calculus, and Lebesgue measure and integration. It is roughly Chapters 1-3 of Folland. The remaining material is in Real Analysis II.

Complex Analysis I, II

I. Winding number, integral along curves. Various definitions of a holomorphic function. Connection with harmonic functions. Cauchy Integral Theorems and Cauchy Integral Formula for closed curves in a domain, and for the boundary of a domain, Poisson Formula. The integral of a holomorphic function and its dependence on the path of integration. Open Mapping Theorem, Inverse Function Theorem, maximum and minimum principle, Liouville's Theorem. Uniform convergence of holomorphic functions. Normal families of holomorphic functions. Montel and Vitali Theorems, Picard's Theorem. Power series, Laurent series. Residues and classification of isolated singularities, meromorphic functions. Divisor of a meromorphic function. Residue Theorem, argument principle, Rouché's Theorem, computation of integrals. Riemann Mapping Theorem, argument principle. Möbius maps. Schwartz Lemma. Theorems of Mittag-Leffler and Weierstrass. Gamma Function, Riemann Zeta Function, Weierstrass $\wp$-function.

II. Definition of complex manifolds and examples. Riemann surfaces. The concepts of divisors, line bundles, differential forms and Chern forms. The Riemann-Roch Theorem. The Dirichlet problem for harmonic functions. The concept of genus of a Riemann surface.

References

Ahlfors, *Complex Analysis*.

Burchkel, *An Introduction to Classical Complex Analysis I*.

Conway, *Functions of One Complex Variable*.

Forster, *Lectures on Riemann Surfaces.*

Gunning, *Lectures on Riemann Surfaces.*

Knopp, *Theory of Functions I, II, and Problem Books*.

Complex Analysis I covers approximately Chapters 1-6 of Ahlfors. Complex Analysis II covers the remaining material.

Topology I, II

I. General Topology

1. Separation properties
Hausdorff, regular, completely regular and normal spaces. Urysohn's Lemma and Tietze's Theorem.

2. Construction of topological spaces
Product and quotient spaces, metric spaces, Baire Category Theorem.

3. Covering properties
Compact and locally compact spaces. Tychonoff Theorem. Paracompactness and partitions of unity. Some metrization theorem.

4. Miscellany
Path connectedness and connectedness. Topology for mapping spaces, Ascoli's Theorem.

References

Cullen, *Introduction to General Topology.*

Dugundji, *Topology.*

Kelley, *General Topology.*

Munkres, *Topology.*

Steen, *Counterexamples in Topology.*

II. Algebraic Topology

1. The fundamental group
Covering spaces, VanKampen's Theorem and calculation of fundamental groups of surfaces.

2. Homology
Singular homology and cohomology theory. Eilenberg-Steenrod axioms. The cohomology ring. Homology calculations via CW complexes. Calculation of the cohomology ring of projective spaces.

3. Homotopy
Exact homotopy sequence of a pair. Hurewicz's Theorem.

4. Manifolds
The Poincaré Duality Theorem.

References

J. Vick, *Homology Theory.*

G. Whitehead, *Homotopy Theory.*

E. Spanier, *Algebraic Topology.*

A. Dold, *Lectures on Algebraic Topology.*

C. Maunder, *Algebraic Topology.*

R. Greenberg and J. Harper, *Lectures on Algebraic Topology.*

A. Hatcher, *Algebraic Topology*.

Topology I covers the material on general topology, plus the first part of algebraic topology - the fundamental group, covering spaces, and VanKampen's Theorem (found in Chapter 1 of Hatcher). The remaining material is in Topology II.

Logic I, II

1. Model Theory

Propositional logic, first order predicate logic. Completeness and Compactness Theorems. Löwenheim-Skolem Theorem, saturated models. Omitting types, $\aleph_0$–categoricity, ω-stability. Quantifier elimination, structure of definable sets. Examples of theories: algebraically closed fields, real closed fields, Presburger arithmetic, dense and discrete linear orderings.

