

Math 10350 – Final Review

1. The graph of the function $f(x)$ is given in Figure 1 below. Find exactly or state that it does not exist each of the following quantity. If it does not exist explain why.

- (a) $\lim_{x \rightarrow 2} f(x)$
- (b) $\lim_{x \rightarrow 1^-} f(x)$
- (c) $\lim_{x \rightarrow 1} f(x)$
- (d) $\lim_{x \rightarrow -1} f(x)$
- (e) $\lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h}$
- (f) $\lim_{h \rightarrow 0} \frac{f(2+h) + 2}{h}$

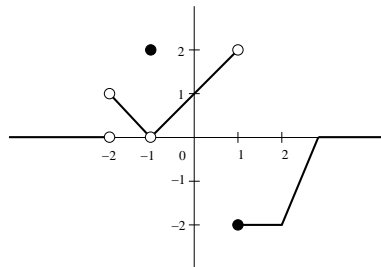


Figure 1

2. If $f'(a) = \lim_{h \rightarrow 0} \frac{\frac{2}{(3+h)^2} - \frac{2}{9}}{h}$, then $f(x) \stackrel{?}{=} \underline{\hspace{2cm}}$ and the value of $a \stackrel{?}{=} \underline{\hspace{2cm}}$.

What is the value of $\lim_{h \rightarrow 0} \frac{\frac{2}{(3+h)^2} - \frac{2}{9}}{h}$? (You shouldn't need to do too much.)

3. Find the values of (i) $\lim_{h \rightarrow 0} \frac{\cos(\pi + h) + 1}{h}$, and (ii) $\lim_{h \rightarrow 0} \frac{e^{2h} - 1}{h}$.

4. Find all horizontal and vertical asymptotes of the graph of $g(x) = \frac{x^2}{x^2 - 1}$. Determine also (a) the values of x for which $g'(x)$ is decreasing, and (b) the values of x for which $g(x)$ is concave up.

(Vert. asympt: $x = -1; x = 1$; Hort. asympt: $y = 1$, (a) Inc: $(-\infty, -1) \cup (-1, 0)$; Dec: $(0, 1) \cup (1, \infty)$, (b) Conc. down: $(-1, 1)$; Conc. up: $(-\infty, -1) \cup (1, \infty)$)

5. Use Newton's method to find an approximation to $\sqrt[3]{100}$.

6. In a certain city the temperature (in $^{\circ}F$) t hours after 9a.m. was approximated by the function

$$T(t) = 50 + 14 \sin \frac{\pi t}{12}.$$

Find the **average rate of change** of temperature during the period from 9a.m. to 9p.m.

7. A particle is moving on a straight line according to the velocity function:

$$v(t) = t(t - 2)(t - 4).$$

Find (a) the displacement and (b) the distance travelled by the particle in the time interval $1 \leq t \leq 6$.

8. Evaluate the following integrals

a. $\int \frac{x^2}{(x-1)^{20}} dx$

d. $\int_0^{\pi/6} \sin^2 u \cos u du$

9. (Review) Gravel is being dumped from a conveyor belt at the rate of $40\pi \text{ ft}^3/\text{min}$ and its coarseness is such that it forms a pile in the shape of a cone whose height is always twice its diameter at the base. How fast is the height of the pile increasing when the pile is 5 feet high? (Answer: $25.6 \text{ ft}/\text{min}$)

10. Let $F(x) = \int_{\sqrt{x}}^3 \frac{\cos t}{t} dt$. Find $F'(x)$.

11. Using limits find the derivative of $f(x) = \sqrt{x+1}$. Write down the linear approximation to $f(x)$ at $x = 3$. Estimate $\sqrt{3.8}$. Draw a graph to illustrate your estimation.

12. Two submarines at 1000 ft below sea level are travelling at 520 mph along straight-line courses that cross at right angles. How fast is the distance between the submarines closing when submarine A is 5 mi from the intersection point and submarine B is 12 mi from the intersection point.

13. The cross-section of a tunnel is a rectangle of height h meters surmounted by a semicircular roof section of radius r meters. If the cross-sectional area is 100 m^2 , determine the dimensions of the cross-section which minimize the perimeter.

