Math 10350 – Exam 03 Review

1. The statement: "$f(x)$ is increasing on $a < x < b$." is the same as:

1a. "$f'(x)$ is _____ on $a < x < b$."

2. The statement: "$f'(x)$ is negative on $a < x < b$." is the same as:

2a. "$f(x)$ is _____ on $a < x < b$."

3. The statement: "The graph of $f(x)$ is concave up on $a < x < b$." is the same as:

3a. "$f''(x)$ is _____ on $a < x < b$." is the same as:

3b. "$f'(x)$ is _____ on $a < x < b$."

4. The statement: "$f'(x)$ is decreasing on $a < x < b$." is the same as:

4a. "$f''(x)$ is _____ on $a < x < b$." is the same as:

4b. "The graph of $f(x)$ is _____ on $a < x < b$."

1. Let $f(x)$ is The figure below is the graph of the **derivative** $f'(x)$ of $f(x)$ for $-8 < x < 4$. Find all intervals on which **the graph of** $f(x)$ is concave up?

(i) Find all values of x in $(-8, 4)$ for which $f(x)$ is increasing.

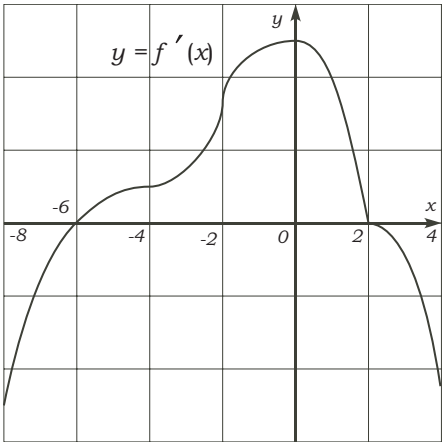
(ii) Find all values of x in $(-8, 4)$ for which $f(x)$ is decreasing.

(iii) Find the critical points of $f(x)$ in $(-8, 4)$. Are these local maximums or minimums?

(iv) Find all intervals on which **the graph of** $f(x)$ is concave up in $(-8, 4)$.

(v) Find all intervals on which **the graph of** $f(x)$ is concave up in $(-8, 4)$.

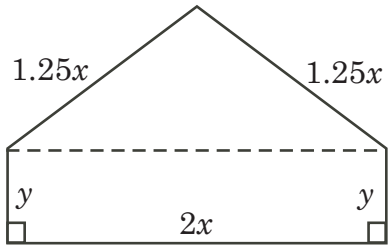
(vi) Find all values of x in $(-8, 4)$ for which $f(x)$ has an inflection point.



2. Find all vertical and horizontal asymptotes of the following functions:

(a) $f(x) = \dfrac{x^2 - 3x - 2}{x^3 - 4x}$; (b) $g(x) = \dfrac{2x^4 + 3x^2 + x + 1}{\sqrt{x^8 - 1}}$; (c) $h(x) = \dfrac{e^{3x} + e^x - 3}{3e^{3x} + 4e^x + 5}$

3. An landscaper has 30 ft of fencing and wishes to enclosed a five sided figure as shown. Find an expression for the area A enclosed in terms of x. Find the values of x and y that maximizes the area A. What is the maximum area that can be enclosed? (Ans: $x = 4$ft, $y = 6$ft, max $A = 60$ft^2)



4. A rectangular tank with an open top must have a volume of 162 cubic meters. The base costs 3 hundred dollars per square meter and the side costs 2 hundred dollars per square meters. Let C be the cost of making such a tank. (a) If the square base is $x \times x$, write down the cost function $C(x)$ in terms of the x. (b) Write down the range of the possible values of x. (c) Using calculus, find the dimensions of the tank that minimizes the cost.

5. Let $f(x) = x + \dfrac{4}{x}$

(a) The derivative of $f(x)$, $f'(x) =$ _____.

(b) Find all critical points of $f(x)$.

(c) By classifying the critical points in Part (b) using first derivative test OR OTHERWISE, fill in the blanks below:

(i) Local maximum occurs at $x =$ _____ Coordinates = _____ (Fill in NA if none).

(ii) Local minimum occurs at $x =$ _____ Coordinates = _____ (Fill in NA if none).

(iii) The function $f(x)$ is **increasing** on the interval(s): _____

(iv) The function $f(x)$ is **decreasing** on the interval(s): _____

(d) Find all values of x for which the function is (a) concave up; (b) concave down. Give the coordinates of the inflection points.

(e) Draw the graph of $y = x + \dfrac{4}{x}$ marking clearly all important features.

6. Find the absolute maximum value and absolute minimum value of the function

$$f(x) = 2x^3 - 3x^2 - 12x$$

on the interval $-2 \leq x \leq 1$, and say where they occur.

7. Find the value of the following limits:

(a) $\lim\limits_{x \to 0+} \left(1 + \dfrac{1}{x^2}\right)^x$ (b) $\lim\limits_{x \to \infty} \dfrac{e^x - 1}{\sin x}$ (c) $\lim\limits_{x \to \pi/2} (\tan x - \sec x)$

8. Find the value(s) of c satisfying the conclusion of the MVT for function $f(x) = x \ln x$ on the interval $[1, 2]$.

9. Draw a picture to illustrate MVT for the function $f(x) = \sqrt{x - 4}$ on the interval $[5, 13]$.

10. Draw a picture to illustrate Rolle's theorem for the function $f(x) = \cos 2x$ on the interval $[-\pi/4, \pi/4]$.

11. (4.6/Q20) Rice production requires both labor and capital investment in equipment and land. Suppose that if x per acre are invested in labor and y dollars per acre are invested in equipment and land, then the yield P of rice per acre is given by the formula $P = 100\sqrt{x} + 150\sqrt{y}$. If a farmer invests \$40/acre, how should he divide the \$40 between labor and capital investment in order to maximize the amount of rice produced.

