

Sample Questions Set 04

1. $f(x) = 1x^3 + 2x^2 - 5x - 6$

Leading coefficient of the leading term = 1.

(Divisors = ± 1)

Constant term = -6 (Divisors = $\pm 1, \pm 2, \pm 3,$

± 6)

Possible rational roots (zeros) of $f(x)$

Divisors of Constant term

Divisors of the leading term

$\pm 1, \pm 2, \pm 3, \pm 6$. ←

$$\begin{aligned} f(-1) &= -1 + 2(1) - 5(-1) - 6 \\ &= -1 + 2 + 5 - 6 = 0 \end{aligned}$$

$x = -1$ is a root and so $(x+1)$ is a factor of $f(x)$.

	x^3	x^2	x	const	
-1	1	2	-5	-6	
+)	↓	↗ -1	↗ -1	↗ 6	
	1	1	-6	0	← rem. (check)
	↙ $x+1$	↙ x^2	↙ x	↙ const.	

$$f(x) = (x+1)(x^2 + x - 6)$$

$$= (x+1)(x-2)(x+3)$$

$$2. \quad g(x) = 2x^3 + 3x^2 - 11x - 6$$

Coefficient of the leading term = 2 (Divisors $\pm 1, \pm 2$)

Constant term = -6 (Divisors $\pm 1, \pm 2, \pm 3, \pm 6$)

Possible rational roots of $g(x)$

Divisors of the constant term
Divisors of the leading term

Using ± 1 divisors of the leading term:

$$\pm 1, \pm 2, \pm 3, \pm 6.$$

Using ± 2 divisors of the leading term:

$$\pm \frac{1}{2}, \pm 1, \pm \frac{3}{2}, \pm 3.$$

Try $g(1) = 2 + 3 - 11 - 6 \neq 0.$

$$\begin{aligned} g(-\frac{1}{2}) &= -2 \cdot \frac{1}{8} + 3 \cdot \frac{1}{4} + \frac{11}{2} - 6 \\ &= -\frac{1}{4} + \frac{3}{4} + \frac{11}{2} - 6 = \frac{1}{2} + \frac{11}{2} - 6 = 0. \end{aligned}$$

$x = -1/2$ is a root so $(x + 1/2)$ is a factor.

$$g(x) = (x + \frac{1}{2})(2x^2 + 2x - 12)$$

$$= 2(x + \frac{1}{2})(x^2 + x - 6)$$

$$= (2x + 1)(x + 3)(x - 2)$$

$-\frac{1}{2}$	2	3	-11	-6
	2	2	-12	0
	2	2	-12	0

$\uparrow x^2 \quad \uparrow x \quad \uparrow \text{const} \quad \text{rem} = 0$

check.

3. $P(x) = 2x^4 - 5x^3 - 5x^2 + 20x - 12$

Divisor = $\pm 1, \pm 2$

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

Possible rational roots of $p(x)$:

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm \frac{1}{2}, \pm \frac{3}{2}$

$\text{Deg}(p(x)) = 4$ ← need 2 roots to apply synthetic divisor.

$p(1) = 2 - 5 - 5 + 20 - 12 = 22 - 22 = 0$

$p(\frac{1}{2}) = 2 \cdot \frac{1}{16} - 5 \cdot \frac{1}{8} + 10 - 12 = \frac{1}{8} - \frac{5}{8} + 10 - 12 \neq 0$

$-\frac{1}{2}$

$p(2) = 2(16) - 5(8) - 5(4) + 40 - 12$
 $= 32 - 20 - 12 = 0$

$x = 1, 2$ are roots of $p(x)$ so $(x-1), (x-2)$ are factors.

$(x+1), (x+2)$

1	2	-5	-5	20	-12	
	↓		2	-3	-8	12
2	2x ³	-3	-8	12	0	✓
	↓		4	2	-12	
	2	1	-6	0	0	✓

$p(x) = (x-1)(x-2)(2x^2+x-6)$
 $= (x-1)(x-2)(2x-3)(x+2)$