14. If $-4 \cos(y) + 3xy^2 = x^7$ find $\frac{dy}{dx}$.

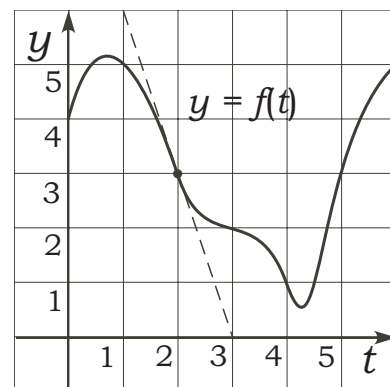
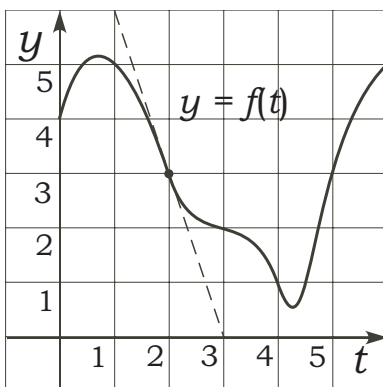
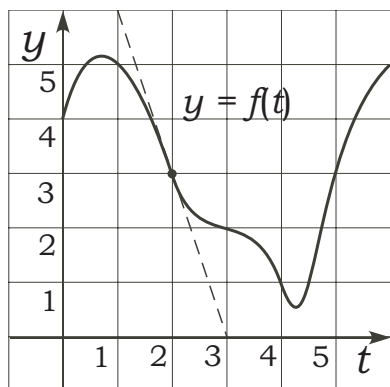
15. Water is flowing into a tank at a rate given by $r = f(t)$ (in m^3/min) whose graph is shown below (three identical ones). Let $V(t)$ denote the volume. The line is the tangent line to the graph of $f(t)$ at $t = 2$.

a. Estimate using (i) left end-point approximation, (ii) right end-point approximation, and (iii) mid-point rule with three equal sub-intervals, the total change in volume over $0 \leq t \leq 6$.

b. Find the $V'(2)$ and $V''(2)$.

c. If $V(3) = 25 \text{ m}^3$, approximate the initial volume. Hint: Use one of the estimates in (a)

d. Is the amount of water in the tank always increasing?



- Be sure that you have all 14 pages of the test.
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- The Honor Code is in effect for this examination, including keeping your answer sheet under cover.

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

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Multiple Choice

1.(6 pts.) Suppose that $g(1) = 2$ and $g'(1) = 5$.

Find the slope of the graph of $P(x) = 2x^3g(x)$ at $x = 1$.

- (a) 2 (b) -2 (c) 12 (d) 30 (e) 22

2.(6 pts.) Suppose that $g(1) = 2$ and $g'(1) = 5$.

Find the derivative of $Q(x) = \frac{g(x)}{2x+1}$ at $x = 1$.

- (a) $\frac{-19}{9}$ (b) $\frac{19}{3}$ (c) $\frac{11}{9}$
(d) $\frac{5}{2}$ (e) $\frac{-11}{9}$

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3.(6 pts.) Find the derivative of $y = \tan^2(3\theta)$.

- (a) $2 \tan(3\theta) \sec^2(3\theta)$
- (b) $6 \tan(3\theta) \sec^2(3\theta)$
- (c) $2 \tan^2(3\theta) \sec(3\theta)$
- (d) None of the given.
- (e) $6 \tan^2(3\theta) \sec(3\theta)$

4.(6 pts.) Evaluate the following limit:

$$\lim_{h \rightarrow 0} \frac{(x+h)^{-2} - x^{-2}}{h}$$

- (a) 0
- (b) $-\frac{2}{x}$
- (c) $-2x^{-3}$
- (d) Does not exist.
- (e) $-\frac{1}{x}$

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5.(6 pts.) Find the value of k for which the function $f(x) = \begin{cases} \frac{x^2 - 3x + 2}{x^2 - 4} & \text{if } x \text{ in } [0, 2) \cup (2, \infty). \\ k & \text{if } x = 2. \end{cases}$

is a continuous for all $x \geq 0$.

