Math 10350 – Example Set 08A

Let $f(x)$ be differentiable at $x = a$. Then the linear approximation for the change Δf in $f(x)$ when x changes from a to $a + \Delta x$:

$$\Delta f = f(a + \Delta x) - f(a) \approx f'(a)\Delta x$$

1. The diameter of a circular disk is given as 10 cm with a maximum error in measurement of 0.2 cm. Use linear approximation to estimate the maximum error (ΔA) and percentage error in the calculated area of the disk. If the disk is made with an expensive titanium sheet that costs \$50 per cm^2, estimate an upper limit for your budget in making a disk of 10 cm diameter (Upper limit for budget is $\$(1250\pi + 50\pi) = \$1300\pi$).

2. A vessel is in the shape of an inverted cone. The radius of the top is 5 cm and the height is 8 cm. Water is poured in to a height of x cm. Find an expression for the volume V of the water in the vessel in terms of x. Use linear approximate to estimate the increased in V when x increases from 4 cm to 4.08 cm. Give units for your answer. (Answer: $\pi/2$ cm^3)

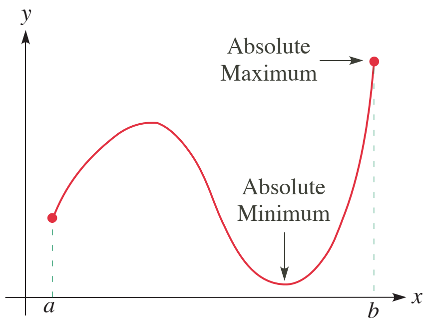

Figure 1

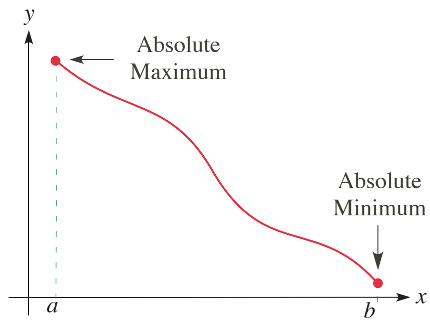

Figure 2

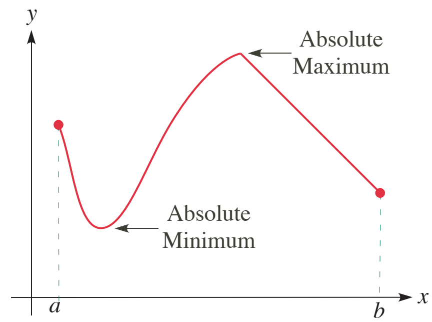

Figure 3

From Figures 1, 2, and 3, we can observe the following fact:

> **The extreme value theorem**
>
> If $f(x)$ is continuous on a closed and bounded interval $[a, b]$ then $f(x)$ takes on a ____________________
>
> and takes on a ____________________ on $[a, b]$.

Q1: If $f(x)$ is continuous, where are the possible places for which absolute maximum and absolute minimum of $f(x)$ occur on $[a, b]$? (See Figures 1, 2, and 3.)

A1: On a closed and bound interval $[a, b]$, a continuous function $f(x)$ attains its absolute maximum and absolute

minimum occur at (1) ____________________, or (2) ____________________, or (3) ____________________.

Definition: Let $f(x)$ be defined at c. Then we say that c is a **critical point** of f if (1) ____________________,

or (2) ____________________.

> **Method for finding absolute maxima and minima of f on $[a, b]$**
> 1. Find all critical points in (a, b).
> 2. Evaluate f at all critical points and at endpoints. Then compare the values of f:
>   **highest** = absolute maximum and **lowest** = absolute minimum.

Q2: What about when the interval is no longer closed and bounded? Draw some graphs to illustrate how the existence of extrema values could differ.

1. Find the absolute (global) maximum and minimum of the function $f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2 + 1$ on the interval $[0, 3]$.

2. Find all critical points of $f(x) = x - \frac{3}{2}x^{2/3}$ for $-1 \leq x \leq 1$. Hence determine the maximum and minimum values of $f(x)$ for $-1 \leq x \leq 1$.