Math 10350 – Example Set 02A
Quadratic Functions: Section 1.6
Basic Exponential Equations: Section 1.6

1. (Completing the Square Review) A particle moving in a straight line has position in meters, measured from a fixed point **O** on the straight line, at time t seconds is given by

$$s(t) = 5 - 4t + 3t^2$$

(i) Sketch the graph of $s(t)$. (ii) Find the time at which the particle is **closest** to the point **O**. (iii) How far can the particle be from the point **O**?

2. (Sect 1.6) Solve the following equations: (a) $4^x = \frac{1}{8}$; (b) $3 \cdot 9^{x+1} = 81^x$.

Math 10350 – Example Set 02B
Exponential & Logarithmic Function: Section 1.6

▶ Graph of $y = a^x$

Case 1: $a > 1$

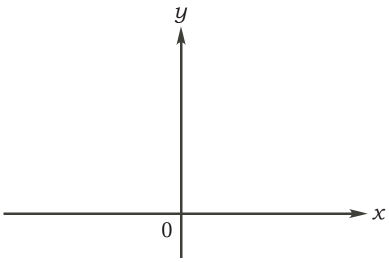
For example, $y = 2^x$.

(i) Complete the table below:

x	-1	-0.5	0	0.5	1
2^x	0.5		1		2

Truncate answers to 2 decimal places

(ii) Plot the points and sketch graph:



(iii) **Properties of a^x when $a > 1$:**

- $a^0 \stackrel{?}{=}$
- domain $\stackrel{?}{=}$ range $\stackrel{?}{=}$
- $\lim\limits_{x\to-\infty} a^x \stackrel{?}{=}$ $\lim\limits_{x\to\infty} a^x \stackrel{?}{=}$
- Asymptote:

Case 2: $0 < a < 1$

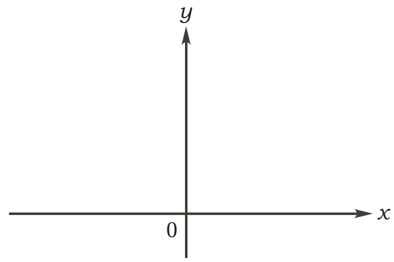
For example, $y = (1/2)^x$.

(i) Complete the table below:

x	-1	-0.5	0	0.5	1
$(1/2)^x$	2		1		0.5

Truncate answers to 2 decimal places

(ii) Plot the points and sketch graph:



(iii) **Properties of a^x when $0 < a < 1$:**

- $a^0 \stackrel{?}{=}$
- domain $\stackrel{?}{=}$ range $\stackrel{?}{=}$
- $\lim\limits_{x\to-\infty} a^x \stackrel{?}{=}$ $\lim\limits_{x\to\infty} a^x \stackrel{?}{=}$
- Asymptote:

1. Sketch the graph of the inverse of a^x for $a > 1$. State its domain and range. We call the inverse of a^x the logarithm function to the base a. Complete the following properties of the logarithm function to the base a below. Could you prove some of them?

i. Change to log base b: $\log_a x =$

ii. $\log_a(a) \stackrel{?}{=}$

iii. $\log_a 1 \stackrel{?}{=}$

iv. $\log_a(xy) \stackrel{?}{=}$

v. $\log_a(x^n) \stackrel{?}{=}$

vi. $\log_a\left(\dfrac{x}{y}\right) \stackrel{?}{=}$

vii. $\log_a(a^x) \stackrel{?}{=}$

viii. $a^{\log_a x} \stackrel{?}{=}$

Commonly used natural log properties: $\ln(e^x) \stackrel{?}{=}$; $e^{\ln x} \stackrel{?}{=}$

2. (Sect 1.6) A quantity y is said to grow or decay exponentially with time t if $y(t) = k \cdot a^t$. A . It is known that the amount of a medication in a patient reduces from an initial amount of 100 mg to 40 mg after three hours. Assuming that the amount of medication decays exponentially, write a formula for the amount of the medication $y(t)$ as a function of time t in hours. What is the half life of the medication in the body? Draw a graph for $y(t)$.