- (a) 0
- (b) No such k .
- (c) All real k except 2.
- (d) 1
- (e) $\frac{1}{4}$

6.(6 pts.) Find **all** horizontal and vertical asymptotes of the function:

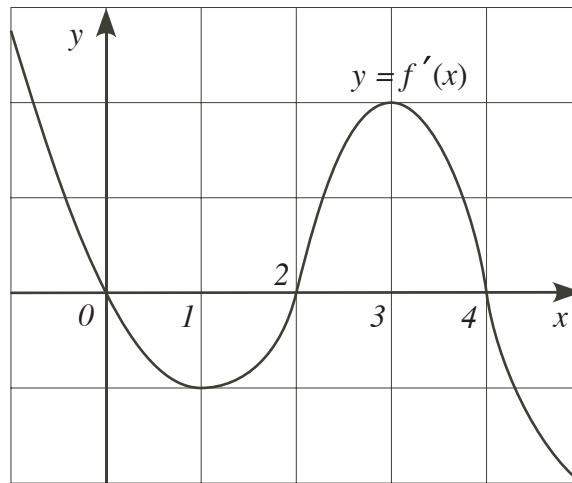
$$g(x) = \frac{\sqrt{x^2 - 4 \cos x + 4}}{x - 2}.$$

- (a) $x = 2$, $y = -1$ and $y = 1$.
- (b) $x = 2$, $y = -2$ and $y = 2$.
- (c) $x = 2$ and $y = 1$.
- (d) $x = 2$ and $y = 2$.
- (e) $x = 1$ and $y = -1$.

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7.(6 pts.) The graph of the **derivative** $f'(x)$ of $f(x)$ for $-1 < x < 5$ is given below.



Derivative of $f(x)$

Find all local extrema of $f(x)$ for $-1 < x < 5$ and classify them?

- (a) Local minimum at $x = 0$ and 4 ; local maximum at $x = 2$.
- (b) Local minimum at $x = 1$; local maximum at $x = 3$.
- (c) All local minimum at $x = 0, 2$ and 4 .
- (d) Local minimum at $x = 2$; local maximum at $x = 0$ and 4 .
- (e) Cannot be determined.

8.(6 pts.) Referring to the same graph of the **derivative** $f'(x)$ of $f(x)$ **ABOVE**, which of the following statements is **TRUE** about the graph of $f(x)$ for $-1 < x < 5$?

- (a) The graph of $f(x)$ is concave downward on the intervals $(2, 4)$ **ONLY**.
- (b) The graph of $f(x)$ is concave upward on the intervals $(1, 3)$ **ONLY**.
- (c) The graph of $f(x)$ is concave upward on the intervals $(-1, 2)$ and $(4, 5)$.
- (d) The graph of $f(x)$ is concave upward on the intervals $(-1, 1)$ and $(3, 5)$.
- (e) The graph of $f(x)$ is concave downward on the intervals $(0, 2)$ and $(4, 5)$.

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9.(6 pts.) The critical numbers of $f(x) = 5 + 8x + 6x^{2/3}$ are

- (a) There are none. (b) -8 and 0 only. (c) $-\frac{1}{8}$ only.
(d) $-\frac{1}{8}$ and 0 only. (e) -8 only.

10.(6 pts.) What are the global maximum and global minimum values of the function $f(x) = x^3 - 12x$ for x in $[0, 3]$?

- (a) The global maximum value is 0 , the global minimum is -9 .
(b) The global maximum value is 16 , the global minimum is -9 .
(c) The global maximum value is 0 , the global minimum is -16 .
(d) The global maximum value is 16 , the global minimum is 0 .
(e) The global maximum value is 9 , the global minimum is 1 .

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11.(6 pts.) Evaluate the integral $\int_0^1 \frac{t+2}{\sqrt{t^2+4t+3}} dt$.

- (a) 2
- (b) $2\sqrt{8} - 2\sqrt{3}$
- (c) $\frac{1}{4}(3^{-3/2} - 8^{-3/2})$
- (d) 1
- (e) $\sqrt{8} - \sqrt{3}$

12.(6 pts.) Find the **linear** approximation (tangent line approximation) of the function $f(x) = (2x + 3)^5 + 3$ at $x = -1$.

- (a) $f(x) \approx 10(2x + 3)^4(x - 1) - 4$ for x near -1 .
- (b) $f(x) \approx 10(2x + 3)^4(x + 1) + 4$ for x near -1 .
- (c) $f(x) \approx 10(x - 4) - 1$ for x near -1 .
- (d) $f(x) \approx 10(x + 1) + 4$ for x near -1 .
- (e) $f(x) \approx 10(x - 1) - 4$ for x near -1 .

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13.(6 pts.) Suppose that the derivative of $f(x)$ is given by

$$f'(x) = 2 \sin x + 6x^2.$$

Find a formula for $f(x)$ if its graph passes through the point $(0, 5)$.