12. Describe the monotonicity and concavity of the function $f(x) = 3x^5 - 20x^3$. State all critical points and classify them. Find all inflection points of $f(x)$.

Math 10350 – Exam 03 Review Answers

1. (i) $(-6, 2)$; (ii) $(-8, -6) \cup (2, 4)$; (iii) $x = -6$ local min., $x = 2$ local max; (iv) $(-8, 0)$; (v) $(0, 2) \cup (2, 4)$, (vi) Inflection point at $x = 0$ but not at $x = 2$.

2. (a) $f(x) = \dfrac{x^2 - 3x + 2}{x^3 - 4x} = \dfrac{(x-1)(x-2)}{x(x-2)(x+2)}$ so vertical asymptotes are $x = -2$, and $x = 0$. Horizontal asymptote is $y = 0$ because $\lim\limits_{x \to -\infty} f(x) = 0 = \lim\limits_{x \to \infty} f(x)$.

(b) $g(x) = \dfrac{2x^4 + 3x^3 + x + 1}{\sqrt{x^8 - 1}} = \dfrac{2x^4 + 3x^3 + x + 1}{\sqrt{(x^4+1)(x^2+1)(x-1)(x+1)}}$. Vertical asymptotes are $x = -1$ and $x = 1$. $\lim\limits_{x \to -\infty} g(x) = 2 = \lim\limits_{x \to \infty} g(x)$ so horizontal asymptote is $y = 2$.

(c) $h(x) = \dfrac{e^{3x} + e^x - 3}{3e^{3x} + 4e^x + 5}$. No vertical asymptotes. $\lim\limits_{x \to -\infty} h(x) = -3/5$ and $\lim\limits_{x \to \infty} g(x) = 1/3$ so horizontal asymptotes are $y = -3/5$ and $y = 1/3$.

3. $x = 4\text{ft}$, $y = 6\text{ft}$, max $A = 60\text{ft}^2$

4. (a) $C(x) = 3x^2 + \dfrac{1296}{x}$; (b) $0 < x < \infty$; (c) Global min at $x = 6$, $C(6) = 324$ hundreds of dollar.

5. (a) $f'(x) = 1 - \dfrac{4}{x^2}$; (b) $x = -2, 2$.

(c) (i) Local maximum occurs at $x = -2$ Coordinates $= (-2, -4)$.

(ii) Local minimum occurs at $x = 2$ Coordinates $= (2, 4)$.

(iii) The function $f(x)$ is **increasing** on the interval(s): $(-\infty, -2) \cup (2, \infty)$

(iv) The function $f(x)$ is **decreasing** on the interval(s): $(-2, 0) \cup (0, 2)$

(d) (a) concave up for $(0, \infty)$; (b) concave down for $(-\infty, 0)$. No inflection points.

(e) Yours!!!

6. End-points: $f(-2) = -4$ and $f(1) = -13$ (Absolute min). Critical point: $f(-1) = 7$ (Absolute max).

7. Find the value of the following limits:

(a) $\lim\limits_{x \to 0+} \left(1 + \dfrac{1}{x^2}\right)^x = 1$ (b) $\lim\limits_{x \to \infty} \dfrac{e^x - 1}{\sin x}$ does not exist. (c) $\lim\limits_{x \to \pi/2} (\tan x - \sec x) = 0$

8. $c = e^{2 \ln 2 - 1}$. **9.** $c = 8$ **10.** $c = 0$.

11. $P(x) = 100\sqrt{x} + 150\sqrt{40 - x}$ where $0 \le x \le 40$. End-points: $P(0) = 300\sqrt{10}$ and $P(40) = 200\sqrt{10}$.

Critical point:
$P(160/13) = 100\sqrt{160/13} + 150\sqrt{360/13} = 400\sqrt{10/13} + 900\sqrt{10/13} = 1300\sqrt{10/13} = 100\sqrt{130}$ (Global max).

12. Increasing on $(-\infty, -2) \cup (2, \infty)$. Decreasing on $(-2, 0) \cup (0, 2)$.

Concave up on $(-\sqrt{2}, 0) \cup (\sqrt{2}, \infty)$. Concave down $(-\infty, -\sqrt{2}) \cup (0, \sqrt{2})$.

Math 10350: Calculus A
Exam III Sample
November 18, 2018

Name: _____

Class Time: _____

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for one hour and 15 minutes.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 11 pages of the test.

Sign the pledge. "On my honor, I have neither given nor received unauthorized aid on this Exam":

_____

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	[a]	[b]	[c]	[d]	[e]
2.	[a]	[b]	[c]	[d]	[e]
3.	[a]	[b]	[c]	[d]	[e]
4.	[a]	[b]	[c]	[d]	[e]
5.	[a]	[b]	[c]	[d]	[e]
6.	[a]	[b]	[c]	[d]	[e]
7.	[a]	[b]	[c]	[d]	[e]
8.	[a]	[b]	[c]	[d]	[e]
9.	[a]	[b]	[c]	[d]	[e]
10.	[a]	[b]	[c]	[d]	[e]
11.	[a]	[b]	[c]	[d]	[e]
12.	[a]	[b]	[c]	[d]	[e]

Please do NOT write in this box.

