Math 10350 – Exam 03 Review

1. The statement: "$f(x)$ is increasing on $a < x < b$." is the same as:

1a. "$f'(x)$ is ____________ on $a < x < b$."

2. The statement: "$f'(x)$ is negative on $a < x < b$." is the same as:

2a. "$f(x)$ is ____________ on $a < x < b$."

3. The statement: "The graph of $f(x)$ is concave up on $a < x < b$." is the same as:

3a. "$f''(x)$ is ____________ on $a < x < b$." is the same as:

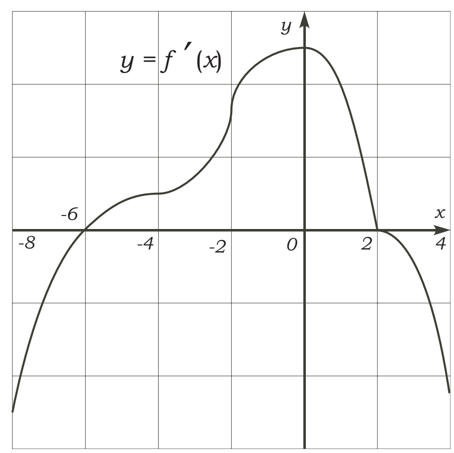
3b. "$f'(x)$ is ____________ on $a < x < b$."

4. The statement: "$f'(x)$ is decreasing on $a < x < b$." is the same as:

4a. "$f''(x)$ is ____________ on $a < x < b$." is the same as:

4b. "The graph of $f(x)$ is ____________ on $a < x < b$."

1. Let $f(x)$ is The figure below is the graph of the **derivative** $f'(x)$ of $f(x)$ for $-8 < x < 4$. Find all intervals on which **the graph of** $f(x)$ is concave up?



(i) Find all values of x in $(-8, 4)$ for which $f(x)$ is increasing.

(ii) Find all values of x in $(-8, 4)$ for which $f(x)$ is decreasing.

(iii) Find the critical points of $f(x)$ in $(-8, 4)$. Are these local maximums or minimums?

(iv) Find all intervals on which **the graph of** $f(x)$ is concave up in $(-8, 4)$.

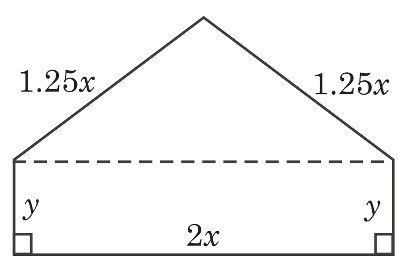
(v) Find all intervals on which **the graph of** $f(x)$ is concave up in $(-8, 4)$.

(vi) Find all values of x in $(-8, 4)$ for which $f(x)$ has an inflection point.

2. Find all vertical and horizontal asymptotes of the following functions:

(a) $f(x) = \dfrac{x^2 - 3x - 2}{x^3 - 4x}$; (b) $g(x) = \dfrac{2x^4 + 3x^2 + x + 1}{\sqrt{x^8 - 1}}$; (c) $h(x) = \dfrac{e^{3x} + e^x - 3}{3e^{3x} + 4e^x + 5}$

3. An landscaper has 30 ft of fencing and wishes to enclosed a five sided figure as shown. Find an expression for the area A enclosed in terms of x. Find the values of x and y that maximizes the area A. What is the maximum area that can be enclosed? (Ans: $x = 4$ft, $y = 6$ft, max $A = 60\text{ft}^2$)



4. A rectangular tank with an open top must have a volume of 162 cubic meters. The base costs 3 hundred dollars per square meter and the side costs 2 hundred dollars per square meters. Let C be the cost of making such a tank. (a) If the square base is $x \times x$, write down the cost function $C(x)$ in terms of the x. (b) Write down the range of the possible values of x. (c) Using calculus, find the dimensions of the tank that minimizes the cost.

5. Let $f(x) = x + \dfrac{4}{x}$

(a) The derivative of $f(x)$, $f'(x) =$ ____________.

(b) Find all critical points of $f(x)$.

(c) By classifying the critical points in Part (b) using first derivative test OR OTHERWISE, fill in the blanks below:

(i) Local maximum occurs at $x =$ ________ Coordinates $=$ ________ (Fill in NA if none).

(ii) Local minimum occurs at $x =$ ________ Coordinates $=$ ________ (Fill in NA if none).

(iii) The function $f(x)$ is **increasing** on the interval(s): ________________________

(iv) The function $f(x)$ is **decreasing** on the interval(s): ________________________

(d) Find all values of x for which the function is (a) concave up; (b) concave down. Give the coordinates of the inflection points.

(e) Draw the graph of $y = x + \dfrac{4}{x}$ marking clearly all important features.

6. Find the absolute maximum value and absolute minimum value of the function

$$f(x) = 2x^3 - 3x^2 - 12x$$

on the interval $-2 \le x \le 1$, and say where they occur.

7. Find the value of the following limits:

(a) $\displaystyle\lim_{x\to 0+} \left(1 + \frac{1}{x^2}\right)^x$ (b) $\displaystyle\lim_{x\to\infty} \frac{e^x - 1}{\sin x}$ (c) $\displaystyle\lim_{x\to\pi/2} (\tan x - \sec x)$

8. Find the value(s) of c satisfying the conclusion of the MVT for function $f(x) = x \ln x$ on the interval $[1, 2]$.

9. Draw a picture to illustrate MVT for the function $f(x) = \sqrt{x-4}$ on the interval $[5, 13]$.

10. Draw a picture to illustrate Rolle's theorem for the function $f(x) = \cos 2x$ on the interval $[-\pi/4, \pi/4]$.

11. (4.6/Q20) Rice production requires both labor and capital investment in equipment and land. Suppose that if x per acre are invested in labor and y dollars per acre are invested in equipment and land, then the yield P of rice per acre is given by the formula $P = 100\sqrt{x} + 150\sqrt{y}$. If a farmer invests \$40/acre, how should he divide the \$40 between labor and capital investment in order to maximize the amount of rice produced.

12. Describe the monotonicity and concavity of the function $f(x) = 3x^5 - 20x^3$. State all critical points and classify them. Find all inflection points of $f(x)$.