(a) $f(x) = -2 \cos x + 2x^3 + 7$

(b) $f(x) = 2 \cos x + 12x + 3$

(c) $f(x) = -2 \cos x + 2x^3 + 5$

(d) $f(x) = 2 \cos x + 2x^3 + 3$

(e) $f(x) = 2 \cos x + 2x^3 + 5$

14.(6 pts.) The volume of a **spherical** balloon is growing at a constant rate of 1 cubic inch per second. How fast is the radius r growing when $r = 2$ inches? (Note that the volume of a sphere of radius r is $V = \frac{4\pi}{3}r^3$.)

(a) $\frac{1}{8\pi}$ inch/sec

(b) $(8\pi - 1)$ inch/sec

(c) $\frac{1}{16\pi}$ inch/sec

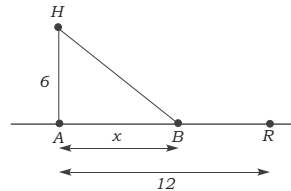
(d) $\frac{1}{4\pi}$ inch/sec

(e) $(4\pi - 1)$ inch/sec

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15.(6 pts.) A house H is located in the woods, 6 miles from the nearest point, A , on a straight road. A restaurant, R , is located 12 miles down the road from A . Jack can ride his bike 2 miles per hour in the woods and 10 miles per hour along the road. He decides to ride the bike through the woods to some intermediate point B , x miles from A , and then ride along the road to R . Since he is starving, he wants to minimize his time. Which of the following is the function to be minimized? **Do not solve the rest of the problem!**



- (a) $12 + 10x$
- (b) $3 + \frac{x}{10}$
- (c) $\frac{\sqrt{36 + x^2}}{2} + \frac{x}{10}$
- (d) $2\sqrt{36 + x^2} + 10(12 - x)$
- (e) $\frac{\sqrt{36 + x^2}}{2} + \frac{12 - x}{10}$

16.(6 pts.) The position function of a ball thrown upward, measured from **ground level**, is given by the function

$$s(t) = -5t^2 + 3t + 2.$$

Here s is in meters and t is in seconds. Find the velocity of the ball the moment it hits the ground.

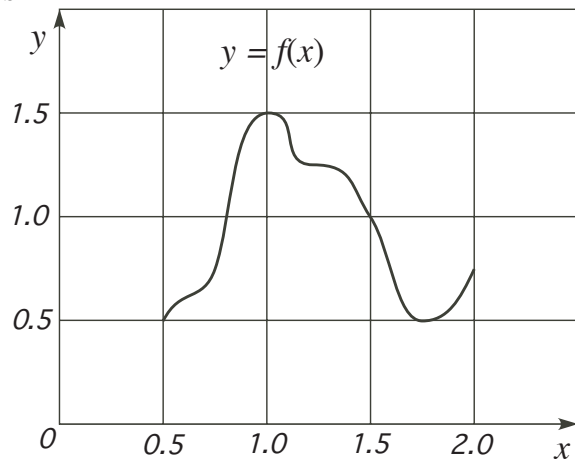
- (a) -7 m/sec
- (b) -1 m/sec
- (c) 1 m/sec
- (d) -10 m/sec
- (e) 0 m/sec

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17.(6 pts.) Find the estimate for the area under the graph of $f(x)$ over $[0.5, 2.0]$, using **left-hand sum** with **three** equal subintervals.

- (a) 1.0
- (b) 3.0
- (c) 4.0
- (d) 2.0
- (e) 1.5



18.(6 pts.) Find the **average rate of change** of the function $f(x) = \sec x \tan x$ for $[0, \pi/4]$.

- (a) $\frac{4}{\pi}(\sqrt{2} - 1)$
- (b) $\frac{4\sqrt{2}}{\pi}$
- (c) 1
- (d) $(\sqrt{2} - 1)$
- (e) $\frac{4}{\pi}$

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19.(6 pts.) Evaluate the integral

$$\int \frac{2x^5 - x^2 + 1}{x^4} dx$$

(a) $\frac{10x^4 - 2x}{4x^3} + C$

(b) $\frac{\frac{x^6}{3} - \frac{x^3}{3} + x + C}{\frac{x^5}{5} + C}$

(c) $\frac{5x}{3} - \frac{5x^{-2}}{3} + 5x + C$

(d) $2 + 2x^{-3} - 4x^{-5} + C$

(e) $x^2 + x^{-1} - \frac{x^{-3}}{3} + C$

20.(6 pts.) Let $f(x)$ be the function whose graph is shown below. Which of the following statements is **FALSE**?

