Total: 25/25

HP3 Math 121 Name: SAMPLE ANSWER

Due: Thursday February 09, 2006

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1. Using the limit criteria for continuity, find the value of k so that $f(x)$ is a continuous function on any interval. (Do not use graphs to support your answer).

$$f(x) = \begin{cases} kx, & 0 < x < 2 \\ 3x^2, & x \geq 2 \end{cases}$$

$f(x)$ is continuous for all $x \in (0, +\infty)$ except possibly at 2.

For $f(x)$ to be continuous at $x = 2$, we need

$$\lim_{x \to 2} f(x) = f(2) \quad \leftarrow \text{— } +1$$

$$\Rightarrow \lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2) \quad \leftarrow \text{— } +1$$

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} kx = 2k \quad \leftarrow \text{— } +2$$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} 3x^2 = 3(2)^2 = 12 = f(2) \quad \leftarrow \text{— } +1$$

Therefore, $2k = 12$ for continuity at $x = 2$

$$\Rightarrow k = 6 \quad \leftarrow \text{— } +1$$

2. Using the limit criteria for continuity, find the value of k so that $g(x)$ is a continuous function on any interval. (Do not use graphs to support your answer).

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$$g(x) = \begin{cases} \dfrac{5x^3 - 10x^2}{x-2}, & x \neq 2 \\ k, & x = 2 \end{cases} = \begin{cases} \dfrac{5x^2(x-2)}{x-2}, & x \neq 2 \\ k, & x = 2 \end{cases}$$

$$= \begin{cases} 5x^2, & x \neq 2 \\ k, & x = 2 \end{cases} \quad +2$$

$g(x)$ is continuous for all x except possibly $x=2$

For continuity at $x=2$, $\lim_{x\to 2} g(x) = g(2)$

$$g(2) = k = \lim_{x\to 2} g(x) = \lim_{x \to 2} 5x^2 \quad +1 \quad +1$$

$$k = 5(2)^2 = 20 \quad +1$$

+1

OR:

$$g(2) = k = \lim_{x\to 2} g(x) = \lim_{x\to 2} \frac{5x^3 - 10x^2}{x-2} \quad +1$$

$$= \lim_{x\to 2} \frac{5x^2\cancel{(x-2)}}{\cancel{(x-2)}} = \lim_{x\to 2} 5x^2 \quad +2$$

+1

$$= 5(2)^2 = 20$$

$$\Rightarrow k = 20 \quad +1$$

Grade correct or incorrect.

3. Using the graphs below, find the values of the limits below if they exists. If the limits do not exist, then state so.

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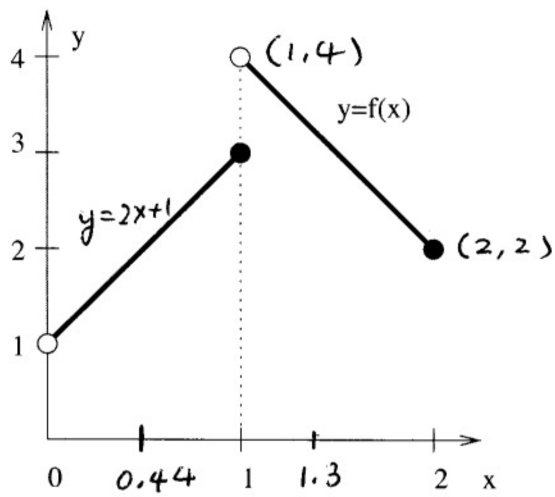

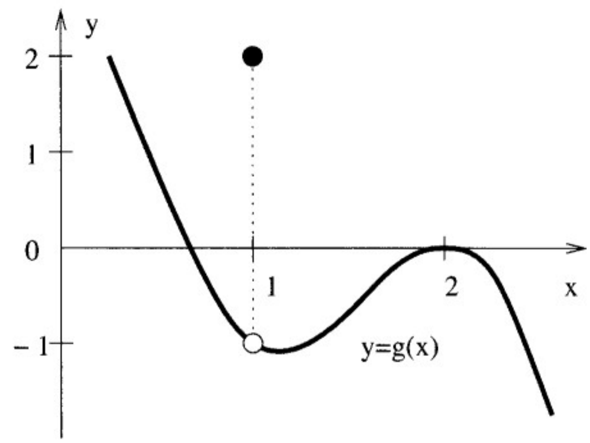

+2 **a.** $\lim_{x\to 1^-}(f(x)+g(x)) = \lim_{x\to 1^-} f(x) + \lim_{x\to 1^-} g(x) = 3-1 = 2$

+2 **b.** $\lim_{x\to 1^+}(f(x)+2g(x)) = \lim_{x\to 1^+} f(x) + 2\lim_{x\to 1^+} g(x)$

$= 4 + 2(-1) = 2$

+2 **c.** $\lim_{x\to 0.44} f(x) = \lim_{x\to 0.44}(2x+1) = 2(0.44)+1 = 1.88$

+2 **d.** $\lim_{h\to 0}\dfrac{f(1.3+h)-f(1.3)}{h} = f'(1.3) =$ slope of line joining $(1,4)$ and $(2,2)$

$= -2$

+2 **e.** $\lim_{x\to 2^-}\dfrac{f(x)}{g(x)} =$ Does not exists because $\lim_{x\to 2^-} g(x) = 0$

+2 **f.** $\lim_{x\to 1^+}\dfrac{g(x)}{f(x)} = \dfrac{\lim_{x\to 1^+} g(x)}{\lim_{x\to 1^+} f(x)} = \dfrac{-1}{4}$