Math 10350 – Exam 03 Review Answers

1. (i) $(-6, 2)$; (ii) $(-8, -6) \cup (2, 4)$; (iii) $x = -6$ local min., $x = 2$ local max; (iv) $(-8, 0)$; (v) $(0, 2) \cup (2, 4)$, (vi) Inflection point at $x = 0$ but not at $x = 2$.

2. (a) $f(x) = \dfrac{x^2 - 3x + 2}{x^3 - 4x} = \dfrac{(x-1)(x-2)}{x(x-2)(x+2)}$ so vertical asymptotes are $x = -2$, and $x = 0$. Horizontal asymptote is $y = 0$ because $\lim\limits_{x \to -\infty} f(x) = 0 = \lim\limits_{x \to \infty} f(x)$.

(b) $g(x) = \dfrac{2x^4 + 3x^3 + x + 1}{\sqrt{x^8 - 1}} = \dfrac{2x^4 + 3x^3 + x + 1}{\sqrt{(x^4+1)(x^2+1)(x-1)(x+1)}}$. Vertical asymptotes are $x = -1$ and $x = 1$. $\lim\limits_{x \to -\infty} g(x) = 2 = \lim\limits_{x \to \infty} g(x)$ so horizontal asymptote is $y = 2$.

(c) $h(x) = \dfrac{e^{3x} + e^x - 3}{3e^{3x} + 4e^x + 5}$. No vertical asymptotes. $\lim\limits_{x \to -\infty} h(x) = -3/5$ and $\lim\limits_{x \to \infty} g(x) = 1/3$ so horizontal asymptotes are $y = -3/5$ and $y = 1/3$.

3. $x = 4\text{ft}$, $y = 6\text{ft}$, max $A = 60\text{ft}^2$

4. (a) $C(x) = 3x^2 + \dfrac{1296}{x}$; (b) $0 < x < \infty$; (c) Global min at $x = 6$, $C(6) = 324$ hundreds of dollar.

5. (a) $f'(x) = 1 - \dfrac{4}{x^2}$; (b) $x = -2, 2$.

(c) (i) Local maximum occurs at $x = -2$ Coordinates $= (-2, -4)$.

(ii) Local minimum occurs at $x = 2$ Coordinates $= (2, 4)$.

(iii) The function $f(x)$ is **increasing** on the interval(s): $(-\infty, -2) \cup (2, \infty)$

(iv) The function $f(x)$ is **decreasing** on the interval(s): $(-2, 0) \cup (0, 2)$

(d) (a) concave up for $(0, \infty)$; (b) concave down for $(-\infty, 0)$. No inflection points.

(e) Yours!!!

6. End-points: $f(-2) = -4$ and $f(1) = -13$ (Absolute min). Critical point: $f(-1) = 7$ (Absolute max).

7. Find the value of the following limits:

(a) $\lim\limits_{x \to 0+} \left(1 + \dfrac{1}{x^2}\right)^x = 1$ (b) $\lim\limits_{x \to \infty} \dfrac{e^x - 1}{\sin x}$ does not exist. (c) $\lim\limits_{x \to \pi/2} (\tan x - \sec x) = 0$

8. $c = e^{2 \ln 2 - 1}$. **9.** $c = 8$ **10.** $c = 0$.

11. $P(x) = 100\sqrt{x} + 150\sqrt{40 - x}$ where $0 \le x \le 40$. End-points: $P(0) = 300\sqrt{10}$ and $P(40) = 200\sqrt{10}$.

Critical point:
$P(160/13) = 100\sqrt{160/13} + 150\sqrt{360/13} = 400\sqrt{10/13} + 900\sqrt{10/13} = 1300\sqrt{10/13} = 100\sqrt{130}$ (Global max).

12. Increasing on $(-\infty, -2) \cup (2, \infty)$. Decreasing on $(-2, 0) \cup (0, 2)$.

Concave up on $(-\sqrt{2}, 0) \cup (\sqrt{2}, \infty)$. Concave down $(-\infty, -\sqrt{2}) \cup (0, \sqrt{2})$.

Math 10350: Calculus A
Exam III Sample
November 18, 2018

Name: ______________________________

Class Time: ________________________

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for one hour and 15 minutes.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 11 pages of the test.

Sign the pledge. "On my honor, I have neither given nor received unauthorized aid on this Exam":

____

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	a	b	c	d	e
2.	a	b	c	d	e
3.	a	b	c	d	e
4.	a	b	c	d	e
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6.	a	b	c	d	e
7.	a	b	c	d	e
8.	a	b	c	d	e
9.	a	b	c	d	e
10.	a	b	c	d	e
11.	a	b	c	d	e
12.	a	b	c	d	e

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Multiple Choice	________
13.	________
14.	________
15.	________
16.	________
Total	________

Name: ______________________________

Class Time: __________________________

Multiple Choice

1.(5 pts.) Which of the following statements is **TRUE** about the graph of

$$f(x) = 3x^{2/3} + 2x - 1?$$

(only one of them is true)

(a) The graph of $f(x)$ is concave upward on the interval $(-\infty, -1) \cup (0, \infty)$ only.

(b) The graph of $f(x)$ is concave downward on the interval $(-1, 0)$ only.

(c) The graph of $f(x)$ is concave upward on the interval $(-1, 0)$ only.

(d) The graph of $f(x)$ is concave upward on the intervals $(-\infty, 0)$ and $(0, \infty)$.

(e) The graph of $f(x)$ is concave downward on the intervals $(-\infty, 0)$ and $(0, \infty)$.

2.(5 pts.) Find the value(s) of x at which the function $f(x) = 3x^5 - 20x^4$ has an inflection point.

(a) $x = 0$ only.

(b) $x = 4$ only.

(c) $x = 0$ and $x = 16/3$ only.

(d) $x = 0$ and $x = 4$ only.

(e) $x = 16/3$ only.

Name: ______________________________

Class Time: __________________________

3.(5 pts.) Find all horizontal asymptotes of the graph of $y = \dfrac{\sqrt{4x^2+2x+1}}{x+2}$.

(a) $y = 4$ and $y = -4$ only.

(b) $y = 2$ and $y = -2$ only.

(c) $y = -4$ only.

(d) $y = 4$ only.

(e) $y = 2$ only.

4.(5 pts.) Find the limit $\displaystyle\lim_{x\to\infty} \frac{2\cos x + x}{\cos x + 5x + 10}$.