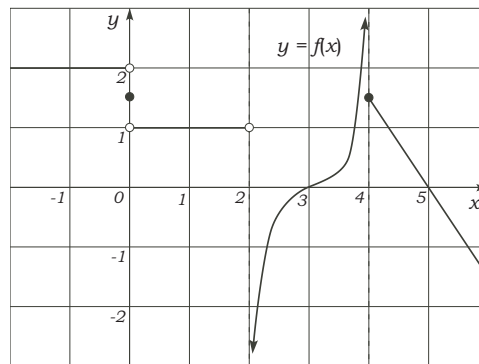
(a) $\lim_{x \rightarrow 2^+} f(x) = -\infty$.

(b) $f(x)$ is not continuous at $x = 0$.

(c) $\lim_{x \rightarrow 4^+} f(x)$ is finite.

(d) $\lim_{x \rightarrow 0} f(x)$ exists.

(e) $f(x)$ is continuous at $x = 3$.



(a)

(b)

(c)

(d)

(e)

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21.(6 pts.) Given that

$$f''(x) = x(x - 2)^2(x - 4)^3.$$

Find all value of x at which the graph of $f(x)$ has an inflection point.

- (a) $x = 0$ and 4 only.
- (b) $x = 0, 2$ and 4.
- (c) $x = 2$ only.
- (d) $x = 0$ only.
- (e) $x = 0$ and 2 only.

22.(6 pts.) Using implicit differentiation, find $\frac{dy}{dx}$ if $y^2 + xy - x^2 = 5$.

- (a) $\frac{dy}{dx} = \frac{2x - y + 5}{2y + x}$
- (b) $\frac{dy}{dx} = \frac{-2x + y}{2y + x}$
- (c) $\frac{dy}{dx} = \frac{-y}{2y - x - 5}$
- (d) $\frac{dy}{dx} = \frac{2x - y}{2y + x}$
- (e) $\frac{dy}{dx} = \frac{1}{3}$

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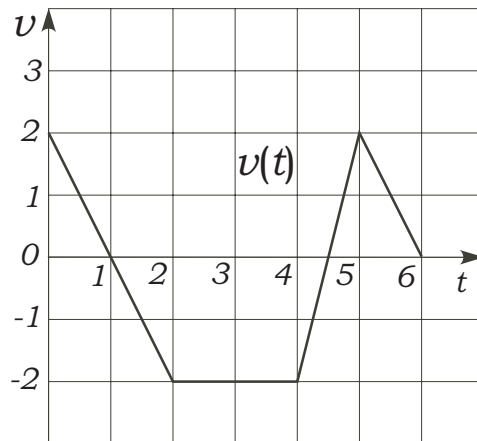
23.(6 pts.) The length (in mm) at time t (in seconds) of a straight metal rod being heated slowly is given by the function

$$L(t) = \sqrt{2t + 1}$$

Using **calculus**, **estimate** the change in length of the rod over the time duration $4 \leq t \leq 4.5$

- (a) $(2\sqrt{10} - 8)$ mm
- (b) Cannot be determined.
- (c) $1/6$ mm
- (d) $(\sqrt{10} - 4)$ mm
- (e) $1/12$ mm

24.(6 pts.) The graph of $v(t)$ is given below:



Find $\int_0^4 v(t) dt$.

- (a) 2
- (b) -4
- (c) 0
- (d) -2
- (e) 4

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25.(6 pts.) Use Newton's method to estimate the solution of

$$x^3 + 2x + 5 = 0.$$

If the initial guess $x_1 = -1$, find the value of the second iterate x_2 .

(a) $x_2 = -\frac{3}{5}$.

(b) $x_2 = -\frac{3}{2}$.

(c) $x_2 = -\frac{2}{5}$.

(d) $x_2 = -\frac{7}{2}$.

(e) $x_2 = -\frac{7}{5}$.

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- Be sure that you have all 14 pages of the test.
- No calculators are to be used.
- The exam lasts for one hour and 15 minutes.
- **When told to begin, remove this answer sheet and keep it under the rest of your test. When told to stop, hand in just this one page.**

Sign the pledge. "On my honor, I have neither given nor received unauthorized aid on this Exam":

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

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Instructor: _____

Multiple Choice

1.(6 pts.) The graph of $g(x)$ and the tangent to the graph at $x = 2$ is given in Figure 1.

