Math105 Final Exam – Fall 2002

1. Life expectancy at birth for males born in the United States in 1950 was 66 years, and in 2000 was 74 years. Find the life expectancy of a boy born in 2025, assuming that this trend continues as a linear function.

   (**a**) 72 (**b**) 76 (**c**) 78 (**d**) 80 (**e**) 84

2. Research data indicate that advances in technology will make it possible to build cars whose gas mileage is modeled by the function

$$M(t) = 250 - \frac{200}{t+1},$$

   where t denotes years from now, and M denotes miles per gallon. What is the limiting value of the gas mileage as time goes on?

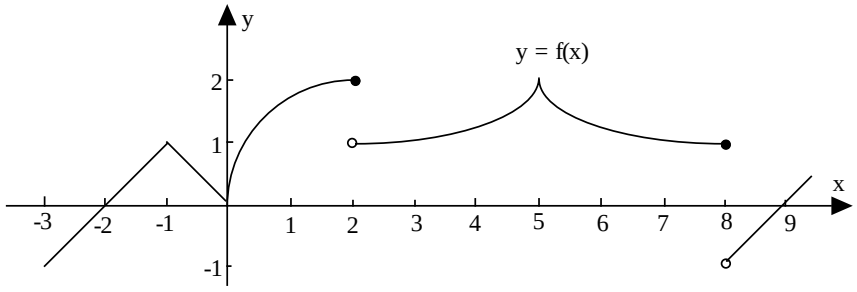
   (**a**) 50 (**b**) 150 (**c**) 200 (**d**) 250 (**e**) 450

3. The world population in 2000 was about 6 billion. Assuming that it grows exponentially with growth constant $r = 0.015$, find the time needed for it to triple its size.

   (**a**) 75 (**b**) $\frac{\ln 3}{0.015}$ (**c**) 45 (**d**) $\frac{\ln 3}{15}$ (**e**) $\frac{\ln 2}{0.015}$

4. The graph of the function $f(x)$ is given in Figure below. The points in the open interval $(-2, 9)$ where $f(x)$ is **not** continuous are:

   (**a**) $2, 8$ (**b**) $2, 5, 8$ (**c**) $-1, 2, 5, 8$ (**d**) $-1, 0, 2, 5, 8$ (**e**) None



5. The graph of the function $f(x)$ is given in Figure above. The points in the open interval $(-2, 9)$ where $f(x)$ is **not** differentiable are:

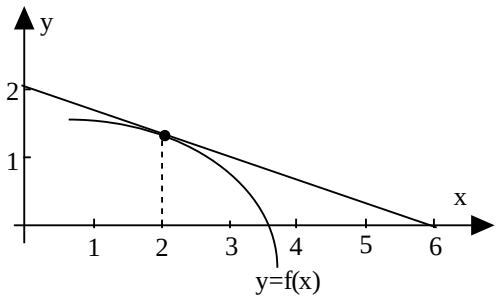
   (**a**) $2, 8$ (**b**) $2, 5, 8$ (**c**) $-1, 2, 5, 8$ (**d**) $-1, 0, 2, 5, 8$ (**e**) None

6. Compute $\lim_{h \to 0} \frac{(4+h)^3 - 4^3}{h}$.

   (**a**) 4 (**b**) 16 (**c**) 0 (**d**) 12 (**e**) 48

7. Compute the derivative of the function $f(x)$ at $x = 2$ whose graph is shown in Figure below.

(**a**) 3 (**b**) -3 (**c**) $\frac{1}{3}$ (**d**) $-\frac{1}{3}$ (**e**) 3.5



8. Let $f(x) = \ln(\frac{1}{x})$. Find $f''(x)$.

(**a**) $\ln(\ln x)$ (**b**) $\frac{1}{x^2}$ (**c**) $-\frac{1}{x^2}$ (**d**) $\frac{1}{\ln x}$ (**e**) $x \ln x$

9. Find all points where the tangent line to the graph of $y = \frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x + 6$ is horizontal.