(a) 2

(b) 1/10

(c) Does not exists

(d) 1/5

(e) $+\infty$

Name: ______________________________

Class Time: __________________________

5.(5 pts.) Let $f(x) = 1 - \dfrac{1}{x}$. What value $x = c$ on the interval $1 \leq x \leq 4$ will satisfy the Mean Value Theorem?

(a) -2 and 2 (b) $\sqrt{3}/2$ (c) No such value.

(d) $1/2$ (e) 2

6.(5 pts.) Find the limit $\displaystyle\lim_{x\to\infty} \frac{\sin(2x^7 + 7x^3 + 3)}{\sqrt{x+1}}$.

(a) Does not exists (b) 7 (c) 0

(d) $+\infty$ (e) 2

Name: ______________________________

Class Time: __________________________

7.(5 pts.) Find the absolute maximum and absolute minimum of the function

$$f(x) = 3x^{2/3} + 2x - 1$$

for x in $[-1, 8]$.

(a) Absolute maximum $= 3$ and Absolute minimum $= 0$

(b) Absolute maximum $= 27$ and Absolute minimum $= 0$

(c) Absolute maximum $= 27$ and Absolute minimum $= -1$

(d) Absolute maximum $= 0$ and Absolute minimum $= -1$

(e) Absolute maximum $= 3$ and Absolute minimum $= 2$

8.(5 pts.) If $s(t)$ represents weekly sales of a product, $s(10) = 20$, $s'(10) = 0$, and $s''(t) > 0$ for all t, which of the following statements is possibly **TRUE**? (only one of them is true)

(a) The rate of sales were decreasing after the 10th week

(b) The sales were decreasing after the 10th week

(c) The sales were increasing before the 10th week

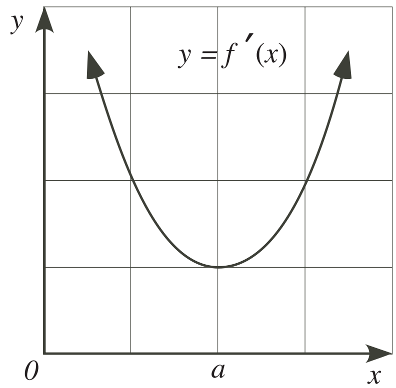
(d) The sales reached a maximum at 10th week

(e) The sales bottomed out (reached a minimum) at 10th week

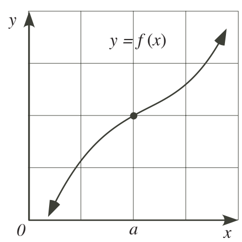
Name: ____________________________

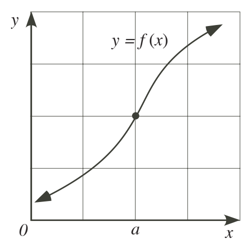
Class Time: ________________________

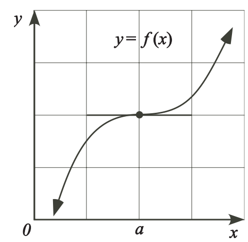
9.(5 pts.) The graph of the **derivative** $f'(x)$ of $f(x)$ is given below.

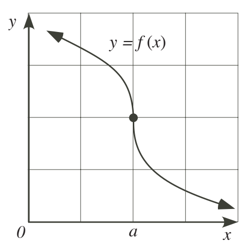


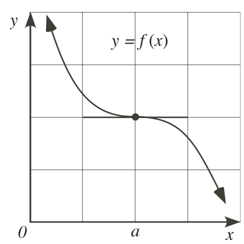
Which of the following best describe the graph of $y = f(x)$?

(a) 

(b) 

(c) 

(d) 

(e) 

10.(5 pts.) Find the equations of all **horizontal** asymptotes of the function

$$y = \frac{5e^{6x} - 2e^{3x} + 4}{2e^{6x} + 3e^{3x} - 2}.$$

(a) $y = -2$ and $y = \frac{5}{2}$

(b) $y = -\frac{1}{2}$ and $y = \frac{2}{5}$

(c) $y = -\frac{2}{3}$ and $y = \frac{5}{2}$

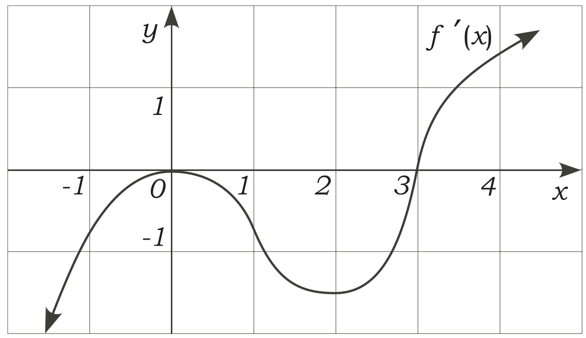
(d) $x = -2$ and $x = \frac{5}{2}$

(e) $x = -\frac{2}{3}$ and $x = \frac{5}{2}$

Name: ______________________________

Class Time: __________________________

11.(5 pts.) The graph of the **derivative** $f'(x)$ of $f(x)$ is given below.



Derivative of $f(x)$

Which of the following best describe the graph of $y = f(x)$?

(a) The graph of $f(x)$ is concave downward on the intervals $(-\infty, 1)$ and $(3, \infty)$.

(b) The graph of $f(x)$ is concave upward on the intervals $(0, 2)$ ONLY.

(c) The graph of $f(x)$ is concave upward on the intervals $(-\infty, 0)$ and $(2, \infty)$.

(d) The graph of $f(x)$ is concave downward on the intervals $(-\infty, 0)$ and $(2, \infty)$.

(e) The graph of $f(x)$ is concave upward on the intervals $(1, 3)$ ONLY.

12.(5 pts.) Find all point(s) of inflection for $f(x)$, whose **second order derivative** is given by $f''(x) = (x-1)(x-3)^2(x-5)^3$.

(a) $x = 5$ only

(b) $x = 3$ only

(c) $x = 1$ only

(d) $x = 1$, $x = 3$ and $x = 5$

(e) $x = 1$ and $x = 5$ only

Name: ____________________________

Class Time: ________________________

Partial Credit

You must show your work on the partial credit problems to receive credit!

13.(12 pts.) Consider the a cylinder **closed** on both ends with surface area 54π m^2.

(a) Write down the volume $V(r)$ in terms of the radius r of the cylinder.

You may use the formulas: $\pi r^2 h$ and $2\pi r h$.