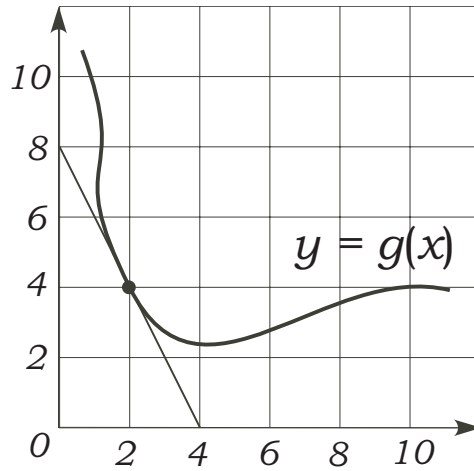


Figure 1.

Find the slope of $P(x) = x^2g(x)$ at $x = 2$.

- (a) -2 (b) -24 (c) 8 (d) -8 (e) 24

2.(6 pts.) Let $F(x) = \int_0^{x^2} f(t) dt$. Find $F'(x)$.

- (a) $x^2f'(x^2)$
(b) $f(x^2)$
(c) $f(x^2) - f(0)$
(d) $2xf'(x^2)$
(e) $2xf(x^2)$

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3.(6 pts.) Find the derivative of $y = \sin^3(2\theta)$.

- (a) $6 \sin^2(2\theta) \cos(2\theta)$ (b) None of the given. (c) $-6 \sin(2\theta) \cos^2(2\theta)$
(d) $-6 \sin^2(2\theta) \cos(2\theta)$ (e) $6 \sin(2\theta) \cos^2(2\theta)$

4.(6 pts.) Compute the following limit:

$$\lim_{h \rightarrow 0} \frac{(x+h)^{10} - x^{10}}{h}$$

- (a) $\frac{x^{11}}{11}$ (b) 0 (c) $10x^{11}$
(d) Does not exist. (e) $10x^9$

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7.(6 pts.) The **derivative** $f'(x)$ of the function $f(x)$ is given below

$$f'(x) = \frac{x}{x^2 - 4} \quad \leftarrow \text{Derivative of } f(x)$$

For what values of x is $f(x)$ **increasing**?

- (a) $(-\infty, -2)$ only.
- (b) $(-2, 0)$ only
- (c) $(-2, 0) \cup (2, \infty)$
- (d) $(-\infty, -2) \cup (0, 2)$
- (e) All x except -2 and 2 .

8.(6 pts.) Referring to the same function considered above with

$$f'(x) = \frac{x}{x^2 - 4} \quad \leftarrow \text{Derivative of } f(x),$$

for what values of x is the graph of $f(x)$ **concave downward**?

- (a) No values of x .
- (b) All x except -2 and 2 .
- (c) $(-\infty, -1/\sqrt{2}) \cup (1/\sqrt{2}, \infty)$ only.
- (d) $(-\infty, -2) \cup (2, \infty)$ only.
- (e) $(-2, 2)$ only.

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9.(6 pts.) Which of the following statements is **TRUE** about the function

$$f(x) = x^3 - 3x^2.$$

- (a) $f(x)$ is always increasing.
- (b) $f(x)$ has a local minimum at $x = 3$ ONLY.
- (c) $f(x)$ neither has a maximum nor minimum at $x = 0$.
- (d) $f(x)$ has a local maximum at $x = 0$
- (e) $f(x)$ has a local maximum at $x = 2$.

10.(6 pts.) Find the **absolute minimum** of the function $f(x) = x^3 - 3x^2 + 10$ for $-2 \leq x \leq 3$.

- (a) -10
- (b) $-\infty$
- (c) -12
- (d) 10
- (e) 6

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11.(6 pts.) Evaluating the integral $\int_0^3 t\sqrt{2t^2 + 1} dt$.

- (a) $\sqrt{3}$
- (b) $\frac{1}{3}(19^{3/2} - 1)$
- (c) $\frac{1}{6}(19^{3/2} - 1)$
- (d) $4\sqrt{3}$
- (e) $\frac{\sqrt{3}}{2}$

12.(6 pts.) Find the **linear** approximation (tangent line approximation) of the function $f(x) = \tan x + 3$ at $x = \pi$.

(You may use: $\tan \pi = 0$ and $\sec \pi = -1$.)