References

Chang and Keisler, *Model Theory.*

Sacks, *Saturated Model Theory.*

Poizat, *Cours de Théorie des Modèles.*

2. Computability Theory

Turing machines, Kleene definition of partial recursive functions, primitive recursive functions. Halting set, recursive sets, computably enumerable sets. Rice's Theorem, Myhill Isomorphism Theorem, index sets. Turing degrees, jumps of sets and degrees. The Hierarchy Theorem, the Recursion Theorem. Computably enumerable degrees, Friedberg-Muchnik Theorem. Gödel's Incompleteness Theorem, Tarski's Undefinability Theorem.

References

R. I. Soare, *Recursively Enumerable Sets and Degrees*.

N. Cutland, *Computability*.

H. Enderton, *A Mathematical Introduction to Logic*.

3. Set Theory
Axioms of ZFC, Schröder-Bernstein Theorem, ordinals, ordinal arithmetic, proof and definition by recursion. Cardinals, cardinal arithmetic, regular and inaccessible cardinals. Well-founded sets and the Levy hierarchy. Absoluteness. The constructible hierarchy. Consistency of GCH and AC.

References

T. Jech, *Set Theory.*

K. Kunen, *Set Theory.*

P. Cohen, *Set Theory and the Continuum Hypothesis.*

Logic I covers either all of the material on model theory and the material on computability through Myhill's Theorem, or else all of the material on computability and the material on model theory through the Compactness Theorem. Logic II covers the remaining material.

Nonlinear Dynamical Systems

Review of linear and nonlinear dynamical systems, such as Duffing's, Van der Pol's, and Lorentz equations, geometry of the phase space, symplectic structures, variational methods, nonlinear Hamiltonian systems, integrable systems, quasiperiodic motion, averaging method, discrete dynamical systems, and the logistic function.

Bifurcation phenomena and transition to chaos and theory of patterns. These include Hamiltonian vector fields, normal forms, stable and unstable manifolds, structural stability, Poincaré maps, Liapunov exponents, power spectra, Hopf bifurcation, Smale diffeomorphism, perturbations of nonlinear systems, the geometric structure of the perturbed phase space, chaos and nonintegrability in Hamiltonian systems, KAM theory, perturbation of homoclinic orbits, Poincaré-Melnikov method; for example, Arnold diffusion, symbolic dynamics, hyperbolic sets, strange attractors, numerical route to chaos. Theory of patterns, including fractals, the Julia and Mandelbrot sets, lattice-based models, pattern dynamics in physics and biology, pattern inference, pattern recognition, and metric pattern theory.

Discrete Mathematics

1. Graph theory: trees and graphs, Eulerian and Hamiltonian graphs, tournaments, graph coloring and Ramsey's theorem. Applications to electrical networks.

2. Enumerative combinatorics: inclusion-exclusion principle, generating functions, Catalan numbers, tableaux, linear recurrences and rational generating functions, and Polya theory.

3. Partially ordered sets: distributive lattices, Dilworth's theorem, Zeta polynomials, Eulerian posets.

4. Projective and combinatorial geometries, designs and matroids.

References

J.H. van Lint and R. M. Wilson, *A Course in Combinatorics*, 2nd ed.

Optimization

Convex sets. Caratheodory and Radon's theorems. Helly's Theorem. Facial structure of convex sets. Extreme points. Krein-Milman Theorem. Separation Theorem. Optimality conditions for convex programming problems. Introduction to subdifferential calculus. Chebyshev approximations.

References

Barvinok, *A Course in Convexity*.

R. Webster, *Convexity*.

Basic PDE

Laplace equations: Green's identity, fundamental solutions, maximum principles, Green's functions, Perron's methods. Parabolic equations: Heat equations fundamental solutions, maximum principles, finite difference and convergence, Stefan Problems. First order equations: Method of characteristics, Cauchy problems, vanishing of viscosity-viscosity solutions. Real analytic solutions: Cauchy-Kowalevski theorem, Holmgren theorem.