(b) Write down the range of the possible values of r. **Answer:** ____________________

(c) Using calculus, find the largest volume that such a cylinder could enclose.

Name: ____________________________

Class Time: ________________________

14.(12 pts.) Sketch the graph of a function defined on $(-2, \infty)$ in the axes below with the following properties:

(1) $f(0) = 2$ and $f(2) = 0$.

(2) $f(x)$ has a vertical asymptotes at $x = -2$.

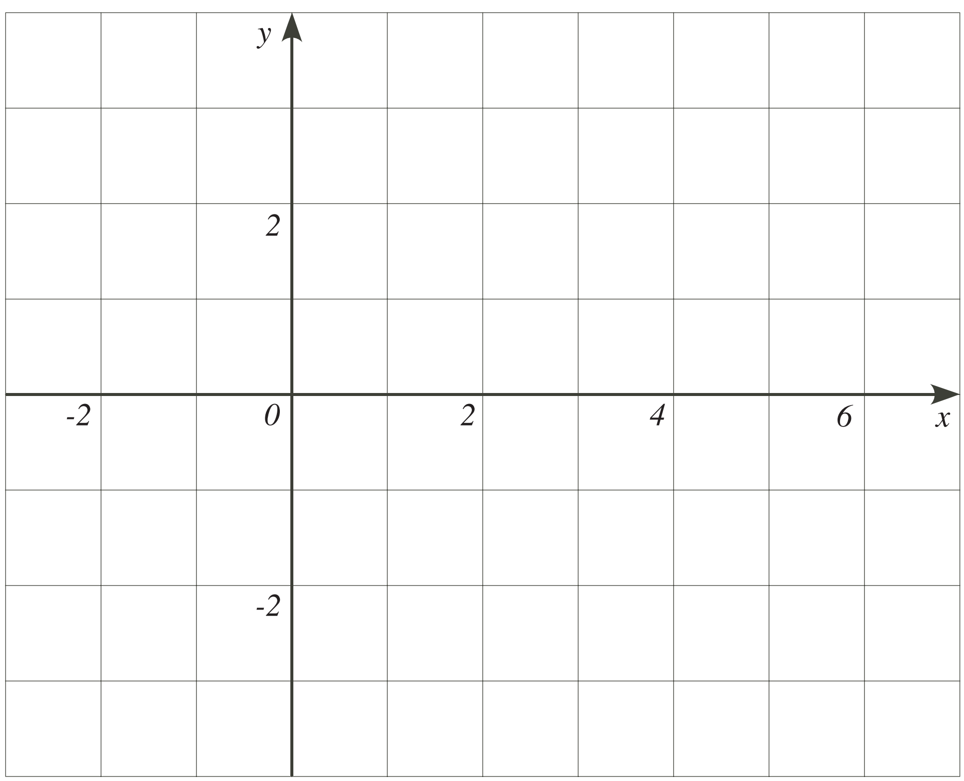
(3) $\lim_{x \to \infty} f(x) = -2$

(4) $f'(x) > 0$ for $-2 < x < 0$.

(5) $f'(x) < 0$ for $0 < x < +\infty$.

(6) $f''(x) < 0$ for $-2 < x < 2$.

(7) $f''(x) > 0$ for $2 < x < +\infty$.



Name: ______________________________

Class Time: __________________________

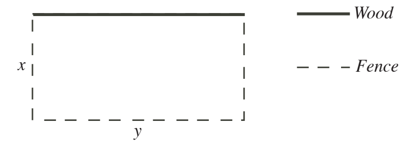
15.(12 pts.) Using calculus, find the point on the graph of $y = 3x + 10$ closest to the origin $(0, 0)$.

Name: ______________________________

Class Time: __________________________

16.(12 pts.) A farmer wishes to build a rectangular enclosure of area 200 sq. ft. Three sides of the enclosure will be fencing at \$1/ft, while the remaining side is wood at \$3/ft. The plan of the enclosure is given in the figure below.

(a) Write down the cost $C(x)$ in terms of the width x of the enclosure.



(b) Write down the range of the possible values of x. **Answer:** ____________________

(c) Using calculus, find the dimensions of the enclosure that minimized the cost C. You must give reason why your answers makes C minimum.

Math 10350: Calculus A
Exam III Sample
November 18, 2018

Name: ______________________________

Class Time: ANSWERS

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for one hour and 15 minutes.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 11 pages of the test.

Sign the pledge. "On my honor, I have neither given nor received unauthorized aid on this Exam":

____

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	a	b	c	d	•
2.	a	•	c	d	e
3.	a	•	c	d	e
4.	a	b	c	•	e
5.	a	b	c	d	•
6.	a	b	•	d	e
7.	a	b	•	d	e
8.	a	b	c	d	•
9.	•	b	c	d	e
10.	•	b	c	d	e
11.	a	b	•	d	e
12.	a	b	c	d	•

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Multiple Choice	________
13.	________
14.	________
15.	________
16.	________
Total	________

Math 10350: Calculus A
Exam III
November 17, 2019

Name: ______________________________

Class Time: __________________________

- The Honor Code is in effect for this examination. All work is to be your own.
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9.	a	b	c	d	e
10.	a	b	c	d	e
11.	a	b	c	d	e

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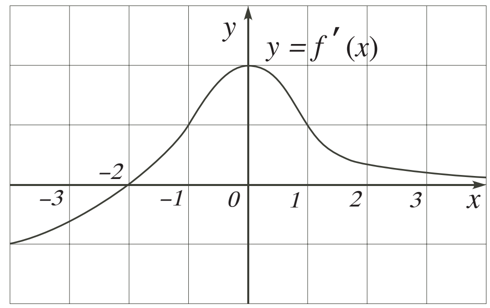
Multiple Choice	________
12.	________
13.	________
14.	________
15.	________
Total	________

Name: ____________________________

Class Time: ________________________

Multiple Choice

1.(5 pts.) The graph of the **derivative** $f'(x)$ of $f(x)$ for $-4 < x < 4$ is given below. Find the values of x in $(-4, 4)$ for which $f(x)$ is **concave up**.



Graph of the Derivative

(a) $(-4, -1) \cup (1, 4)$

(b) $(0, 4)$

(c) $(-1, 1)$

(d) $(-2, 4)$

(e) $(-4, 0)$

2.(5 pts.) For the same function $f(x)$ above, which of the following statements is **TRUE**?