(**a**) $x = 2$ (**b**) $x = 1$ (**c**) $x = 0$ (**d**) $x = 0$ and $x = 1$ (**e**) $x = 1$ and $x = 2$

10. Find the derivative of the function $f(x) = \frac{g(x)}{x^2+1}$ at $x = 0$, given that $g(0) = 1$ and $g'(0) = 3$.

(**a**) 4 (**b**) 3 (**c**) 2 (**d**) -2 (**e**) 0

11. Find the derivative of the function $f(x) = g(x)e^{2x}$ at $x = 0$, given that $g(0) = -4$ and $g'(0) = 2$.

(**a**) -4 (**b**) 2 (**c**) -6 (**d**) 10 (**e**) -2

12. Find y' if $\ln(1 + y^2) - e^x = -1$. (You may use implicit differentiation.)

(**a**) $y' = \frac{e^x}{1+y^2}$ (**b**) $y' = (1 + y^2)e^x$ (**c**) $y' = ye^x$

(**d**) $y' = -\frac{2y}{1+y^2} + e^x$ (**e**) $y' = \frac{(1+y^2)e^x}{2y}$

13. Find the derivative of $y = 5^{x^2}$. (Hint: $5^{x^2} = e^{(\ln 5)x^2}$)

(**a**) $(\ln 5)x^2 5^{x^2}$ (**b**) $(\ln 5)x5^{x^2}$ (**c**) $2(\ln 5)5^{x^2}$ (**d**) $(\ln 5)5^{x^2}$ (**e**) $2(\ln 5)x5^{x^2}$

14. Use the method of linear approximation to estimate $(1.01)^{10}$. In your work, use $f(x) = x^{10}$ and $a = 1$.

(**a**) $(1.01)^{10} \approx 3$ (**b**) $(1.01)^{10} \approx 1.1$ (**c**) $(1.01)^{10} \approx 1.2$ (**d**) $(1.01)^{10} \approx 2$

(**e**) $(1.01)^{10} \approx 10$

15. A spherical balloon is being filled with helium at a rate of 100 cubic centimeters per minute. At what rate is the radius r of the balloon expanding at the moment when $r = 5$ centimeters? (The volume of a ball of radius r is $\frac{4}{3}\pi r^3$.)

(**a**) $\frac{1}{\pi}$ cm/min (**b**) 20π cm/min (**c**) $\frac{1}{5\pi}$ cm/min (**d**) 100πcm/min

(**e**) $\frac{10}{\pi}$ cm/min

16. The position of a train traveling on a straight track is given by the function $s(t) = \frac{1}{3}t^3 - 2t^2 + 7t + 12$, where s is measured in feet and t is measured in seconds. At what time is the acceleration of the train equal to zero?

(**a**) $t = 1$ (**b**) $t = 0$ (**c**) $t = \sqrt{2}$ (**d**) $t = 2$ (**e**) None of the above

17. A particle which is initially at rest travels with acceleration at time t given by $a(t) = e^t - 2t + 1$. What is its velocity at $t = 1$?