Reference

L. Evans, *Partial Differential Equations*.

Numerical Analysis I

1. Polynomial interpolation (including generalized Hermite interpolation), two dimensional interpolation, splines, trigonometric interpolation.

2. Least squares and the basic theory of orthogonal functions.

3. Numerical integration in one variable, including adaptive methods, Romberg integration, Gauss quadrature and the relations to orthogonal functions.

4. Numerical linear algebra

(a) direct methods and analysis of error based on the condition number,

(b) basic numerical factorizations of matrices and the singular value decomposition,

(c) iterative methods; eg., the Jacobi method and the method of successive over-relaxation,

(d) brief introduction to Krylov methods,

(e) methods to find eigenvalues and eigenvectors such as the QR Method, power method, and inverse power method.

5. Methods to solve systems of nonlinear equations; e.g., Newton-like methods and constrained Newton methods such as homotopy continuation.

6. Numerical solution of ordinary differential equations by marching methods, multistep methods, shooting, finite differences, and the finite element method (including such variants as the Galerkin and Rayleigh-Ritz method).

7. Solution of some simple partial differential equations by difference methods.

References

P.J. Davis, *Interpolation and Approximation*.

C. De Boor, *A Practical Guide to Splines.*

E. Isaacson and H. G. Keller, *Analysis of Numerical Methods*.

J. Stoer and R. Bulirsch, *Introduction to Numerical Analysis*, 2nd ed.

E. E. Tyrtyshnikov, *A Brief Introduction to Numerical Analysis*.

Numerical Methods in PDE

This is part of two-semester sequence, with Numerical Analysis I as prerequisite. Finite difference methods for time dependent equations and systems of equations.

1. Interpolation (particularly interpolation using trig functions), grid functions, and approximation of derivatives.

2. Examples of systems of partial differential equations arising in engineering and science, and the stability and convergence of their solutions.

3. High order accurate difference methods, and Fourier methods.

4. Well posed problems and general solutions for a variety of types of systems of equations with constant coefficients.

5. Stability and convergence: for constant coefficient systems, for variable coefficients.

6. Hyperbolic systems of equations with constant coefficients and then with variable coefficients in one and then several space variables, the method of lines, the finite volume method, and the Fourier method.

Reference

A. Gustafsson, H.O. Kreiss, and J. Oliger, *Time Dependent Problems and Difference Methods*.

Chapters 1-6 cover the material.

Differentiable Manifolds

Differentiable manifolds, vector fields, differential forms, and tensor analysis; inverse and implicit function theorems, transversality, Sard's theorem, Morse theory, integration on manifolds, Stokes' Theorem, de Rham cohomology.

Nonlinear Analysis

Elements of variational calculus, with application to: theory of interfaces, existence of solitons, vortices and bubbles, image segmentation, control theory. Implicit function and fixed-point theorems, with application to: Bose-Einstein condensation, existence of discrete breathers, existence of small data solutions of nonlinear Schroedinger, heat, and wave equations, economics. Gradient and Hamiltonian systems: energy conservation versus energy dissipation, stability of stationary solutions and traveling waves, stability of periodic solutions and Floquet theory.

References

L. Johsson, M. Merkli, and I. M. Sigal, *Lectures on Applied Analysis*, (draft notes).

Linear Control

Introduction to linear system theory. Linear-quadratic control, H-infinity control, introduction to robust control based on matrix cube theorem, linear matrix inequalities and interior-point algorithms.

References

A. Ben-Tal and A. Nemirovski, *Lectures on Modern Convex Optimization*.

P. Lancaster and L. Rodman, *Algebraic Riccati Equations*.

S. Boyd et. al. *Linear Matrix Inequalities in System and Control Theory*.

Probability

1. Elements of measure and integration theory.

2. Basic setup of probability theory (including sample spaces, conditional probability, independence). Random variables, the "law of large numbers."