(a) $f(x)$ has a local minimum at $x = -2$.

(b) $f(x)$ has a local minimum at $x = 0$.

(c) $f(x)$ has a local maximum at $x = -2$.

(d) $f(x)$ has a local maximum at $x = 1$.

(e) $f(x)$ has a local maximum at $x = 0$.

Name: ____________________________

Class Time: ________________________

3.(5 pts.) Find all horizontal asymptote(s) for $f(x) = \dfrac{2x^3 + x^2 + 1}{\sqrt{3x^6 + 7x^3 - 11x + 2}}$

(a) $y = \dfrac{2}{\sqrt{3}}$ only

(b) $y = 0$ only

(c) $y = -\dfrac{2}{\sqrt{3}}$ only

(d) $y = \dfrac{2}{\sqrt{3}}$ and $y = -\dfrac{2}{\sqrt{3}}$

(e) There is no horizontal asymptote

4.(5 pts.) Find the value $\lim\limits_{x\to\infty} x^{1/x}$.

Hint: You will need to use log and exponential functions.

(a) Does not exist.

(b) 1

(c) 0

(d) e

(e) $+\infty$

Name: ____________________________

Class Time: ________________________

5.(5 pts.) Find all **vertical** asymptotes of the function $y = \dfrac{x-2}{x^2+x-6}$

(a) $x = 2$

(b) $x = 3$

(c) $x = -3$

(d) $x = -3$ and $x = 2$

(e) $x = -2$ and $x = 3$

6.(5 pts.) A piece of wire 30 meter long is cut into two pieces. One piece is bent into a square and the other is bent into an equilateral triangle. If x is the length of the wire bent into a square, which of the following functions below represents the total area enclosed by both the square and equilateral triangle

(a) $\dfrac{x^2}{4} + \dfrac{(30-x)^2\sqrt{3}}{9}$

(b) $\dfrac{x^2}{16} + \dfrac{(30-x)^2}{27}$

(c) $\dfrac{x^2}{16} + \dfrac{(30-x)^2\sqrt{3}}{36}$

(d) $\dfrac{x^2}{16} + \dfrac{(30-x)^2}{54}$

(e) $\dfrac{x^2}{16} + \dfrac{(30-x)^2\sqrt{3}}{9}$

Name: ______________________________

Class Time: __________________________

7.(5 pts.) Suppose $f(x)$ is a function defined for all values of x such that its **second derivative** is

$$f''(x) = x^3 - 4x^2.$$

Find all values of x for which $f(x)$ has an inflection point.

(a) $x = 0$ and $x = 4$

(b) No such value exist.

(c) $x = 0$ and $x = 8/3$.

(d) $x = 0$ only.

(e) $x = 4$ only.

8.(5 pts.) If the derivative of $g(x)$ is given by $g'(x) = \dfrac{x-2}{x+2}$, find the values of x for which $g(x)$ is **increasing**.

(a) $(-2, 2)$ only.

(b) $(-\infty, -2) \cup (2, \infty)$ only.

(c) $(-\infty, -2)$ only.

(d) For all real values except $x = 2$.

(e) $(2, \infty)$ only.

Name: ______________________________

Class Time: __________________________

9.(5 pts.) Find the value of $\displaystyle\lim_{x\to-\infty} \frac{2x}{\sqrt{x^2+2x+2}}$.

(a) 2

(b) -2

(c) -1

(d) 0

(e) 1

10.(5 pts.) Let $f(x) = \dfrac{1}{x}$. Find all values $x = c$ in the interval $2 \leq x \leq 4$ that satisfy the Mean Value Theorem.

(a) -2 and 2

(b) 2

(c) 3

(d) $\sqrt{8}$

(e) $-\sqrt{8}$ and $\sqrt{8}$

Name: ______________________________

Class Time: __________________________

11.(5 pts.) Suppose $x = 1$ is a critical point of $f(x)$ such that $f'(1) = 0$, and

$$f''(x) = 3x^2 - x.$$

Using second derivative test, which of one of the following statements could you conclude?

(a) $f(x)$ has a **global maximum** at $x = 1$.

(b) $f(x)$ has a **global minimum** at $x = 1$.

(c) $f(x)$ is **increasing** at $x = 1$.

(d) $f(x)$ has a **local maximum** at $x = 1$.

(e) $f(x)$ has a **local minimum** at $x = 1$.

Name: ____________________________

Class Time: ________________________

Partial Credit

You must show your work on the partial credit problems to receive credit!

12.(12 pts.) Find the absolute maximum and absolute minimum of the function

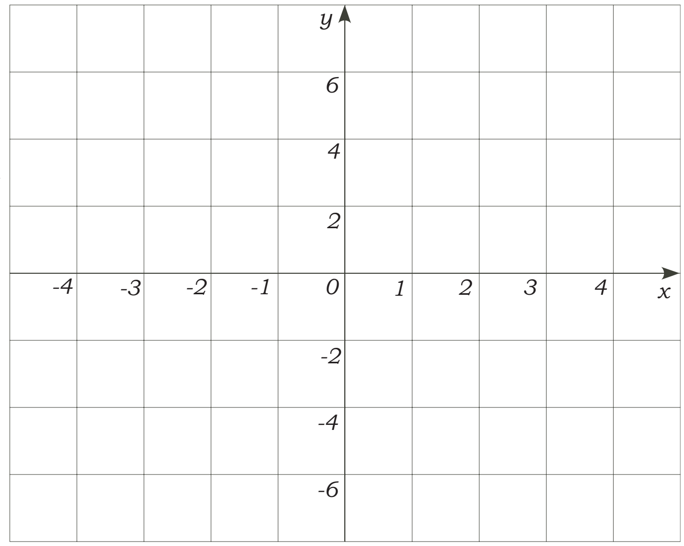
$$f(x) = e^{-x^2+2x}$$

on the interval $-1 \leq x \leq 2$.

Name: ____________________________

Class Time: ________________________

13.(12 pts.) **Part (A)** Sketch the graph of a function in the axes below with the following properties:



(1) $f(0) = 0$

(2) $f(x)$ has vertical asymptotes at $x = -2$ and $x = 2$.

(3) $\lim_{x \to -\infty} f(x) = 2 = \lim_{x \to \infty} f(x)$

(4) $f'(x) > 0$ for all values of x except -2 and 2.

