Vertical Asymptote.
Let c be a real number. If $\lim_{x \to c^-} f(x) = \pm\infty$ or $\lim_{x \to c^+} f(x) = \pm\infty$.

Then $y = f(x)$ has a ________________________ asymptote at $x = c$.

Horizontal Asymptote.
If $\lim_{x \to \infty} f(x) = A$ (finite number) or $\lim_{x \to -\infty} f(x) = A$.

Then $y = f(x)$ has a ________________________ asymptote at $y = A$.

1. Draw a graph with horizontal asymptotes $y = 1$ and $y = -4$.

2. Find the equations of all horizontal asymptotes of $y = \dfrac{3e^{3x} + 4e^x + 5}{2e^{3x} + e^x + 3}$.

L'Hopital's Rule: If both $f(x)$ and $g(x)$ are differentiable functions such that:

(a) $\lim_{x \to c} f(x) = 0 = \lim_{x \to c} g(x)$ such that $\lim_{x \to c} \dfrac{f'(x)}{g'(x)}$ exists then $\lim_{x \to c} \dfrac{f(x)}{g(x)} = \lim_{x \to c} \dfrac{f'(x)}{g'(x)}$.

(b) $\lim_{x \to c} f(x) = \pm\infty = \lim_{x \to c} g(x)$ such that $\lim_{x \to c} \dfrac{f'(x)}{g'(x)}$ exists then $\lim_{x \to c} \dfrac{f(x)}{g(x)} = \lim_{x \to c} \dfrac{f'(x)}{g'(x)}$.

Here $x \to c$ could mean limit to a number like $x \to 4$, or left-right limit notations like $x \to 0^-$ and $x \to 0^+$, or limit to infinity ($x \to \infty$ and $x \to -\infty$).

3. Evaluate the following limits using L'Hopital's Rule where necessary.

(A) <u>$0/0$ - type, ∞/∞ - type and $0 \cdot \infty$ - type</u>

(i) $\lim_{x \to \infty} \dfrac{\ln(1+x)}{x}$.

(ii) $\lim_{x \to \infty} \dfrac{\sin(x) + \sin(2x)}{x^2 + 1}$.

(iii) $\lim_{x \to 0^+} x \ln(x)$.

(B) <u>1^∞ - type, ∞^0 - type and 0^0 - type</u>

(iv) $\lim_{x \to \infty} (1+x)^{1/x}$.

(v) $\lim_{x \to \infty} \left(1 - \dfrac{2}{x}\right)^x$.

(vi) $\lim_{x \to 0^+} x^x$.

(C) <u>$\infty - \infty$ - type</u>

(vii) $\lim_{x \to 0^+} (\csc x - \cot x)$.

Math 10350 – Example Set 11B

1. Sketch the graph of $g(x) = xe^{-x^2}$ by completing the steps below.

a. Find all x-intercepts and y-intercept of the graph of $g(x)$ whenever possible.

b. Find coordinates of all critical points, vertical asymptotes, and places where $g(x)$ are undefined. $\left(g'(x) = (1 - 2x^2)e^{-x^2}\right)$

c. Determine where $g(x)$ is increasing and where it is decreasing.

d. Determine the concavity and coordinates of inflection points of $g(x)$. $\left(g''(x) = (4x^3 - 6x)e^{-x^2}\right)$

e. Find all asymptotes and limit at infinity whenever applicable.

f. Sketch the graph below labeling all important features. Your picture should be large and clear.

Math 10350 – Example Set 11C

1. Find the equations of all vertical and horizontal asymptotes of $y = \dfrac{3x^2 + 2x - 5}{2x^2 + x - 3}$.

2. Sketch the graph of $f(x) = \dfrac{e^x + 1}{e^x - 1}$ by completing the steps below.

a. Find all x-intercepts and y-intercept of the graph of $f(x)$ whenever possible.

b. Find coordinates of all critical points, vertical asymptotes, and places where $f(x)$ are undefined. $\left(f'(x) = -\dfrac{2e^x}{(e^x - 1)^2} \right)$

c. Determine where $f(x)$ is increasing and where it is decreasing.

d. Determine the concavity and coordinates of inflection points of $f(x)$. $\left(f''(x) = \dfrac{2e^x(1 + e^x)}{(e^x - 1)^3} = \dfrac{2e^x(1 + e^x)}{(e^x - 1)^2} \cdot \dfrac{1}{e^x - 1} \right)$

e. Find all asymptotes and limit at infinity whenever applicable.

f. Sketch the graph below labeling all important features. Your picture should be large and clear.