3. Discrete random variables (including random walks).

4. Continuous random variables, the basic distributions, sums of random variables.

5. Generating functions, branching processes, basic theory of characteristic functions, central limit theorems.

6. Markov chains (embedding, birth and death processes, Poisson processes).

7. Monte Carlo simulations.

8. More "laws of large numbers," including the law of the iterated logarithm, Martingales, filtered sigma algebras, and the simplest martingale convergence theorems.

9. Various stochastic processes, including Brownian motion, queues, and applications.

10. Martingales, including stopping times and optimal stopping.

11. The rudiments of stochastic integration (including Ito's formula and the Black-Scholes differential equation).

References

G. Grimmett and D. Strizaker, *Probability and Random Processes*.

L. Rogers and D. Williams, *Diffusions, Markov Processes, and Martingales*, vol. 1.

D. Williams, *Probability and Martingales*.

Stochastic Analysis

This is the second part of a two-semester sequence, with Probability as a prerequisite.

Introduction to stochastic modeling and the underlying theory, with application to models from science and engineering.

1. Stochastic versus deterministic models.

2. Diffusion processes in physics, biology, population dynamics, and epidemiology.

3. Discrete and continuous Markov chain models, with applications.

4. The long run behavior of Markov chains.

5. Poisson processes, with applications.

6. Brownian motion and related processes.

7. The Ornstein-Uhlenbeck Process.

8. Elements of stochastic dynamical systems.

9. Numerical methods for stochastic processes.

References

H. M. Taylor and S. Karlin, *An Introduction to Stochastic Modeling*, 3rd ed.

A. Lasota, M. C. Mackey, *Chaos, Fractals, and Noise: Stochastic Aspects of Dynamics*, 2nd ed.

GRADUATE BULLETIN EXCERPTS

The following excerpts from the *Graduate Bulletin* are included for information. You are responsible for following all of the material in the Bulletin.

Academic Policies of the Graduate School

Degree Eligibility

The student must fulfill all doctoral requirements, including the dissertation and its defense, within eight years from the time of matriculation. Failure to complete any of the Graduate School or departmental requirements within the prescribed period results in forfeiture of degree eligibility.

Enrollment

Once admitted, all degree and non degree graduate students must enroll before each semester at the times and locations announced by the University Registrar. Enrollment dates are published in the Graduate School Calendar. Any admitted student who fails to enroll for one semester or more must apply for readmission upon return. (See "Continuous Enrollment," below.) All degree-seeking students are expected to maintain full-time status and to devote full time to graduate study. No degree student may hold a job, on or off campus, without the express permission of his or her department and the Graduate School.

Continuous Enrollment

All students must enroll each semester in the academic year to maintain student status. Continuous enrollment is met normally by enrollment in the University and registration in a graduate-level course relevant to the student's program. A student who is concurrently pursuing degrees in the Graduate School and in another school in the University meets the continuous enrollment requirement by registering for a course in either program. Any exception to this rule, including a leave of absence, must be approved by the Graduate School. (See "Leave of Absence," below.) Degree students who have completed the credit-hour requirement for their degree must register for at least one credit hour per semester, including the final semester or summer session in which they receive their degree. These students may be considered full-time students whether or not they are in residence. Students not in residence and taking one credit hour pursuant to continuous enrollment requirements are charged a special registration fee.

A student who fails to enroll for one semester or more must apply for readmission upon return.

Continuing degree-seeking students (i.e., degree students who are eligible to continue their studies in the fall semester) may have access to University facilities and services from May through August without registering and enrolling for academic credit in the Summer Session.