(5) $f''(x) > 0$ for $-\infty < x < -2$ and $0 < x < 2$.

(6) $f''(x) < 0$ for $-2 < x < 0$ and $2 < x < \infty$.

Part (B) The **derivative** of a function $g(x)$ is given by:

$$g'(x) = \frac{x-2}{x-1}$$

For what values of x is $g(x)$ concave up? what about concave down?

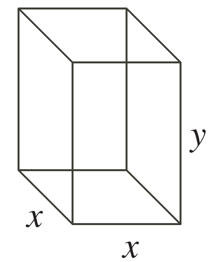
Answer: Concave up on ________________; Concave down on ________________

Name: ______________________________

Class Time: __________________________

14.(12 pts.) A **closed** rectangular box with square based is such that its volume is 8 m^3. If the dimensions of the base is $x \times x$, and the height of the box is y, answer the question below.

(a) Write down the total surface area $A(x)$ in terms of the width x of the box.



(b) Write down the range of the possible values of x. **Answer:** ____________________

(c) Using calculus, find the value of x that minimizes the area A of the box. You must give reason why your answer makes A minimum.

Name: ______________________________

Class Time: __________________________

15.(12 pts.) Sketch the graph of a **differentiable** function defined on $(-\infty, 2)$ in the axes below with the following properties:

(1) $f(0) = 0$ and $f(-2) = 2$.

(2) $f(x)$ has a vertical asymptotes at $x = 2$.

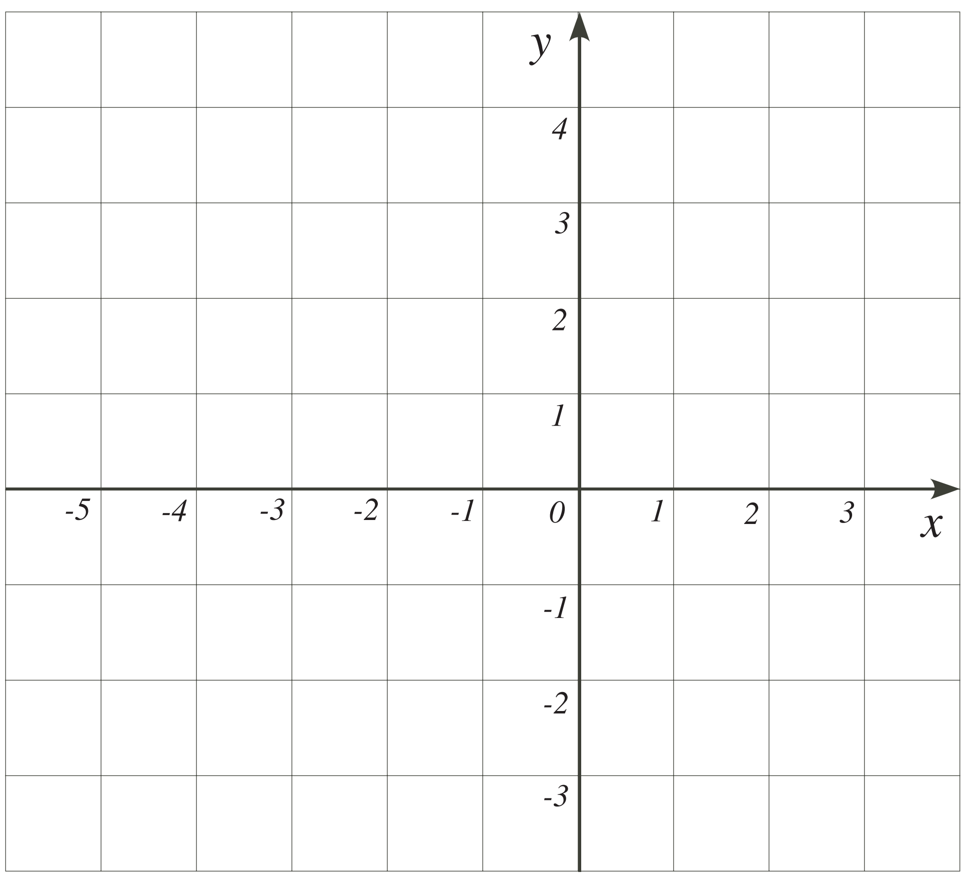
(3) $\lim_{x \to -\infty} f(x) = 4$

(4) $f'(0) = 0$.

(5) $f'(x) < 0$ for $(-\infty, 0) \cup (0, 2)$.

(6) $f''(x) < 0$ for $(-\infty, -2) \cup (0, 2)$.

(7) $f''(x) > 0$ for $(-2, 0)$.



Math 10350: Calculus A
Exam III
November 17, 2019

Name: ______________________________

Class Time: ANSWERS

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for one hour and 15 minutes.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 11 pages of the test.

Sign the pledge. "On my honor, I have neither given nor received unauthorized aid on this Exam":

____

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	a	b	c	d	•
2.	•	b	c	d	e
3.	a	b	c	•	e
4.	a	•	c	d	e
5.	a	b	•	d	e
6.	a	b	•	d	e
7.	a	b	c	d	•
8.	a	•	c	d	e
9.	a	•	c	d	e
10.	a	b	c	•	e
11.	a	b	c	d	•

Please do NOT write in this box.

Multiple Choice	________
12.	________
13.	________
14.	________
15.	________
Total	________

Nov 18, 2018
Sample Exam
Answer Key

Name: ________________________

Class Time: ________________________

Partial Credit

You must show your work on the partial credit problems to receive credit!

13.(12 pts.) Consider the a cylinder **closed** on both ends with surface area 54π m^2.

(a) Write down the volume $V(r)$ in terms of the radius r of the cylinder.

You may use the formulas: $\pi r^2 h$ and $2\pi rh$.

$SA = 2\pi rh + 2\pi r^2 = 54\pi$

$rh + r^2 = 27$

$h + r = \frac{27}{r}$

$h = \frac{27}{r} - r$

$V = \pi r^2 h$

$= \pi r^2 \left(\frac{27}{r} - r\right)$

$\boxed{V(r) = 27\pi r - \pi r^3}$

0 < r ≤ sqrt(27)

(b) Write down the range of the possible values of r. **Answer:** XXXXXXXXXXX

(c) Using calculus, find the largest volume that such a cylinder could enclose.