Multiple Choice	_____
13.	_____
14.	_____
15.	_____
16.	_____
Total	_____

Name: _____

Class Time: _____

Multiple Choice

1.(5 pts.) Which of the following statements is **TRUE** about the graph of

$$f(x) = 3x^{2/3} + 2x - 1?$$

(only one of them is true)

(a) The graph of $f(x)$ is concave upward on the interval $(-\infty, -1) \cup (0, \infty)$ only.

(b) The graph of $f(x)$ is concave downward on the interval $(-1, 0)$ only.

(c) The graph of $f(x)$ is concave upward on the interval $(-1, 0)$ only.

(d) The graph of $f(x)$ is concave upward on the intervals $(-\infty, 0)$ and $(0, \infty)$.

(e) The graph of $f(x)$ is concave downward on the intervals $(-\infty, 0)$ and $(0, \infty)$.

2.(5 pts.) Find the value(s) of x at which the function $f(x) = 3x^5 - 20x^4$ has an inflection point.

(a) $x = 0$ and $x = 4$ only.

(b) $x = 4$ only.

(c) $x = 16/3$ only.

(d) $x = 0$ and $x = 16/3$ only.

(e) $x = 0$ only.

Name: _____

Class Time: _____

3.(5 pts.) Find all horizontal asymptotes of the graph of $y = \dfrac{\sqrt{4x^2+2x+1}}{x+2}$.

(a) $y = 4$ and $y = -4$ only.

(b) $y = 2$ and $y = -2$ only.

(c) $y = -4$ only.

(d) $y = 4$ only.

(e) $y = 2$ only.

4.(5 pts.) Find the limit $\lim\limits_{x\to\infty} \dfrac{2\cos x + x}{\cos x + 5x + 10}$.

(a) 2

(b) 1/10

(c) Does not exists

(d) 1/5

(e) $+\infty$

Name: _____

Class Time: _____

5.(5 pts.) Let $f(x) = 1 - \dfrac{1}{x}$. What value $x = c$ on the interval $1 \leq x \leq 4$ will satisfy the Mean Value Theorem?

(a) -2 and 2 (b) $\sqrt{3}/2$ (c) No such value.

(d) $1/2$ (e) 2

6.(5 pts.) Find the limit $\lim\limits_{x \to \infty} \dfrac{\sin(2x^7 + 7x^3 + 3)}{\sqrt{x+1}}$.

(a) Does not exists (b) 7 (c) 0

(d) $+\infty$ (e) 2

Name: _____

Class Time: _____

7.(5 pts.) Find the absolute maximum and absolute minimum of the function

$$f(x) = 3x^{2/3} + 2x - 1$$

for x in $[-1, 8]$.

(a) Absolute maximum $= 3$ and Absolute minimum $= 0$

(b) Absolute maximum $= 27$ and Absolute minimum $= 0$

(c) Absolute maximum $= 27$ and Absolute minimum $= -1$

(d) Absolute maximum $= 0$ and Absolute minimum $= -1$

(e) Absolute maximum $= 3$ and Absolute minimum $= 2$

8.(5 pts.) If $s(t)$ represents weekly sales of a product, $s(10) = 20$, $s'(10) = 0$, and $s''(t) > 0$ for all t, which of the following statements is possibly **TRUE**? (only one of them is true)

(a) The rate of sales were decreasing after the 10th week

(b) The sales were decreasing after the 10th week

(c) The rate of sales were increasing before the 10th week

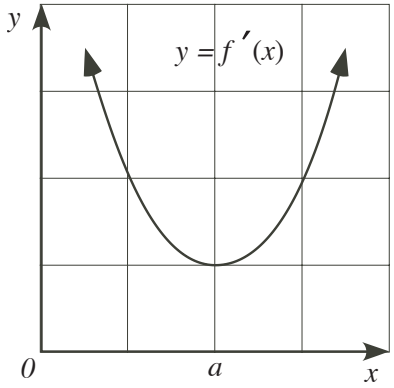
(d) The sales reached a maximum at 10th week

(e) The sales bottomed out (reached a minimum) at 10th week

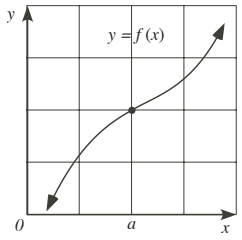
Name: _____

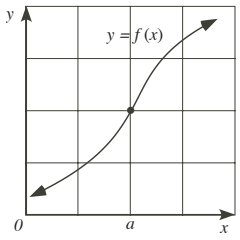
Class Time: _____

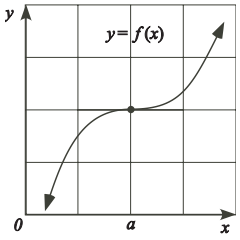
9.(5 pts.) The graph of the **derivative** $f'(x)$ of $f(x)$ is given below.

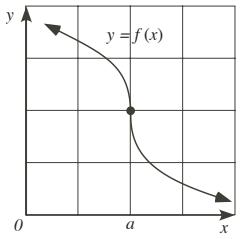


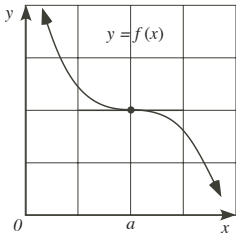
Which of the following best describe the graph of $y = f(x)$?