Leave of Absence

For exceptional reasons and on the recommendation of the department, a student in academic good standing may request a leave of absence for a maximum of two consecutive semesters. A request for a leave of absence must be made before the semester in which the leave is taken and all leaves of absence must be approved by the Graduate School. If, for some urgent reason, a student is allowed to leave the University after the beginning of the semester, the withdrawal procedure below must be followed. If at the end of the leave of absence period the student does not return, the student is considered terminated. Application for readmission is required if the student wishes to return. In the case of a medical leave of absence, clearance from the University Health Center is required prior to readmission.

Withdrawal from the Program

To withdraw from the University before the end of the semester, a student must inform the department and the Graduate School as well as complete the notice of withdrawal in the Office of the Registrar. Upon approval of the withdrawal, the University enters a grade of "W" for each course in which the student was registered. If a student drops out of the University without following the procedure described above, a grade of "F" is recorded for each course. The credit for any course or examination will be forfeited if the student interrupts his or her program of study for five years or more. The University reserves the right to require the withdrawal of any student when academic performance, health status or general conduct may be judged clearly detrimental to the best interests of either the student or the University community.

Maximal Registration

During the academic year, a graduate student may not register for more than 12 credit hours of graduate courses, i.e., the 500-, 600- and 700-level courses, each semester. An additional three credit hours of 400-level courses may be taken if authorized by the department chair and approved by the Graduate School. In the Summer Session, a graduate student may not register for more than 10 credit hours.

Changes in Student Class Schedules

A student may add courses only during the first seven class days of the semester. Students may add courses after this time only on recommendation of the department and with approval of the Graduate School. A student may drop courses during the first seven class days of the semester. To drop a course after this period and up to the midsemester point (see the Graduate School Calendar for the exact date), a student must have the approval of the chair of the department offering the course, the student's adviser and the Graduate School. A course may be dropped after the midsemester point only in cases of serious physical or mental illness. Courses dropped after this date will be posted on the student's permanent record with the grade of "W." A course taken for credit can be changed to an audit course after the midsemester point only in cases of serious physical or mental illness.

Course Numbers

No graduate credit is allowed for courses below the 400 level.

The advanced undergraduate courses numbered 400 through 499 may, with the approval of the department chair and the Graduate School, be taken to satisfy up to 10 hours of graduate credit requirements. Departments may place additional constraints on the use of 400-level courses to meet their degree requirements.

Courses numbered 500 through 599 are first-level graduate courses into which qualified advanced undergraduates may be admitted with the permission of the instructor and the approval of the chair. Courses numbered 600 and above are advanced graduate courses open only to those who have completed the undergraduate and graduate prerequisites.

Graduate Grades

Listed below are graduate grades and the corresponding number of quality points per credit hour.

A	4	
A-	3.667	
B+	3.333	
B	3	
B-	2.667	
C+	2.333	
C	2	
F	0	
I	0	(Until Incomplete is removed)
NR	None	No grade reported
S	None	Satisfactory
U	None	Unsatisfactory
V	None	Auditor (graduate students only)
W	None	Withdrew

Quality point values are used to compute the student's grade point average (G.P.A.). The G.P.A. is the ratio of accumulated earned quality points to the accumulated earned semester credit hours. G.P.A. computation takes into account only those grades earned in Notre Dame graduate courses by students with graduate status at Notre Dame. For courses taken in a department or college in the University but outside the Graduate School, or taken outside the University, the grade will not be included in the G.P.A. computation.

If a grade of "C-" or "D" is given to a graduate student for a course taken in any department or college in the University, the grade will be considered equivalent to an "F."

A student receives the temporary grade of "I" when, for acceptable reasons, he or she has not completed the requirements for a 500 or higher level graduate course within the semester or Summer Session. No grade of "I" can be given for courses below the 500 level or to graduating students in the final semester or final Summer Session.

The student then must complete the course work for a grade prior to the beginning of the final examination period of the next semester in which the student is enrolled. If a student receives an "I" for a Summer Session course, he or she must complete the course work for a grade before the final examination period begins for the next semester or Summer Session (whichever comes first) in which the student is enrolled.