$V' = 27\pi - 3\pi r^2$

$V' = 0$ if $27\pi = 3\pi r^2$

$9 = r^2$

$r = 3$ ($r = -3$ not needed)

$V(3) = 27\pi(3) - \pi(3)^3 = 81\pi - 27\pi = 54\pi$

Endpoints:

$V(\sqrt{27}) = 27\pi(\sqrt{27}) - \pi(\sqrt{27})^3 = 0$

$\lim_{r\to 0} V(r) = \lim_{r \to 0} 27\pi r - \pi r^3 = 0 - 0 = 0$

so $\boxed{54\pi \text{ is the maximum volume}}$

Name: ______________________________

Class Time: __________________________

14.(12 pts.) Sketch the graph of a function defined on $(-2, \infty)$ in the axes below with the following properties:

(1) $f(0) = 2$ and $f(2) = 0$.

(2) $f(x)$ has a vertical asymptotes at $x = -2$.

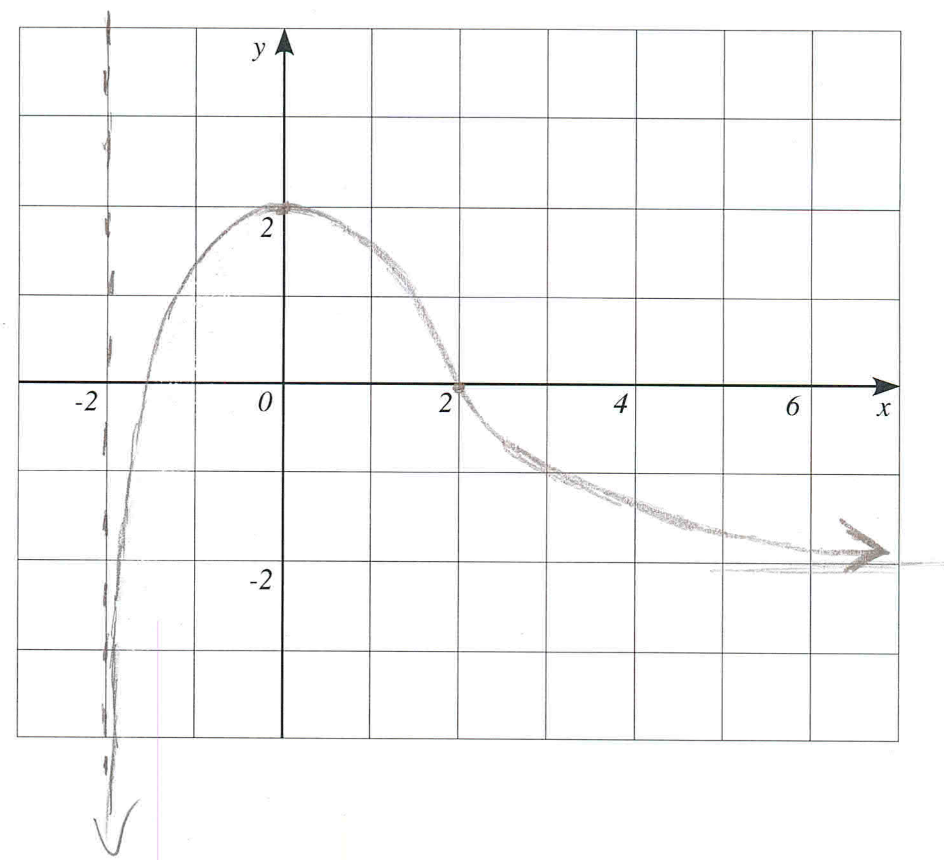
(3) $\lim_{x \to \infty} f(x) = -2$

(4) $f'(x) > 0$ for $-2 < x < 0$.

(5) $f'(x) < 0$ for $0 < x < +\infty$.

(6) $f''(x) < 0$ for $-2 < x < 2$.

(7) $f''(x) > 0$ for $2 < x < +\infty$.



Name: ____________________

Class Time: ____________________

15.(12 pts.) Using calculus, find the point on the graph of $y = 3x + 10$ closest to the origin $(0, 0)$.

Constraint: $y = 3x + 10$

to minimize the function

$$d = \sqrt{x^2 + y^2},$$

we instead minimize

$D = x^2 + y^2$, because they will achieve their mins at the same location.

$$\begin{aligned} D &= x^2 + y^2 \\ &= x^2 + (3x + 10)^2 \\ &= x^2 + 9x^2 + 60x + 100 \\ &= 10x^2 + 60x + 100 \end{aligned}$$

$D'(x) = 20x + 60$

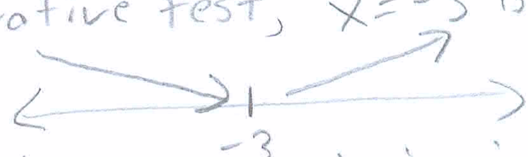
$D'(x) = 0$ if $20x = -60$

$x = -3$

if $x < -3$ then $20x < 60$, so $D'(x) < 0$

if $x > -3$ then $20x > 60$ so $D'(x) > 0$

so by the first derivative test, $x = -3$ is the minimum



so the point closest to the origin is

$x = -3$ $y = 3(-3) + 10 = 1$

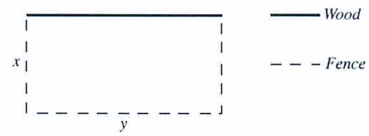
$\boxed{(-3, 1)}$

Name: ______________________

Class Time: ______________________

16.(12 pts.) A farmer wishes to build a rectangular enclosure of area 200 sq. ft. Three sides of the enclosure will be fencing at \$1/ft, while the remaining side is wood at \$3/ft. The plan of the enclosure is given in the figure below.

(a) Write down the cost $C(x)$ in terms of the width x of the enclosure.



Constraint:

$xy = 200 \qquad y = \frac{200}{x}$

Cost

$C = x + x + y + 3y = 2x + 4y = \boxed{2x + \frac{800}{x}}$

(b) Write down the range of the possible values of x. **Answer:** $0 < x < \infty$

(c) Using calculus, find the dimensions of the enclosure that minimized the cost C. You must give reason why your answers makes C minimum.