(a) 

(b) 

(c) 

(d) 

(e) 

10.(5 pts.) Find the equations of all **horizontal** asymptotes of the function

$$y = \frac{5e^{6x} - 2e^{3x} + 4}{2e^{6x} + 3e^{3x} - 2}.$$

(a) $y = -2$ and $y = \dfrac{5}{2}$

(b) $y = -\dfrac{1}{2}$ and $y = \dfrac{2}{5}$

(c) $y = -\dfrac{2}{3}$ and $y = \dfrac{5}{2}$

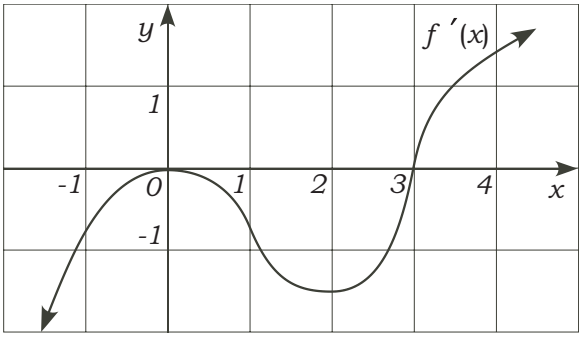
(d) $x = -2$ and $x = \dfrac{5}{2}$

(e) $x = -\dfrac{2}{3}$ and $x = \dfrac{5}{2}$

Name: _____

Class Time: _____

11.(5 pts.) The graph of the **derivative** $f'(x)$ of $f(x)$ is given below.



Derivative of $f(x)$

Which of the following best describe the graph of $y = f(x)$?

(a) The graph of $f(x)$ is concave downward on the intervals $(-\infty, 1)$ and $(3, \infty)$.

(b) The graph of $f(x)$ is concave upward on the intervals $(0, 2)$ ONLY.

(c) The graph of $f(x)$ is concave upward on the intervals $(-\infty, 0)$ and $(2, \infty)$.

(d) The graph of $f(x)$ is concave downward on the intervals $(-\infty, 0)$ and $(2, \infty)$.

(e) The graph of $f(x)$ is concave upward on the intervals $(1, 3)$ ONLY.

12.(5 pts.) Find all point(s) of inflection for $f(x)$, whose **second order derivative** is given by $f''(x) = (x-1)(x-3)^2(x-5)^3$.

(a) $x = 5$ only

(b) $x = 3$ only

(c) $x = 1$ only

(d) $x = 1$, $x = 3$ and $x = 5$

(e) $x = 1$ and $x = 5$ only

Name: _____

Class Time: _____

Partial Credit

You must show your work on the partial credit problems to receive credit!

13.(12 pts.) Consider the a cylinder **closed** on both ends with surface area 54π m^2.

(a) Write down the volume $V(r)$ in terms of the radius r of the cylinder.

You may use the formulas: $\pi r^2 h$ and $2\pi r h$.

(b) Write down the range of the possible values of r. **Answer:** _____

(c) Using calculus, find the largest volume that such a cylinder could enclose.

Name: _____

Class Time: _____

14.(12 pts.) Sketch the graph of a function defined on $(-2, \infty)$ in the axes below with the following properties:

(1) $f(0) = 2$ and $f(2) = 0$.

(2) $f(x)$ has a vertical asymptotes at $x = -2$.

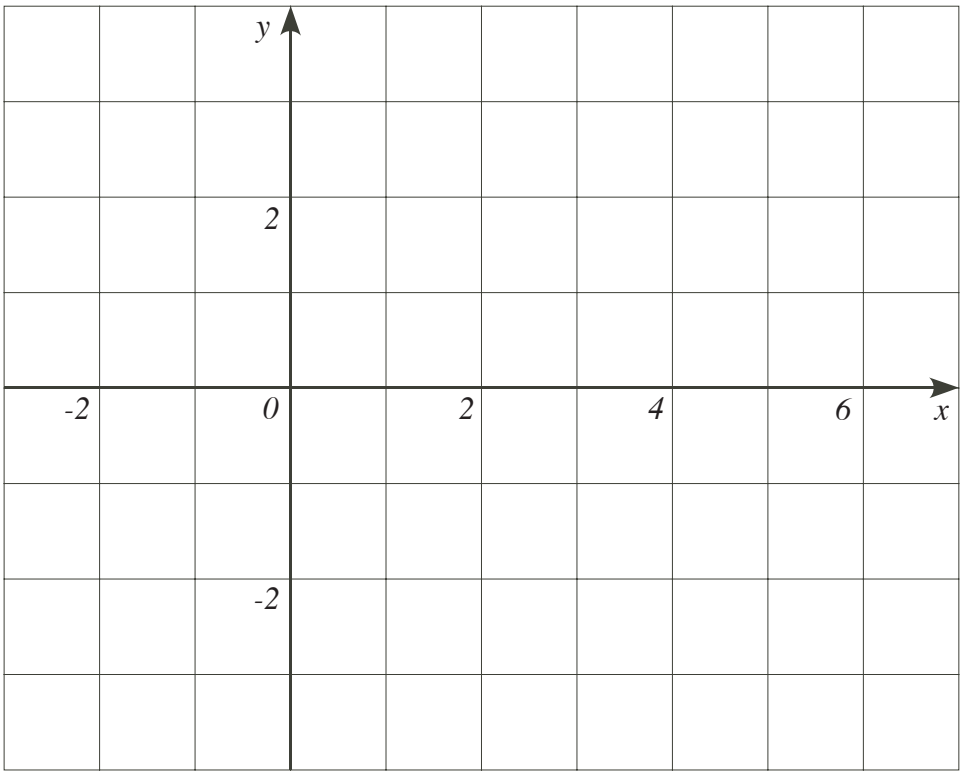
(3) $\lim_{x \to \infty} f(x) = -2$

(4) $f'(x) > 0$ for $-2 < x < 0$.