The University temporarily computes this grade as the equivalent of an "F" in calculating the G.P.A. When the student fulfills the above requirements, the "I" is replaced by the new grade. Should the student not complete the course work as required, the "I" remains on the academic record and is computed in the G.P.A. as equivalent to an "F."

The department and the Graduate School will review a student who receives more than one "I" in a semester or an "I" in two or more consecutive semesters, to determine his or her eligibility for continued support and enrollment.

The grades of "S" and "U" are used in courses without semester credit hours, as well as in research courses, departmental seminars, colloquia, workshops, directed studies, field education and skill courses. These courses, if given the grade of "S," do figure in a student's earned semester credit-hour total but do not figure in the computation of the G.P.A. A grade of "U" will not count toward the student's earned semester credit-hour total, nor will it figure in the computation of the G.P.A.

The grade of "V" has neither quality-point nor credit-hour values. It is the only grade available to the registered auditor who requests at the beginning of the semester that it be made part of his or her permanent record and who attends the course throughout the entire semester. The grade of "V" cannot be changed to a credit-earning grade.

The grade of "W" is given for a course that a student is allowed to drop after the midsemester point.

Examinations

Unexcused absence from a scheduled final examination results in an "F." An absence excused in advance results in an "I" (incomplete).

Academic Integrity

Integrity in scholarship and research is an essential characteristic of academic life and social structure in the University. Any activity that compromises the pursuit of truth and the advancement of knowledge besmirches the intellectual effort and may undermine confidence in the academic enterprise. A commitment to honesty is expected in all academic endeavors, assistants, associates and colleagues by mentors and academic leaders. The procedures for ensuring academic integrity in the Graduate School are distinct from those in the Undergraduate Honor Code. Violations of academic integrity may occur in classroom work and related academic functions or in research/scholarship endeavors. Classroom-type misconduct includes the use of information obtained from another student's paper during an examination, plagiarism, submission of work written by someone else, falsification of data, etc. Violation of integrity in research/ scholarship is deliberate fabrication, falsification or plagiarism in proposing, performing or reporting research or other deliberate misrepresentation in proposing, conducting, reporting or reviewing research. Misconduct does not include errors of judgment, errors in recording, selection or analysis of data, differences in opinions involving interpretation, nor conduct unrelated to the research process. Misconduct includes practices that materially and adversely affect the integrity of scholarship and research.

If an individual suspects that a violation of academic integrity has occurred, he or she should discuss the matter confidentially with the department chair or appropriate director. If there appears to be a reasonable basis for further inquiry, the chair will select an impartial panel consisting of three members, one of whom may be a graduate student, to investigate the matter. The chair will inform the accused of the charges. The panel will determine initially whether to proceed directly to a hearing, to further investigate the case, or to dismiss the charges. If the panel decides to proceed directly to a hearing, the hearing will be held within 10 days of the original notification. If the panel decides that further investigation is necessary, it shall immediately notify the chair. If it decides that a hearing is not warranted, all information gathered for this investigation will be destroyed. The utmost care will be taken to minimize any negative consequence to the accused. The accused party must be given the opportunity to respond to any and all allegations and supporting evidence at the hearing. The response will be made to the appointed panel. The panel will make a final judgment, recommend appropriate disciplinary action, and report to the chair in writing. The report will include all of the pertinent documentation and will be presented within 30 days after meeting with the accused. Copies of the report are to be made available to the accused, the chair, and the vice president. If a violation is judged to have occurred, this might be grounds for dismissal from the University; research/scholarship violations might be reported to the sponsor of the research effort (e.g., NSF, NIH, Lilly Foundation, etc.), if appropriate.

If the student chooses to appeal, he or she must address the appeal in writing to the Vice President for Graduate Studies and Research within 10 days. The student has the right to appear before the vice president or his or her delegate. The vice president may decide to appoint an ad hoc committee to handle this appeal, if deemed necessary.