(**a**) $e + 1$ (**b**) e (**c**) $e - 1$ (**d**) $e - 2$ (**e**) $e + 2$

18. Find the global maximum and global minimum of the function

$$f(x) = 3x^4 - 4x^3 - 12x^2 + 2$$

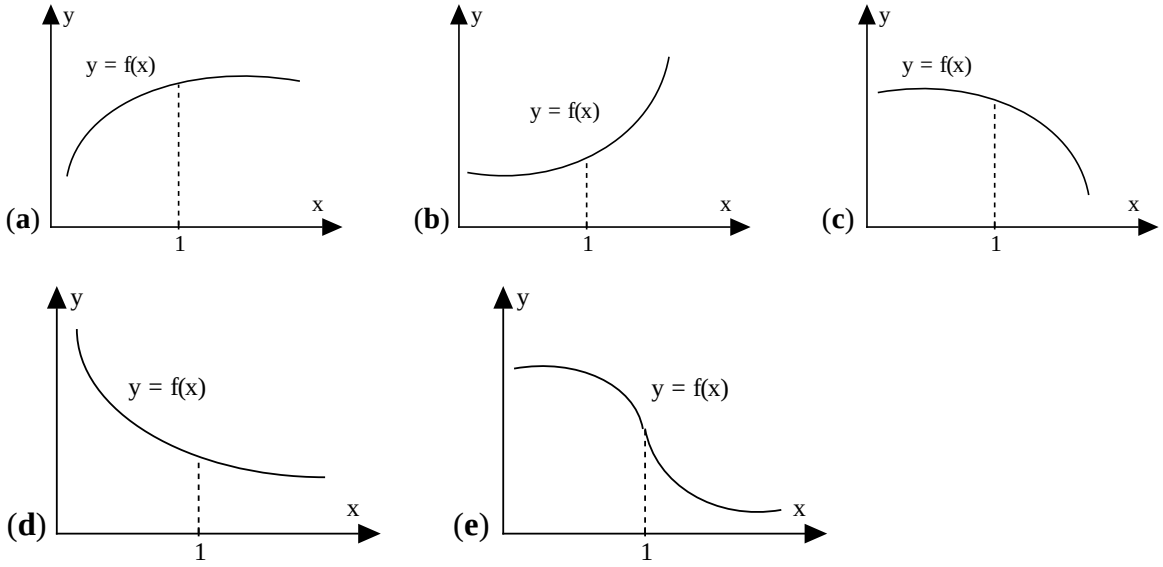
on the interval $[-2, 1]$.

(**a**) Max at $x = -2$, Min at $x = 1$ (**b**) Max at $x = -2$, Min at $x = 2$

(**c**) Max at $x = 0$, Min at $x = 2$ (**d**) Max at $x = 0$, No global min

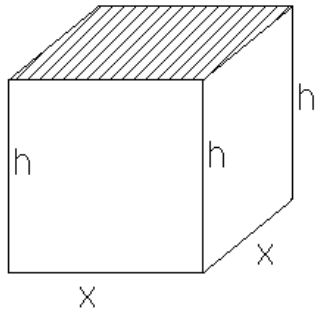
(**e**) Min at $x = -1$, No global max

19. Suppose that $f'(x) > 0$ and $f''(x) < 0$ near $x = 1$. Which of the following graph best describes $f(x)$?



20. Assume that you need to construct a top open rectangular box with a square base and volume 4 cubic feet. Find the smallest surface area of such a box. (The surface area is the sum of the areas of the five faces of the box, excluding the top face.)

(**a**) 24 ft^2 (**b**) 20 ft^2 (**c**) 16 ft^2 (**d**) 12 ft^2 (**e**) 8 ft^2



21. If $f'(a) = 0$ and $f''(a) = 1$, then

(**a**) $f(x)$ has a local minimum at $x = a$ (**b**) $f(x)$ has a local maximum at $x = a$

(**c**) $f(x)$ is increasing near $x = a$ (**d**) $f(x)$ has a point of inflection at $x = a$

(**e**) $f(x)$ has a vertical asymptote at $x = a$

22. If $\dfrac{dy}{dx} = e^{2x} + \dfrac{1}{2}$ and $y(0) = 1$, then $y(1) =$

(**a**) $\dfrac{e^2 - 1}{2}$ (**b**) $\ln(e^2 - \dfrac{1}{2})$ (**c**) $\dfrac{e^4}{2} - 2$ (**d**) $\ln 2 - 3$ (**e**) $\dfrac{1}{2}e^2 + 1$

23. For which of the following integrals would it help to do a substitution with $u = x^2 + 1$?

(a) $\displaystyle\int \frac{dx}{\sqrt{x^2+1}}$ **(b)** $\displaystyle\int (x^2+1)^5 dx$ **(c)** $\displaystyle\int \frac{x\,dx}{\sqrt{x^2+1}}$ **(d)** $\displaystyle\int x^2(x^2+1)^6 dx$