$C'(x) = 2 - \frac{800}{x^2}$

$C'(x) = 0$ if $2 = \frac{800}{x^2}$

$2x^2 = 800$

$x^2 = 400$

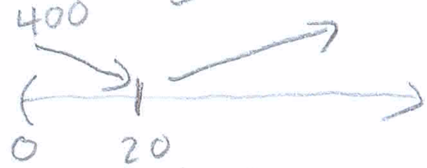
$x = 20$ (don't need $x = -20$)

if $0 < x < 20$, then $x^2 < 400$, so $\frac{800}{x^2} > \frac{800}{400} = 2$

so $C'(x) < 0$

if $x > 20$, then $x^2 > 400$, so $\frac{800}{x^2} < \frac{800}{400} = 2$

so $C'(x) > 0$



so by the first derivative test, the cost is minimized at $x = 20$.

$\boxed{x = 20,\ y = \frac{200}{20} = 10}$

Name: ______________________

Class Time: ______________________

Partial Credit

You must show your work on the partial credit problems to receive credit!

12.(12 pts.) Find the absolute maximum and absolute minimum of the function

$$f(x) = e^{-x^2+2x}$$

on the interval $-1 \leq x \leq 2$.

$f'(x) = (-2x+2)e^{-x^2+2x}$

$f'(x) = 0 \rightarrow (-2x+2)e^{-x^2+2x} = 0$

since e^{-x^2+2x} is never 0, we must have $-2x+2=0 \rightarrow x=1$

critical points : 1

ent points : -1 , 2

$f(1) = e \longrightarrow$ absolute maximum

$f(-1) = e^{-3} \longrightarrow$ absolute minimum

$f(2) = 1$

Name: ______________________

Class Time: ______________________

13.(12 pts.) **Part (A)** Sketch the graph of a function in the axes below with the following properties:

(1) $f(0) = 0$

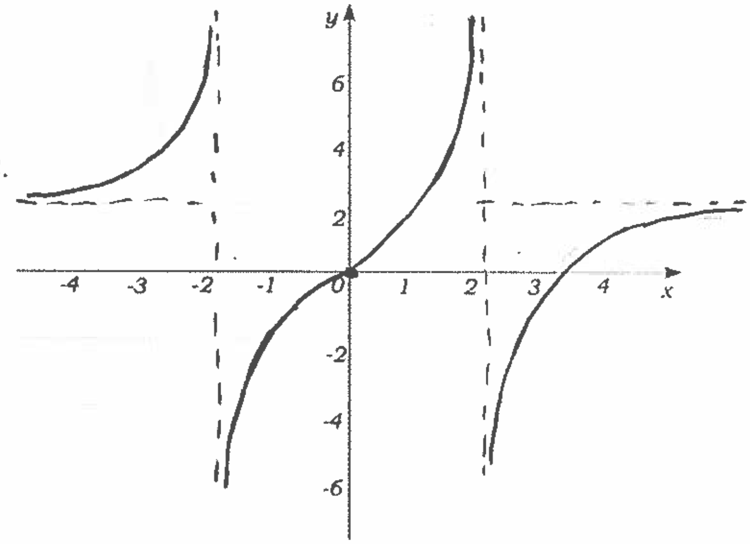
(2) $f(x)$ has vertical asymptotes at $x = -2$ and $x = 2$.

(3) $\lim_{x \to -\infty} f(x) = 2 = \lim_{x \to \infty} f(x)$

(4) $f'(x) > 0$ for all values of x except -2 and 2.

(5) $f''(x) > 0$ for $-\infty < x < -2$ and $0 < x < 2$.

(6) $f''(x) < 0$ for $-2 < x < 0$ and $2 < x < \infty$.



Part (B) The **derivative** of a function $g(x)$ is given by:

$$g'(x) = \frac{x-2}{x-1}$$

For what values of x is $g(x)$ concave up? what about concave down?

$$g''(x) = \frac{(x-1)-(x-2)}{(x-1)^2} = \frac{\cancel{x}-1-\cancel{x}+2}{(x-1)^2} = \frac{1}{(x-1)^2}$$

$g''(x) > 0$ for every $x \neq 1$

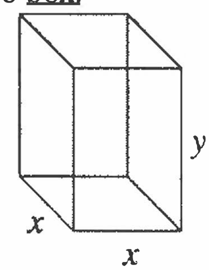
Answer: Concave up on $(-\infty, 1) \cup (1, \infty)$; Concave down on $\emptyset$

Name: ______________________________

Class Time: ______________________________

14.(12 pts.) A **closed** rectangular box with square based is such that its volume is 8 m^3. If the dimensions of the base is $x \times x$, and the height of the box is y, answer the question below.

(a) Write down the total surface area $A(x)$ in terms of the width x of the box.



$$V = x^2 y \quad , \quad V = 8 \longrightarrow x^2 y = 8 \rightarrow y = \frac{8}{x^2}$$

$$A = 2x^2 + 4xy$$

$$A = 2x^2 + 4x\left(\frac{8}{x^2}\right) = 2x^2 + \frac{32}{x}$$

(b) Write down the range of the possible values of x. **Answer:** $(0, \infty)$

(c) Using calculus, find the value of x that minimizes the area A of the box. You must give reason why your answer makes A minimum.

$$A' = 4x - \frac{32}{x^2}$$

$$A' = 0 \longrightarrow 4x - \frac{32}{x^2} = 0 \rightarrow 4x = \frac{32}{x^2} \rightarrow 4x^3 = 32$$

$$\rightarrow x^3 = 8 \longrightarrow x = 2$$

$$A' = 4x - \frac{32}{x^2} = \frac{4x^3 - 32}{x^2} = \frac{4(x^3 - 8)}{x^2}$$

x	0		2	
A'	not defined	$-$	0	$+$
A		$\searrow$	local min	$\nearrow$

$$\lim_{x \to 0^+} A(x) = \infty$$

$$\lim_{x \to \infty} A(x) = \infty$$

So $A(2)$ is the global min.

Name: ____________________

Class Time: ____________________

15.(12 pts.) Sketch the graph of a **differentiable** function defined on $(-\infty, 2)$ in the axes below with the following properties:

(1) $f(0) = 0$ and $f(-2) = 2$.

(2) $f(x)$ has a vertical asymptotes at $x = 2$.

(3) $\lim_{x \to -\infty} f(x) = 4$

(4) $f'(0) = 0$.

(5) $f'(x) < 0$ for $(-\infty, 0) \cup (0, 2)$.

(6) $f''(x) < 0$ for $(-\infty, -2) \cup (0, 2)$.

(7) $f''(x) > 0$ for $(-2, 0)$.