Policies on Harassment and Other Aspects of Student Life

Sexual and discriminatory harassment are prohibited by the University. Definitions and policies regarding sexual harassment, discriminatory harassment and other aspects of student life and behavior are described in du Lac, which is the University's description of student life policies and procedures. Students in the Graduate School must abide by those portions of du Lac which explicitly refer to graduate students or to the Graduate School. Copies of du Lac are mailed to all continuing students at the beginning of the fall semester, and may be obtained from the Office of Residence Life, 315 Main Building. The Vice President for Graduate Studies and Research has appointed an academic counselor in the Graduate School to be available to graduate students who want to discuss confidentially problems they are having in their programs. The counselor can help a student decide how to resolve the problem. The Graduate School's academic counselor is Dr. Barbara M. Turpin, associate dean.

Discriminatory Harassment

(Approved at the April 16, 1991 meeting of the Academic Council)

I. Policy

The University of Notre Dame believes in the intrinsic value of all human beings. It is, moreover, committed to the full peaceable participation of all its members in the educational endeavor it fosters. This is the reason that the University prohibits discriminatory harassment as defined below. The University is also committed to the free expression and advocacy of ideas; it wishes to maintain the integrity of this commitment as well. For this reason, cases of verbal harassment are defined here with great caution. Harassment in general is prohibited elsewhere in the University's regulations.

II. Definition

For purposes of this policy:

A. Harassment
Harassment is any physical conduct intentionally inflicting injury on the person or property of another, or any intentional threat of such conduct, or any hostile intentional, and persistent badgering, addressed directly at another, or small group of others, that is intended to intimidate its victim(s) from any University activity, or any verbal attack, intended to provoke the victim to immediate physical retaliation.

B. Discriminatory Harassment
Conduct as described in A., above, constitutes discriminatory harassment, if, in addition, it is accompanied by intentionally demeaning expressions concerning the race, gender, religion, sexual orientation, or national origin of the

victim(s).

III. Prohibition

All discriminatory harassment is prohibited.

IV. Administration of Policy

It is appropriate to report any allegation of discriminatory harassment to the authorities of the University. The ways available for doing this are as follows:

A. Students
An alleged incident of discriminatory harassment by a student toward another student that occurs outside a residence hall is to be reported to the Office of Residence Life and shall be handled in the same manner as other violations of University rules and regulations. (See section of du Lac entitled University Disciplinary Procedures.)

Likewise, any alleged incident of discriminatory harassment by a student toward a faculty member or staff member is to be reported to the Office of Residence Life. Any alleged incident of discriminatory harassment by a student toward any other student that occurs in a residence hall is to be reported first to the Rector and in consultation with the Office of Residence Life, a determination shall be made as to whether the allegation should be handled at the hall level or whether the matter should be referred to the Office of Residence Life.

B. Faculty
An alleged incident of discriminatory harassment by a faculty member is to be reported to the chair of the academic department, or, in cases involving the chair, to the dean of the college. If the matter cannot be resolved at the department or college level, it is to be referred to the Provost's Office.

C. Staff
An alleged incident of discriminatory harassment by a staff member is to be reported to the Director of Human Resources, and shall be handled by the Office of Human Resources in the same manner as any other violation of University rules and regulations as outlined in the University Human Resource Manual.

D. Administration
An alleged incident of discriminatory harassment by an administrator is to be reported to the appropriate superior officer of the person involved.

E. Ombudsperson
Notwithstanding the above, a person alleging discriminatory harassment may choose to report the incident to the University ombudsperson. This is to be a member of the University community selected by the President, in consultation with the other officers of the University, for that task. The ombudsperson, after taking information of the incident, is to help the complainant handle the matter, either by informal conciliation, or by helping the complainant proceed with the reporting procedure described above. [Note: Prof. Dwight King is currently the University ombudsperson.]