(5) $f'(x) < 0$ for $0 < x < +\infty$.

(6) $f''(x) < 0$ for $-2 < x < 2$.

(7) $f''(x) > 0$ for $2 < x < +\infty$.



Name: _____

Class Time: _____

15.(12 pts.) Using calculus, find the point on the graph of $y = 3x + 10$ closest to the origin $(0, 0)$.

Name: _____

Class Time: _____

16.(12 pts.) A farmer wishes to build a rectangular enclosure of area 200 sq. ft. Three sides of the enclosure will be fencing at \$1/ft, while the remaining side is wood at \$3/ft. The plan of the enclosure is given in the figure below.

(a) Write down the cost $C(x)$ in terms of the width x of the enclosure.

(b) Write down the range of the possible values of x. **Answer:** _____

(c) Using calculus, find the dimensions of the enclosure that minimized the cost C. You must give reason why your answers makes C minimum.

Math 10350: Calculus A
Exam III Sample
November 18, 2018

Name: _____

Class Time: ANSWERS

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for one hour and 15 minutes.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 11 pages of the test.

Sign the pledge. "On my honor, I have neither given nor received unauthorized aid on this Exam":

_____

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	[a]	[b]	[c]	[d]	[•]
2.	[a]	[•]	[c]	[d]	[e]
3.	[a]	[•]	[c]	[d]	[e]
4.	[a]	[b]	[c]	[•]	[e]
5.	[a]	[b]	[c]	[d]	[•]
6.	[a]	[b]	[•]	[d]	[e]
7.	[a]	[b]	[•]	[d]	[e]
8.	[a]	[b]	[c]	[d]	[•]
9.	[•]	[b]	[c]	[d]	[e]
10.	[•]	[b]	[c]	[d]	[e]
11.	[a]	[b]	[•]	[d]	[e]
12.	[a]	[b]	[c]	[d]	[•]

Please do NOT write in this box.

Multiple Choice _____

13. _____

14. _____

15. _____

16. _____

Total _____

Math 10350: Calculus A
Exam III
November 17, 2019

Name: _____

Class Time: _____

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4.	[a]	[b]	[c]	[d]	[e]
5.	[a]	[b]	[c]	[d]	[e]
6.	[a]	[b]	[c]	[d]	[e]
7.	[a]	[b]	[c]	[d]	[e]
8.	[a]	[b]	[c]	[d]	[e]
9.	[a]	[b]	[c]	[d]	[e]
10.	[a]	[b]	[c]	[d]	[e]
11.	[a]	[b]	[c]	[d]	[e]

Please do NOT write in this box.

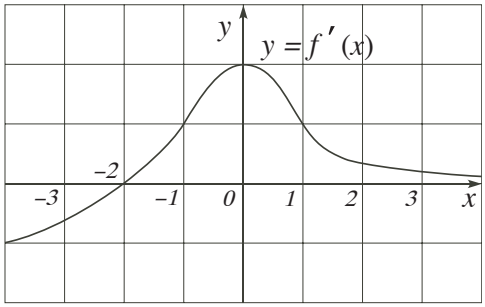
Multiple Choice	_____
12.	_____
13.	_____
14.	_____
15.	_____
Total	_____

Name: _____

Class Time: _____

Multiple Choice

1.(5 pts.) The graph of the **derivative** $f'(x)$ of $f(x)$ for $-4 < x < 4$ is given below. Find the values of x in $(-4, 4)$ for which $f(x)$ is **concave up**.



Graph of the Derivative

(a) $(-4, -1) \cup (1, 4)$

(b) $(0, 4)$

(c) $(-1, 1)$

(d) $(-2, 4)$

(e) $(-4, 0)$

2.(5 pts.) For the same function $f(x)$ above, which of the following statements is **TRUE**?

(a) $f(x)$ has a local minimum at $x = -2$.

(b) $f(x)$ has a local minimum at $x = 0$.

(c) $f(x)$ has a local maximum at $x = -2$.

(d) $f(x)$ has a local maximum at $x = 1$.

(e) $f(x)$ has a local maximum at $x = 0$.

Name: _____

Class Time: _____

3.(5 pts.) Find all horizontal asymptote(s) for $f(x) = \dfrac{2x^3 + x^2 + 1}{\sqrt{3x^6 + 7x^3 - 11x + 2}}$

(a) $y = \dfrac{2}{\sqrt{3}}$ only

(b) $y = 0$ only

(c) $y = -\dfrac{2}{\sqrt{3}}$ only

(d) $y = \dfrac{2}{\sqrt{3}}$ and $y = -\dfrac{2}{\sqrt{3}}$

(e) There is no horizontal asymptote

4.(5 pts.) Find the value $\lim\limits_{x \to \infty} x^{1/x}$.

Hint: You will need to use log and exponential functions.

(a) Does not exist.