- (a) $f(x) \approx (x - 3) - \pi$ for x near π .
- (b) $f(x) \approx (\sec^2 x)(x - \pi) + 3$ for x near π .
- (c) $f(x) \approx (\sec^2 x)(x + \pi) - 3$ for x near π .
- (d) $f(x) \approx (x + \pi) - 3$ for x near π .
- (e) $f(x) \approx (x - \pi) + 3$ for x near π .

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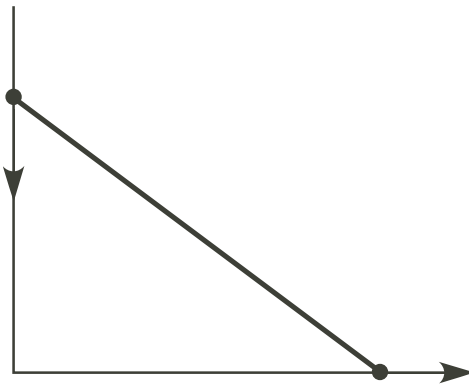
13.(6 pts.) Voltage of a capacitor $V(t)$ (in volts) changes according to the rate

$$\frac{dV}{dt} = \cos t - \sin t; \quad V(0) = 20$$

where t is time in seconds. What is the voltage when $t = \pi$?

- (a) $V(\pi) = 18$
- (b) $V(\pi) = 19$
- (c) $V(\pi) = 22$
- (d) $V(\pi) = 20$
- (e) $V(\pi) = 21$

14.(6 pts.) Consider a 5 feet plank leaning against a vertical wall as shown. If the top of the plank is moving **down** at a rate of $\frac{1}{2}$ ft/sec, at what rate is the bottom of the plank moving when the top is 3 feet **above the ground**.



- (a) 2 ft/sec
- (b) $-\frac{3}{8}$ ft/sec
- (c) $-\frac{2}{3}$ ft/sec
- (d) $\frac{3}{8}$ ft/sec
- (e) $\frac{2}{3}$ ft/sec

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15.(6 pts.) A manufacturer wishes to make a cylindrical can closed at **both ends** with volume $4\pi \text{ m}^3$. The material for the top and bottom costed $\$ 2/\text{m}^2$, and the material of the side costs $\$ 1/\text{m}^2$. Write the formula for the cost function $C(r)$ in terms of the radius of the cylinder r .

(a) $C(r) = 4\pi r^2 + \frac{6\pi}{r}$

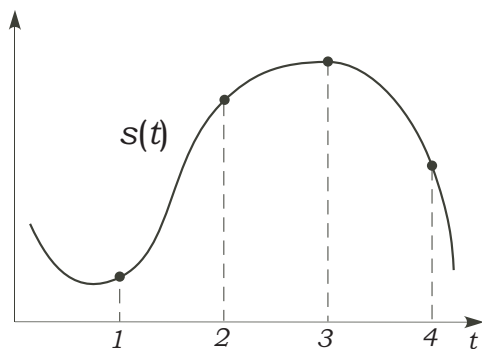
(b) $C(r) = 4\pi r^2 + \frac{8\pi}{r}$

(c) $C(r) = 4\pi r^2 + \frac{4\pi}{r}$

(d) $C(r) = 2\pi r^2 + \frac{8\pi}{r}$

(e) $C(r) = 4\pi r^2 + \frac{6\pi}{r^2}$

16.(6 pts.) Let O be a point on a straight line path. The graph of the position $s(t)$, measured from O , of a particle P moving on the straight path is given below. Let $v(t)$ be the velocity of P , and $a(t)$ be the acceleration of P .



Only ONE of the following statement is **TRUE**. Which is it?

- (a) P returns to the origin twice in the duration $0 < t < 4$.
- (b) $v(t)$ is **greatest** at $t = 3$.
- (c) P is **decelerating** at $t = 1$.
- (d) $v(2)$ and $a(2)$ are both **positive**.
- (e) $v(4)$ and $a(4)$ are both **negative**.

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17.(6 pts.) Find the estimate for the area under the graph of $f(x) = x^2$ over $[1, 3]$, using **right-hand sum** with **four** equal subintervals.