(e) $\displaystyle\int (x^3+x^2+1)^8 dx$

24. Use integration by parts to find the indefinite integral $\displaystyle\int x^3 \ln x\, dx$.

(a) $\frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 + c$ **(b)** $x^4 \ln x - \frac{1}{4}x^4 + c$ **(c)** $\frac{1}{4}x^4 \ln x - \frac{1}{20}x^5 + c$

(d) $\frac{1}{4}x^4 \ln x + \frac{1}{16}x^4 + c$ **(e)** $3x^2 \ln x + x^2 + c$

25. Find the following indefinite integral with the method of partial fractions

$$\int \frac{2}{(x-1)(x-2)}\, dx.$$

(a) $2\ln|x-1| - 2\ln|x-2| + c$ **(b)** $2\ln|x-2| - 2\ln|x-1| + c$

(c) $\frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x-2| + c$ **(d)** $\frac{1}{2}\ln|x-2| - \frac{1}{2}\ln|x-1| + c$

(e) $2\ln|x-1| + 2\ln|x-2| + c$

26. Suppose that the population of a region is modeled by

$$P(t) = 20e^{0.05t}$$

with P in millions and the time t in years. Find the average population of this region in the first 10 years.

(a) $40(e^{0.5}+1)$ million **(b)** $40e^{0.5}$ million **(c)** $20(e^{0.5}-1)$ million

(d) $20(e^{0.5}+1)$ million **(e)** $40(e^{0.5}-1)$ million

27. Suppose that the marginal profit of producing and selling x units of a certain product per day in a company is given by

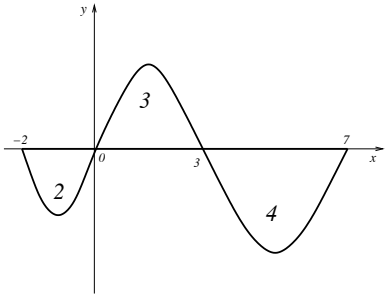
$$MP(x) = -40x + 1,000 \quad \text{for } 10 \leqslant x \leqslant 30$$

with P in dollars. Find the total increase in profit if the company increases its production from 10 units per day to 30 units per day.

(a) $\$\,1,000$ **(b)** $\$\,2,000$ **(c)** $\$\,3,000$ **(d)** $\$\,4,000$ **(e)** $\$\,5,000$

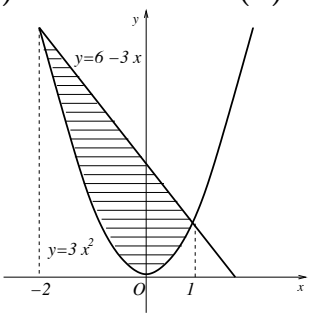
28. The following figure shows the regions and their areas enclosed by the graph of some continuous function $y = f(x)$ for $-2 \leqslant x \leqslant 7$ and the x-axis. Find the value of the definite integral $\displaystyle\int_{-2}^{7} f(x)\,dx$.

(a) -5 **(b)** 5 **(c)** -3 **(d)** 3 **(e)** 24



29. Find the area between the curves $y = 3x^2$ and $y = 6 - 3x$.

(a) 13.5 **(b)** 12.5 **(c)** 13 **(d)** 12 **(e)** 14



30. Suppose that $f(x)$ is a continuous function on the interval $[-2, 2]$ with its values

x	-2	-1	0	1	2
$f(x)$	-3	-1	2	3	1

Use the trapezoidal rule with $n = 4$ to estimate the definite integral $\displaystyle\int_{-2}^{2} f(x)\,dx$.

(a) 1 **(b)** 2 **(c)** 3 **(d)** $\dfrac{10}{3}$ **(e)** 5