(b) 1

(c) 0

(d) e

(e) $+\infty$

Name: _____

Class Time: _____

5.(5 pts.) Find all **vertical** asymptotes of the function $y = \dfrac{x-2}{x^2+x-6}$

(a) $x = 2$

(b) $x = 3$

(c) $x = -3$

(d) $x = -3$ and $x = 2$

(e) $x = -2$ and $x = 3$

6.(5 pts.) A piece of wire 30 meter long is cut into two pieces. One piece is bent into a square and the other is bent into an equilateral triangle. If x is the length of the wire bent into a square, which of the following functions below represents the total area enclosed by both the square and equilateral triangle

(a) $\dfrac{x^2}{4} + \dfrac{(30-x)^2\sqrt{3}}{9}$

(b) $\dfrac{x^2}{16} + \dfrac{(30-x)^2}{27}$

(c) $\dfrac{x^2}{16} + \dfrac{(30-x)^2\sqrt{3}}{36}$

(d) $\dfrac{x^2}{16} + \dfrac{(30-x)^2}{54}$

(e) $\dfrac{x^2}{16} + \dfrac{(30-x)^2\sqrt{3}}{9}$

Name: _____

Class Time: _____

7.(5 pts.) Suppose $f(x)$ is a function defined for all values of x such that its **second derivative** is

$$f''(x) = x^3 - 4x^2.$$

Find all values of x for which $f(x)$ has an inflection point.

(a) $x = 0$ and $x = 4$

(b) No such value exist.

(c) $x = 0$ and $x = 8/3$.

(d) $x = 0$ only.

(e) $x = 4$ only.

8.(5 pts.) If the derivative of $g(x)$ is given by $g'(x) = \dfrac{x-2}{x+2}$, find the values of x for which $g(x)$ is **increasing**.

(a) $(-2, 2)$ only.

(b) $(-\infty, -2) \cup (2, \infty)$ only.

(c) $(-\infty, -2)$ only.

(d) For all real values except $x = 2$.

(e) $(2, \infty)$ only.

Name: _____

Class Time: _____

9.(5 pts.) Find the value of $\lim_{x \to -\infty} \dfrac{2x}{\sqrt{x^2 + 2x + 2}}$.

(a) 2

(b) -2

(c) -1

(d) 0

(e) 1

10.(5 pts.) Let $f(x) = \dfrac{1}{x}$. Find all values $x = c$ in the interval $2 \leq x \leq 4$ that satisfy the Mean Value Theorem.

(a) -2 and 2

(b) 2

(c) 3

(d) $\sqrt{8}$

(e) $-\sqrt{8}$ and $\sqrt{8}$

Name: _____

Class Time: _____

11.(5 pts.) Suppose $x = 1$ is a critical point of $f(x)$ such that $f'(1) = 0$, and

$$f''(x) = 3x^2 - x.$$

Using second derivative test, which of one of the following statements could you conclude?

(a) $f(x)$ has a **global maximum** at $x = 1$.

(b) $f(x)$ has a **global minimum** at $x = 1$.

(c) $f(x)$ is **increasing** at $x = 1$.

(d) $f(x)$ has a **local maximum** at $x = 1$.

(e) $f(x)$ has a **local minimum** at $x = 1$.

Name: _____

Class Time: _____

Partial Credit

You must show your work on the partial credit problems to receive credit!

12.(12 pts.) Find the absolute maximum and absolute minimum of the function

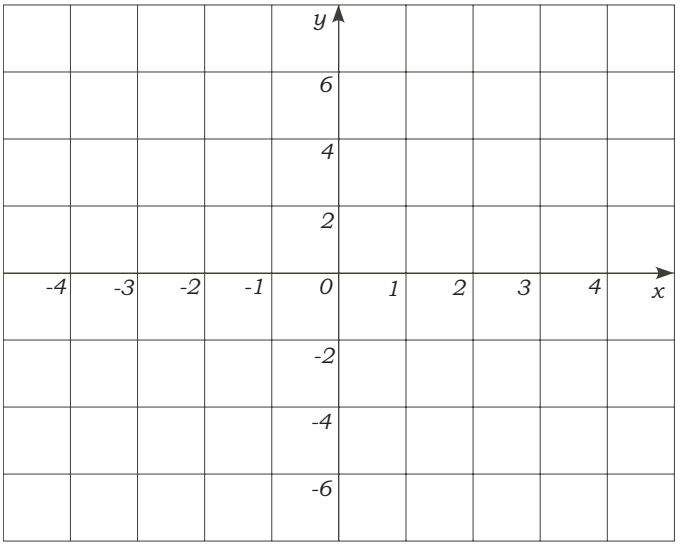
$$f(x) = e^{-x^2+2x}$$

on the interval $-1 \leq x \leq 2$.

Name: _____

Class Time: _____

13.(12 pts.) **Part (A)** Sketch the graph of a function in the axes below with the following properties:



(1) $f(0) = 0$

(2) $f(x)$ has vertical asymptotes at $x = -2$ and $x = 2$.

(3) $\lim_{x\to-\infty} f(x) = 2 = \lim_{x\to\infty} f(x)$

(4) $f'(x) > 0$ for all values of x except -2 and 2.

(5) $f''(x) > 0$ for $-\infty < x < -2$ and $0 < x < 2$.

(6) $f''(x) < 0$ for $-2 < x < 0$ and $2 < x < \infty$.

Part (B) The **derivative** of a function $g(x)$ is given by:

$$g'(x) = \frac{x-2}{x-1}$$

For what values of x is $g(x)$ concave up? what about concave down?

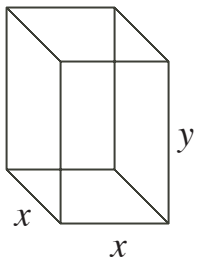
Answer: Concave up on _____; Concave down on _____

Name: _____

Class Time: _____

14.(12 pts.) A **closed** rectangular box with square based is such that its volume is 8 m^3. If the dimensions of the base is $x \times x$, and the height of the box is y, answer the question below.

