Math 10350 – Example Set 05A
Sections 3.1 & 3.2
Product Rule & Quotient Rule

Product and Quotient Rule. Let $f(x)$ and $g(x)$ be differentiable functions. Derive formulas for the derivatives of $p(x) = f(x) \cdot g(x)$ and $q(x) = \dfrac{f(x)}{g(x)}$.

Product Rule: $\dfrac{d}{dx}(f(x)g(x)) =$ ______________________________

Quotient Rule: $\dfrac{d}{dx}\left(\dfrac{f(x)}{g(x)}\right) =$ ____

1. The stationary points in the domain of a function $f(x)$ are the values of x such that $f'(x) = 0$. What can you say about the tangent line at stationary points? Find the stationary points of the following functions:

1a. $f(x) = (x^2 - 3)e^x$.

1b. $y = \dfrac{2x - 1}{x^2 + 1}$.

2. Let $p(x) = (x^3 - 5x + 1)g(x)$ and $q(x) = \dfrac{f(x)}{g(x) + 1}$. Given that $f(2) = 2$, $g(2) = 3$, $f'(2) = -1$ and $g'(2) = -4$, find the following values:

a. The instantaneous rate of change of $p(x)$ at $x = 2$.

b. The slope of the tangent line to the graph of $y = q(x)$ when $x = 2$.

Math 10350 – Example Set 05B
Section 3.5 Higher Derivatives
Section 3.6 Trigonometric Functions

1. Consider the function $f(t) = t^4 - 2e^t + 2$.

a. Find the following derivatives of $f(t)$: (i) $f'(t)$, (ii) $f''(t) = \dfrac{d^2 f}{dt^2}$, (iii) $f'''(t)$, and (iv) $\dfrac{d^4 f}{dt^4}$.

b. If $f(t)$ represents the position of a particle moving on a straight line, what would $f'(t)$ and $f''(t)$ mean physically?

2. Define the trigonometric functions:

$$\tan x = \frac{\sin x}{\cos x}, \quad \cot x = \frac{\cos x}{\sin x}, \quad \sec x = \frac{1}{\cos x} \quad \text{and} \quad \csc x = \frac{1}{\sin x}.$$

Use the fact that $\dfrac{d}{dx}(\sin x) = \cos x$ and $\dfrac{d}{dx}(\cos x) = -\sin x$ to show that

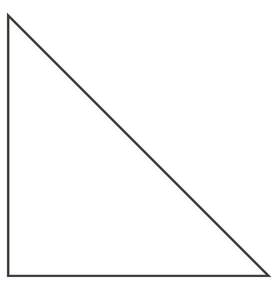
a. $\dfrac{d}{dx}(\tan x) = \sec^2 x$

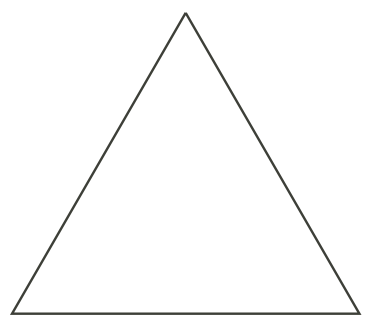
b. $\dfrac{d}{dx}(\cot x) = -\csc^2 x$

c. $\dfrac{d}{dx}(\sec x) = \sec x \tan x$

d. $\dfrac{d}{dx}(\csc x) = -\csc x \cot x$

3. Using the equilateral triangle and right isosceles triangle, determine all trigonometric ratios of the special angles $\pi/6$, $\pi/4$ and $\pi/3$.





4. A piece if wood floating on the surface of a pond is bobbing up and down according to the position function

$$s(t) = \cos(t) + \sin(t) \quad \text{cm}$$

where t is in seconds.

(a) Find formulas for both its velocity and acceleration at time t seconds.

(b) Find smallest time at which the velocity of the piece of wood is zero.

5. Assuming that $\displaystyle\lim_{x\to 0} \frac{\sin x}{x} = 1$ and $\displaystyle\lim_{x\to 0} \frac{1-\cos x}{x} = 0$, answer the questions below:

a. Find the values of (i) $\displaystyle\lim_{x\to 0} \frac{\sin 7x}{3x}$ and (ii) $\displaystyle\lim_{x\to 0} \frac{\tan x}{2x}$

b. Show that the derivative of $\sin x$ is $\cos x$. You will need the identity $\sin(A+B) = \sin A \cos B + \cos A \sin B$.