(a) $\left[\left(\frac{3}{2}\right)^2 + 2^2 + \left(\frac{5}{2}\right)^2 + 3^2 \right]$

(b) $\left[1^2 + \left(\frac{3}{2}\right)^2 + 2^2 + \left(\frac{5}{2}\right)^2 \right]$

(c) $\frac{3}{2} \left[\left(\frac{3}{2}\right)^2 + 2^2 + \left(\frac{5}{2}\right)^2 + 3^2 \right]$

(d) $\frac{1}{2} \left[\left(\frac{3}{2}\right)^2 + 2^2 + \left(\frac{5}{2}\right)^2 + 3^2 \right]$

(e) $\frac{1}{2} \left[1^2 + \left(\frac{3}{2}\right)^2 + 2^2 + \left(\frac{5}{2}\right)^2 \right]$

18.(6 pts.) Given that

$$\int_1^3 f(x) dx = 5$$

$$\int_2^3 f(x) dx = -1$$

Find the value of $\int_1^2 (2 - f(x)) dx$.

(a) -4

(b) 8

(c) -6

(d) 6

(e) -2

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19.(6 pts.) Evaluate the integral

$$\int \frac{x-2}{x^3} dx$$

- (a) $\frac{x^2}{2} + x^{-2} + C$ (b) $\frac{\frac{x^2}{2} - 2x + C}{\frac{x^4}{4} + C}$ (c) $-x^{-1} - 2x + C$
- (d) $-x^{-1} + x^{-2} + C$ (e) $-2x^{-2} + 6x^{-4} + C$

20.(6 pts.) Evaluate the limit

$$\lim_{x \rightarrow \infty} (x - \sqrt{x^2 + x})$$

- (a) $\frac{1}{2}$ (b) Does not exist. (c) $-\frac{1}{2}$
- (d) 0 (e) $-\infty$

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21.(6 pts.) The graph of the function $y = f(x)$, and the size of the area enclosed by it and x -axis are shown below.

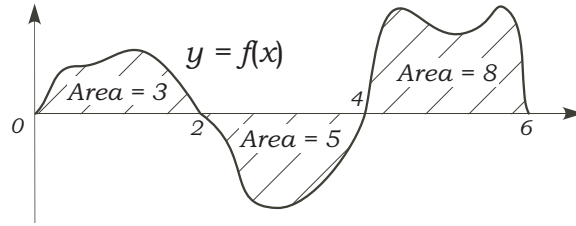


Figure 3.

Find the value of $\int_0^6 f(x) dx$.

- (a) -16 (b) 6 (c) 16
 (d) -6 (e) 0

22.(6 pts.) Find $\frac{dy}{dx}$ if $x^3 - xy + y^2 = 4$

- (a) $\frac{dy}{dx} = \frac{3x^2}{2y - x}$ (b) $\frac{dy}{dx} = \frac{-3x^2}{2y - 1}$ (c) $\frac{dy}{dx} = \frac{y - 3x^2 + 4}{2y - x}$
 (d) $\frac{dy}{dx} = \frac{-y - 3x^2}{2y - x}$ (e) $\frac{dy}{dx} = \frac{y - 3x^2}{2y - x}$

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23.(6 pts.) The critical numbers of $f(x) = 5x^{4/5} - 4x$ are

- (a) There are none. (b) 0 only (c) 1 only
(d) 0 and 1 (e) -1 only

24.(6 pts.) Find the **coordinates** of the points on the curve $f(x) = x^3 + 11$ where the tangent lines are parallel to the line $12x - y + 5 = 0$

- (a) $(-1, 10)$ and $(1, 12)$
(b) None exists.
(c) $(-2, 3)$ and $(2, 19)$
(d) $(2, 19)$ only.
(e) $(1, 12)$ only.

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25.(6 pts.) A student attempts to use Newton's method to estimate the solution of

$$x^3 + x + 1 = 0.$$

If her initial guess $x_0 = 0$. Find the values of the first two iterates x_1 and x_2 .

(a) $x_1 = -1$ and $x_2 = -\frac{1}{2}$.

(b) $x_1 = 1$ and $x_2 = \frac{1}{4}$.

(c) $x_1 = 1$ and $x_2 = \frac{7}{4}$.

(d) $x_1 = -1$ and $x_2 = 2$.

(e) $x_1 = -1$ and $x_2 = -\frac{3}{4}$.

- Be sure that you have all 14 pages of the test.
- No calculators are to be used.
- The exam lasts for one hour and 15 minutes.
- **When told to begin, remove this answer sheet and keep it under the rest of your test. When told to stop, hand in just this one page.**

Sign the pledge. "On my honor, I have neither given nor received unauthorized aid on this Exam":

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

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