(a) Write down the total surface area $A(x)$ in terms of the width x of the box.



(b) Write down the range of the possible values of x. **Answer:** _____

(c) Using calculus, find the value of x that minimizes the area A of the box. You must give reason why your answer makes A minimum.

Name: _____

Class Time: _____

15.(12 pts.) Sketch the graph of a **differentiable** function defined on $(-\infty, 2)$ in the axes below with the following properties:

(1) $f(0) = 0$ and $f(-2) = 2$.

(2) $f(x)$ has a vertical asymptotes at $x = 2$.

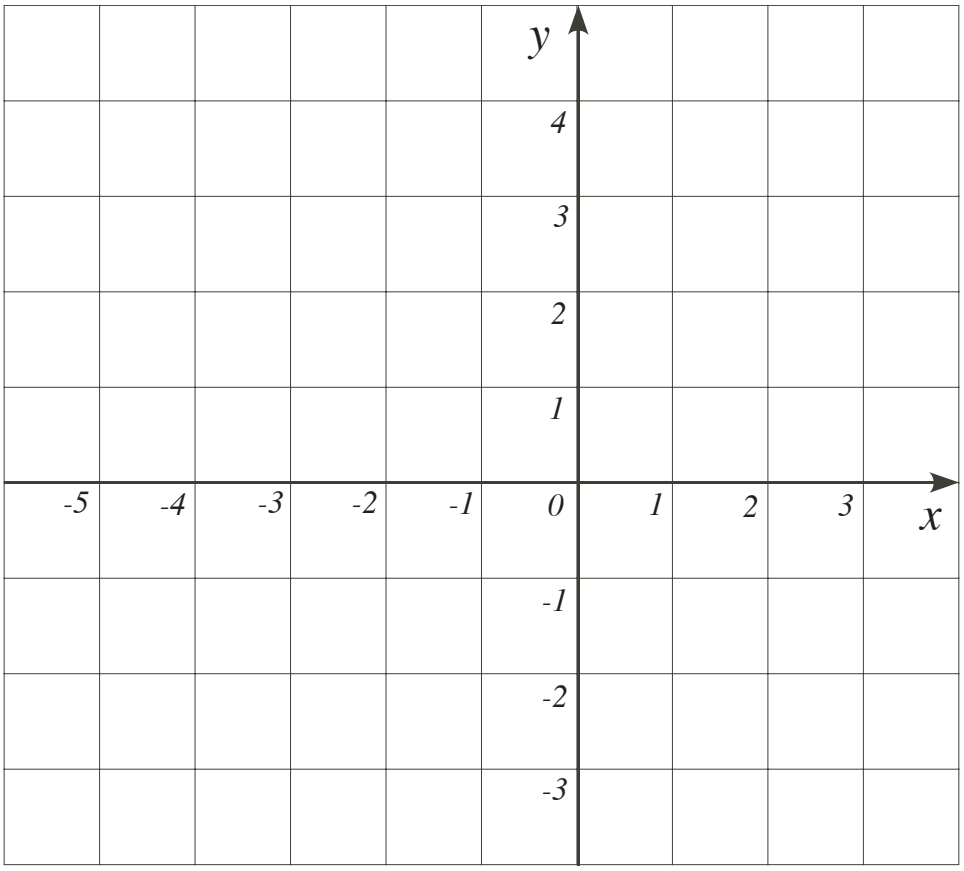
(3) $\lim_{x \to -\infty} f(x) = 4$

(4) $f'(0) = 0$.

(5) $f'(x) < 0$ for $(-\infty, 0) \cup (0, 2)$.

(6) $f''(x) < 0$ for $(-\infty, -2) \cup (0, 2)$.

(7) $f''(x) > 0$ for $(-2, 0)$.



Math 10350: Calculus A
Exam III
November 17, 2019

Name: _____

Class Time: ANSWERS

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for one hour and 15 minutes.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 11 pages of the test.

Sign the pledge. "On my honor, I have neither given nor received unauthorized aid on this Exam":

_____

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	[a]	[b]	[c]	[d]	[•]
2.	[•]	[b]	[c]	[d]	[e]
3.	[a]	[b]	[c]	[•]	[e]
4.	[a]	[•]	[c]	[d]	[e]
5.	[a]	[b]	[•]	[d]	[e]
6.	[a]	[b]	[•]	[d]	[e]
7.	[a]	[b]	[c]	[d]	[•]
8.	[a]	[•]	[c]	[d]	[e]
9.	[a]	[•]	[c]	[d]	[e]
10.	[a]	[b]	[c]	[•]	[e]
11.	[a]	[b]	[c]	[d]	[•]

Please do NOT write in this box.

Multiple Choice _____

12. _____

13. _____

14. _____

15. _____

Total _____

Exam 03 Review Solutions

1a. positive **2a.** decreasing **3a.** positive **3b.** increasing **4a.** negative **4b.** concave down

1. The graph of $f(x)$ is concave up if $f'(x)$ is increasing (equivalently where $f''(x) > 0$). So, $f(x)$ concave up: $(-8, 0)$.

(i) $f(x)$ increases if $f'(x) > 0$: $(-6, 2)$

(ii) $f(x)$ decreases if $f'(x) < 0$: $(-8, -6), (2, 4)$

(iii) CP of $f(x)$ if $f'(x) = 0$ or $f'(x)$ DNE: $x = -6$ which is a Minimum and $x = 2$ which is a Maximum by First Derivative Test.