V. Existing University Rules and Regulations

This policy is intended to be an addition to existing University rules and regulations and does not alter or modify any existing University rule or regulation.

Sexual Harassment

I. **Policy**

The University of Notre Dame prohibits sexual harassment by all faculty, staff and students. Sexual harassment by any faculty, staff or student is a barrier to the educational, scholarly and research purposes of the University of Notre Dame and is a violation of law and University policy. The University of Notre Dame affirms its commitment to maintaining a learning and working environment which is fair, respectful and free from sexual harassment. To these ends, the following sexual harassment policy has been adopted.

II. Definition

The determination of what constitutes sexual harassment will vary with the particular circumstances, but may be described generally as: Unwelcome sexual advances, requests for sexual favors, and other verbal or physical conduct of a sexual nature when:

1. submission to or rejection of such conduct is made either explicitly or implicitly a term or condition of instruction, employment, or participation in other University activity;

2. submission to or rejection of such conduct by an individual is used as a basis for evaluation in making academic or personnel decisions affecting an individual; or

3. such conduct has the purpose or effect of unreasonably interfering with an individual's performance or creating an intimidating, hostile, or offensive University environment.

III. Administration of Policy

A. Students
An alleged incident of sexual harassment by a student toward any other student or faculty or staff member, should be reported to the Vice President for Student Affairs and shall be handled by the Office of Residence Life in the same manner as any other violations of University rules and regulations.

B. Faculty

Any incident of sexual harassment by a faculty member toward any student, staff personnel or other faculty member shall be reported to the Provost's Office, and shall be handled by the Provost's Office. If a formal charge is to be filed, it shall be administered in the same manner as a charge for Serious Cause for Dismissal as outlined in the Academic Articles.

C. Staff
Any incident of sexual harassment by a staff member toward a student, faculty member or other staff member, shall be reported to the Director of Human Resources, and shall be handled by the Human Resources Office in the same manner as any other violation of University rules and regulations as outlined in the *University Human Resources Manual*.

D. Confidentiality
Sexual harassment is a particularly sensitive issue which may affect any member of the University community. The right to confidentiality of all parties involved in a sexual harassment charge shall be strictly adhered to insofar as it does not interfere with the University's legal obligation to investigate allegations of sexual harassment when brought to the University's attention, and to take corrective action.

E. Resolution
A sexual harassment charge may result in a finding that no action is warranted, or may be handled by: 1) informal resolution, 2) reprimand, 3) disciplinary sanction, or 4) termination or expulsion.

F. Non-Retaliation
Any attempt by a member of the student body, staff or faculty to penalize in any way, a person bringing a sexual harassment charge, or any other form of retaliation, is prohibited and will be treated as a separate incident to be reviewed in its own right.

G. Protection of the Accused
(a) In cases under III(B.) or III(C.) during the investigation and before formal charges, the accused will be informed of the allegations, the identity of the complainant, the facts surrounding the allegations, and given the opportunity to respond.

(b) In the event the allegations are not substantiated, all reasonable steps will be taken to restore the reputation of the accused if it was damaged by the proceeding.

(c) A complainant found to have been intentionally dishonest in making the allegations or to have made them maliciously is subject to the full range of the University's disciplinary procedures from official reprimand to dismissal.

IV. Consensual Relationships

Because of the unique relationships between student and faculty members, with the faculty member serving as educator, counselor and evaluator, and the possibility of abuse of this relationship or the appearance of abuse, the University views it as unacceptable if faculty members (including all those who teach at the University, graduate students with teaching responsibilities and other instructional personnel) engage in amorous relations with students enrolled in their classes or subject to their supervision, even when both parties appear to have consented to the relationship. In keeping with this philosophy of the University, if charges of sexual harassment are made, it shall not be a defense to allege that the relationship was consensual.

Rev. 4/21/04