

Math 10350 Example Set 14A

1. Write down all integration (anti-differentiation) formula for the trigonometric functions.

2. A small piece of wood is bobbing up and down on the surface of a pond with its acceleration given by

$$a(t) = (\sin t - \cos t) \text{ m/s}^2.$$

Given that the cork has velocity 1 m/s and position -2 m when $t = \pi$ seconds, answer the following questions:

a. If $s(t)$ is the position of the cork, write in terms of s and its derivatives, a differential equation, and initial value conditions modeling the position of the cork.

b. Solve the equation in (a) for $s(t)$ by first finding for $s'(t)$.

3. Evaluate the following indefinite integrals:

a. $\int \frac{\tan \theta}{\cos \theta} d\theta$

b. $\int \frac{1 + \cos^2 \theta}{\cos^2 \theta} d\theta$

Summation Notation (Section 5.1)

1. Assuming that the pattern in the sums below, write down (a) the formula for the general term, and (b) the sum using summation notation.

a. $\frac{5}{1+1^2} + \frac{5}{1+2^2} + \frac{5}{1+3^2} + \dots + \frac{5}{1+15^2}$

b. $\frac{5}{1+5^2} + \frac{5}{1+6^2} + \frac{5}{1+7^2} + \dots + \frac{5}{1+13^2}$

c. $\left(\frac{1}{n}\right) \sqrt{1 - \left(\frac{0}{n}\right)^2} + \left(\frac{1}{n}\right) \sqrt{1 - \left(\frac{1}{n}\right)^2} + \left(\frac{1}{n}\right) \sqrt{1 - \left(\frac{2}{n}\right)^2} + \dots + \left(\frac{1}{n}\right) \sqrt{1 - \left(\frac{n-1}{n}\right)^2}$

Properties of summation notation:

$$(A) \sum_{k=1}^n (a_k + b_k) = \sum_{k=1}^n a_k + \sum_{k=1}^n b_k \quad (B) \sum_{k=1}^n (c \cdot a_k) = c \cdot \left(\sum_{k=1}^n a_k \right) \quad (C) \sum_{k=1}^n (c) = c \cdot n$$

Why are these properties true?

2. If $a_0 = 2$, $a_1 = 0$, $a_2 = -1$, $a_3 = -2$, and $a_4 = 0$. Find the value of the sums:

(a) $\sum_{j=2}^4 (2a_j + 3)$ (b) $\sum_{n=0}^2 \cos(a_n \pi)$.