3. Solve the following equations:

(a) $3(4^{x-1}) = 5$ (Give your answer in base e), **(b)** $4e^{x-2} = 3e^{2x}$

4. Given that $\ln(x) = p$ and $\ln(y) = q$, write the following expressions in term of p and q

(i) $\ln(5x^2y^3) \stackrel{?}{=}$

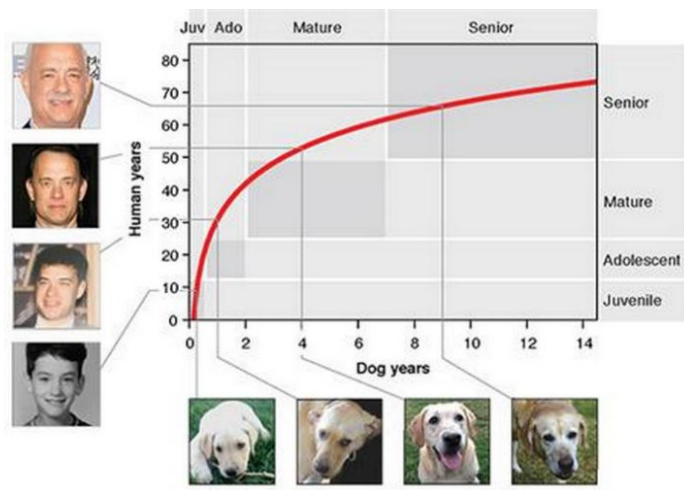
(ii) $\ln\left(\sqrt[5]{\frac{e^4x}{y}}\right) \stackrel{?}{=}$

Math 10350 – Example Set 02C
Exponential & Logarithmic Function: Section 1.6

1. Solve for x for each of the following equations: (a) $2\ln(x-3)=3$, (b) $\log_2(2x+3)=\log_2(x+1)+2$.

2a. It is said that one dog year equals seven human years. Chart a graph of dog years D verses human years H. What kind of function is H in terms of D?

2b. A new genomics research that studies aging in mammals (**) says that the equation between dog years and humans years is much more complicated: $H = 16\ln(D)+31$. (i) What is the human years of a dog when it is one year old? (ii) What is the dog age of a 21 year old human? (**): Quantitative Translation of Dog-to-Human Aging by Conserved Remodeling of the DNA Methylome



Doubling time and Half life.

The **doubling time** of a quantity growing exponentially as time progress is the amount of time needed for

The **half life** of a quantity decaying exponentially as time progress is the amount of time needed for

3. Recent experiments on viability of the coronavirus indicates that it reduces exponentially on various surfaces. The half life of the coronavirus on glass is estimated to be about 14 hours. (a) Starting with 100% initially, find a formula in the form $A\cdot b^t$ for the percentage of the virus on glass after t hours. (b) If we consider the virus no longer infectious (or viable) after it is reduced to 1% or less, estimate how long will the virus remain infectious on glass. $(100(1/2)^{t/14})$

Reference:
Aerosol and Surface Stability of SARS-CoV-2 as Compared with SARS-CoV-1, N Engl J Med April 2020
Stability of SARS-CoV-2 in different environmental conditions, Lancet April 2020.

Discrete Compound Interest. If P dollars (principal) is deposited in an account at annual interest rate r (in decimal form) compounded n times a year, then the balance of the account A after t years is given by

$$\boxed{A(t) = P\left(1 + \frac{r}{n}\right)^{nt}}$$

4. Find a formula for the balance A for an investment with principle \$1000 earning interest at a rate of 5% compounded (a) daily, (b) weekly, (c) monthly and (d) quarterly. Compare the balances when the interest are compounded daily, weekly and quarterly after 10 years. What could you say?

Continuous Compound Interest. If P dollars is deposited in an account at annual interest rate r (in decimal form) compounded **continuously**, then the balance of the account A after t years is given by

$$\boxed{A(t) = Pe^{rt}}$$

5. Write down a formula for the balance A for an investment with principle \$1000 earning interest at a rate of 5% compounded continuously after t years. Find the doubling time of the account in years and months (to the nearest integer)? Does the doubling time depends on the principal? ($1000e^{0.05t}$)

6. You like to see your investment triple every 20 years. At what rate should your investment be growing per annum if interest is (a) compounded quarterly and (b) continuous?

(a) $4(3^{1/80} - 1) \approx 0.0553$ ie 5.53% , (b) $\frac{\ln 3}{20} \approx 0.0549$ ie 5.49%