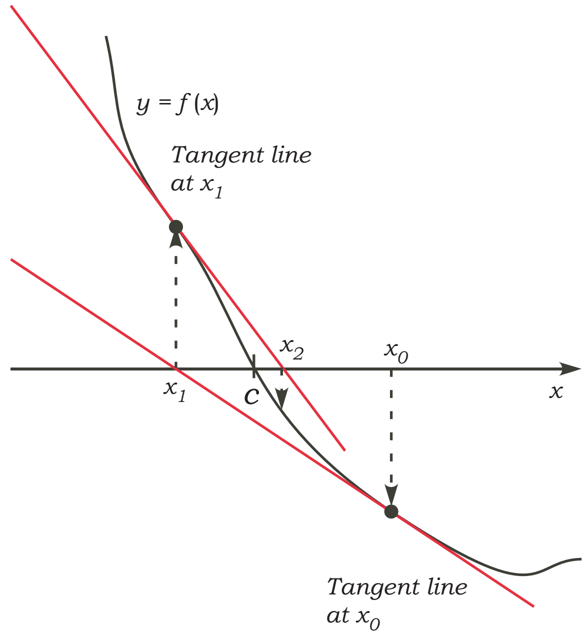

Newton's Method

Steps to applying Newton's method to approximate the solution of $f(x) = 0$:

(1) Make an initial guess x_0 near to the zero you wish to find.

(2) Determine the new approximations x_1, x_2, ... :

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

(3) Check $|x_n - x_{n+1}| \to 0$ as $n \to \infty$ for convergence to required zero.

1a. Find $f'(x)$ if $f(x) = x^3 + x + 1$. Explain why we could see that $f(x)$ has a unique zero in the interval $[-1, 0]$.

1b. Apply Newton's Method with $x_0 = -0.5$ to estimate the zero of $f(x)$ up to three decimal places.

2. Estimate all solutions of $x^2 = \cos x$ up to four decimal places. (Hint: Sketch some graphs to see where the roots are located. Make your first guess for the root. You only need to find one.)

Math 10350 Example Set 13B

▶ **Antiderivatives** (Reversing differentiation – Section 4.9)

Definition: We say that $F(x)$ is an **antiderivative** of $f(x)$ provided ________________.

Example 1 Verify that $x^2 + 5$ is an antiderivative of $2x$. Can you write down a few more antiderivative of $2x$? What did you notice? Explain graphically.

Remark: We denote the family of antiderivatives of $2x$ by ________________.

From Example 1, we see that

> **Theorem:** If $F(x)$ and $G(x)$ are antiderivatives of the same function throughout an interval, then they differ by a constant c over that interval; that is, for $a < x < b$
>
> $F'(x) = G'(x) \iff$ ________________ for some number C.

Notation: If $F(x)$ is an antiderivative of $f(x)$, that is, $F'(x) = f(x)$. Then we may write

$$\int f(x)dx = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

We call $\int f(x)dx$ the **indefinite integral**.

▶ **Basic indefinite integral formulas**

- For any constant k: $\int k\,dx \stackrel{?}{=}$. For Example: $\int 100\,dx \stackrel{?}{=}$
- Power Rule when $k \neq -1$: $\int x^k dx \stackrel{?}{=}$. For Example: $\int x^9 dx \stackrel{?}{=}$
- Power Rule when $k = -1$: $\int \frac{1}{x}\,dx =$.
- Constant Multiple Rule: $\int kf(x)dx = k\int f(x)dx,$ any k. For Example: $\int \frac{8}{x^2}\,dx \stackrel{?}{=}$
- Sum Rule: $\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx.$
- General Exponential function : $\int a^x dx \stackrel{?}{=}$. For Example: $\int 10^x dx \stackrel{?}{=}$
- Exponential base e: $\int e^x dx \stackrel{?}{=}$.
- Exponential function: $\int e^{ax} dx \stackrel{?}{=}$. For Example: $\int e^{3x} dx \stackrel{?}{=}$

1. Evaluate the following indefinite integrals:

a. $\int (1 + e^{2x} + e^2 + 3x - x^2)\, dx$

b. $\int \frac{2u^2 - 5u + \sqrt[3]{u}}{u^2}\, du$

2. Find the antiderivative F of function f satisfying the given condition:

$$f(x) = (e^x + 1)^2; \qquad F(0) = 3$$

In other words, solve the initial value problem:

$$\frac{dF}{dx} = (e^x + 1)^2; \qquad F(0) = 3$$

3. A ball is projected upward from the ground with an initial velocity of 3 m/sec. Using calculus, write the velocity and position for the ball at time t. You may assume that the acceleration due to gravity is 10 m/s^2.