Math 10350 – Example Set 12A

1a. Find the absolute (global) maximum and minimum of $f(x) = xe^{-x}$ on the interval $[0.5, 2]$. Write down the range of the values of $f(x)$ for $0.5 \le x \le 2$.

1b. Using the steps below, find the global maximum and minimum of $f(x) = xe^{-x}$ on $[0.5, \infty)$.

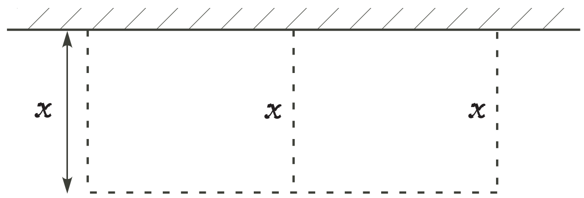
Step 1: Find all critical points in the domain of $f(x)$ and the values of $f(x)$ there. Classify them using first derivative test.

Step 2: Find the values of $f(x)$ at the end-points (if any) of its domain. ____________________

Step 3: If end-point not included, or $\pm\infty$, find all limits of $f(x)$ towards end of interval.

Step 4: Give a schematic sketch (ignore concavity) of the graph of $f(x)$ clearly indicating where the global maximum and minimum are. State the global maximum and minimum of $f(x)$ on $[0.5, \infty)$ if any. Find the range of $f(x)$ for x in $0.5 \le x < \infty$.

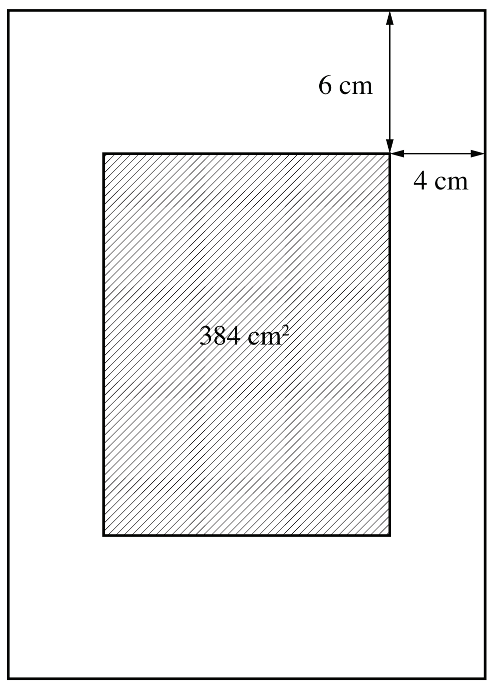
2. A landscaper plans to use 120 m of fencing and a very wide straight wall to make two rectangular enclosures with the same dimensions as shown.



a. Write down the possible values of x.

b. Find the maximum value of the total area of the enclosures. What are the dimensions of each enclosure when maximum occurs?

3. The top and bottom margins of a poster are each 6 cm and the side margins are each 4 cm. If the area of printed material on the poster is fixed at 384 cm^2, find the dimensions of the poster with the smallest area.



Math 10350 – Example Set 12B

1. A cylindrical can without a top is made to contain 100 cm^3 of liquid. Find the dimension that will minimize the cost of the material to make the can if the material for the side costs \$2/cm^2 and the material for the base costs \$3/cm^2.

2. Find the area of the largest rectangle that can be inscribed in a right triangle with legs of lengths 3cm and 4cm if two sides of the rectangle lie along the legs.

3. Show that of all the isosceles triangles with a perimeter of $30\,cm$, the one with the largest area is equilateral.

Math 10350 – Example Set 12C

1. Find the dimensions of a **closed** cylindrical can with maximum volume if the surface area is 200π cm^2.

At least set up the following optimization problems. Be sure to give the domain for the optimization problem.

2. A 100 meter long wire is to be bent into a shape consisting of a semi-circular side and two equal straight sides. Find the dimensions of the shape if the area enclosed is to be maximized. Your answer should give the radius of the semi-circular side and the length of the two equal straight sides.

3. Consider a shape consisting of a semi-circular side and two equal straight sides which encloses an area of 100 sq. meters. Find the dimensions of the shape if its perimeter is to be shortest. Your answer should give the radius of the semi-circular side and the length of the two equal straight sides.