Differentiable Functions Summary

Definition 1 A function $f(x)$ is said to be differentiable at $x = c$ provided the following limit exist:

Theorem 1 If $f(x)$ be differentiable at $x = c$, then $f(x)$ is ____________ at $x = c$.

Derivative of a function. The derivative of the function $f(x)$ is given by the following limit:

$$f'(x) =$$

Setting $\Delta x = h$ and $\Delta y = f(x + h) - f(x)$ gives the notation:

$$f'(x) = \underline{\qquad\qquad\qquad} = \underline{\qquad\qquad}$$

Notation: If $y = f(x)$ is a differentiable function. Write down all standard notations of the derivative of $y = f(x)$.

Derivative Properties and Formulas Summary

Basic Properties of Derivatives:

$[f(x) + g(x)]' \stackrel{?}{=}$

$[f(x) - g(x)]' \stackrel{?}{=}$

$[c \cdot f(x)]' \stackrel{?}{=}$

Product/Quotient/Chain Rule. Let $f(x)$ and $g(x)$ be differentiable functions. Derive formulas for the derivatives of $p(x) = f(x) \cdot g(x)$ and $q(x) = \dfrac{f(x)}{g(x)}$.

Product Rule: $\dfrac{d}{dx}(f(x)g(x)) = (f(x)g(x))' =$

Quotient Rule: $\dfrac{d}{dx}\left(\dfrac{f(x)}{g(x)}\right) = \left(\dfrac{f(x)}{g(x)}\right)' =$

Chain Rule: $\dfrac{d}{dx}\left(f(g(x))\right) = [f(g(x))]' =$

Some Common Derivatives. For any numbers k and n:

$\dfrac{d}{dx}(k) \stackrel{?}{=}$

$\dfrac{d}{dx}(x^n) \stackrel{?}{=}$ (Power Rule)

$\dfrac{d}{dx}(e^x) \stackrel{?}{=}$

$\dfrac{d}{dx}(a^x) \stackrel{?}{=}$ $(a > 0)$

$\dfrac{d}{dx}(\ln(x)) \stackrel{?}{=}$

$\dfrac{d}{dx}(\log_a(x)) \stackrel{?}{=}$ $(a > 0)$

Some Common Derivatives. For any numbers k and n:

$\frac{d}{dx}(\sin(x)) \stackrel{?}{=}$

$\frac{d}{dx}(\cos(x)) \stackrel{?}{=}$

$\frac{d}{dx}(\tan(x)) \stackrel{?}{=}$

$\frac{d}{dx}(\cot(x)) \stackrel{?}{=}$

$\frac{d}{dx}(\sec(x)) \stackrel{?}{=}$

$\frac{d}{dx}(\csc(x)) \stackrel{?}{=}$

Implicit Differentiation. Can you find $\frac{dy}{dx}$ if $e^{xy} = x^3 - y^2 + 5$?

Logarithmic Differentiation. For each of the derivatives below which one do you apply logarithmic differentiation? Find the derivative

$\frac{d}{dx}\left(e^{x^2+5}\right) \stackrel{?}{=}$

$\frac{d}{dx}\left(x^{x^2+5}\right) \stackrel{?}{=}$

$\frac{d}{dx}\left(x^{e^2+5}\right) \stackrel{?}{=}$

1. Find the equation of the tangent line to the graph of $y = \dfrac{2x+3}{4x+5}$ at $x = -1$.

2. Find the stationary points of the function $e^{\frac{x^3}{3} - \frac{x^2}{2} - 6x + 5}$.

3. Find their derivatives of the following functions.

a. $y = (2x^2 + 5)^4$

b. $y = 4^{x^2+3}$

c. $y = \ln\left(\dfrac{2e^{2x} + 3}{3e^{2x} + 4}\right)$

d. $y = (\sin(x) + \cos^2(2x))^3$

e. $y = \tan^3(e^{3x^2+4})$

f. $y = \sqrt[3]{e^{\sec(x)} + \sec(e^x)}$

4. Let $f(x)$ be a differentiable function such that $f(2) = 1$ and $f'(2) = -1$.

4a. Find the slope of the graph of $y = f(x)e^{f(x)}$ at $x = 2$.

4b. Find instantaneous rate of change of $y = \dfrac{f(x)}{f(x) + 3}$ at $x = 2$.

5. Find the equation of the tangent line to the curve $x\cos(1+2xy) = 2y^3 - 2$ at the point $(0, 1)$.

6. Find the derivative of y with respect to x if $y = \dfrac{5-x}{(2x-3)^3}$. Completely simplify your answer giving it in the form $\dfrac{Ax+B}{(2x-3)^n}$

Parametric Equations. The slope formula $\frac{dy}{dx}$ for the parametric equations $x = f(t)$ and $y = g(t)$.

$$\frac{dy}{dx} = \underline{\hspace{4cm}} = \underline{\hspace{4cm}}$$

Linearization of a function. The linearization of a function $f(x)$ at $x = c$ is the same as the linear function

gives the equation of the ____ to the curve $y = f(x)$ at $x = c$

The linearization of a function $f(x)$ at $x = c$ or linear approximation of $f(x)$ at $x = c$ is given by

$$f(x) \approx \underline{\hspace{8cm}}$$

The linear approximation of the change in $f(x)$ when x changes from $x = c$ to $x = c + \Delta x$ is given by

$$\Delta y = f(c + \Delta x) - f(c) \approx \underline{\hspace{4cm}}$$

Estimating Derivatives from Data Points. Suppose a differentiable function $f(x)$ has known values at $x = a,\quad b\quad$ and $\quad c$.

x	a	b	c
$f(x)$	$f(a)$	$f(b)$	$f(c)$

Give the three difference formula for estimating the derivative of $f(x)$ at $x = b$.

Forward Difference formula: $f'(b) \approx$ ____________________

Central Difference formula: $f'(b) \approx$ ____________________

Backward Difference formula: $f'(b) \approx$ ____________________

We only have __________________________ difference to estimate $f'(a)$.

We only have __________________________ difference to estimate $f'(c)$.