(iv) Done in 1.

(v) We are meant to find where $f(x)$ it is concave **down**: $(0, 2), (2, 4)$

(vi) IP of $f(x)$ if $f'(x) = 0$ and concavity changes: $x = 0$

2.

(a) $f(x) = \dfrac{x^2 - 3x - 2}{x^3 - 4x} = \dfrac{(x-2)(x-1)}{x(x-2)(x+2)} = \dfrac{x-1}{x(x+2)}$

VA

$x(x+2) = 0 \Rightarrow x = 0$ or $x = -2$ AND $\lim\limits_{x\to 0^-} \dfrac{x-1}{x(x+2)} = \infty$ $\lim\limits_{x\to 0^+} \dfrac{x-1}{x(x+2)} = -\infty$ $\lim\limits_{x\to -2^-} \dfrac{x-1}{x(x+2)} = -\infty$ $\lim\limits_{x\to -2^+} \dfrac{x-1}{x(x+2)} = \infty$

The VA of $f(x)$ are $x = 0$ and $x = -2$.

HA

$$\lim_{x\to\infty} \frac{x^2 - 3x - 2}{x^3 - 4x} = \lim_{x\to\infty} \frac{x^2}{x^3} = \lim_{x\to\infty} \frac{1}{x} + 0$$
$$\lim_{x\to-\infty} \frac{x^2 - 3x - 2}{x^3 - 4x} = \lim_{x\to\infty} \frac{x^2}{x^3} = \lim_{x\to-\infty} \frac{1}{x} = 0$$

The HA of $f(x)$ is $y = 0$.

(b) VA

$x^8 - 1 = 0 \Rightarrow x = -1$ or $x = 1$ AND

$\lim_{x \to 1+} g(x) = \infty$

$\lim_{x \to -1^-} g(x) = \infty$

The VA of $f(x)$ are $x = -1$ and $x = 1$.

HA

$$\lim_{x \to \infty} \frac{2x^4 + 3x^2 + x + 1}{\sqrt{x^8 - 1}} = \lim_{x \to \infty} \frac{2x^4}{\sqrt{x^8}} = \lim_{x \to \infty} \frac{2x^4}{x^4} = 2$$

$$\lim_{x \to \infty} \frac{2x^4 + 3x^2 + x + 1}{\sqrt{x^8 - 1}} = \lim_{x \to \infty} \frac{2x^4}{\sqrt{x^8}} = \lim_{x \to \infty} \frac{2x^4}{(-x)^4} = 2$$

The HA of $f(x)$ is $y = 2$.

(c) VA

$3e^{3x} + 4e^x + 5$ is never 0. Therefore, there are no VA.

HA

$$\lim_{x \to \infty} \frac{e^{3x} + e^x - 3}{3e^{3x} + 4e^x + 5} = \lim_{x \to \infty} \frac{e^{3x} + e^x - 3}{3e^{3x} + 4e^x + 5} \cdot \frac{e^{-3x}}{e^{-3x}} = \lim_{x \to \infty} \frac{1 + e^{-2x} - 3e^{-3x}}{3 + 4e^{-2x} + 5e^{-3x}} = \frac{1}{3}$$

$$\lim_{x \to -\infty} \frac{e^{3x} + e^x - 3}{3e^{3x} + 4e^x + 5} = \lim_{x \to \infty} \frac{e^{-3x} + e^{-x} - 3}{3e^{-3x} + 4e^{-x} + 5} = \frac{-3}{5}$$

The HA of $f(x)$ are $y = \frac{-3}{5}$ and $y = \frac{1}{3}$

3. We follow the usual steps:

I. Control equation: $30 = 2\left(\frac{5}{4}\right)x + 2y + 2x \Rightarrow y = 15 - \frac{9}{4}x$

II. Equation to optimize (in terms of x): $A(x) = A_{\square} + A_{\triangle} = 2xy + \frac{1}{2}(2x)(h)$

Since h can be found using Pythagora's Theorem: $h^2 + x^2 = \left(\frac{5}{4}x\right)^2 \Rightarrow h = \frac{3}{4}x$ we have:

$A(x) = 2x\left(15 - \frac{9}{4}x\right) + x\left(\frac{3}{4}x\right) = 30x - \frac{15}{4}x^2$

III. Domain: $0 \leq x \leq \frac{20}{3}$

IV. Find CP: $A'(x) = 0 \Rightarrow 30 - \frac{15}{2}x = 0 \Rightarrow x = 4$

V. Check: $A(0) = 0$, $A(4) = 60$, $A\left(\frac{20}{3}\right) = \frac{100}{3}$

VI. Answer: $x = 4ft$, $y = 6ft$, $A = 60ft^2$

4. We follow the usual steps:

I. Control equation: $162 = x^2y \Rightarrow y = \frac{162}{x^2}$

II. Equation to optimize (in terms of x): $C(x) = 3x^2 + 2(4xy) = 3x^2 + \frac{1296}{x}$

III. Domain: $0 < x < \infty$

IV. Find CP: $C'(x) = 0 \Rightarrow 6x - \frac{1296}{x^2} = 0 \Rightarrow \frac{6x^3 - 1296}{x^2} = 0 \Rightarrow x = 6$

V. Check: $\lim_{x \to 0^+} 3x^2 + \frac{1296}{x} = \infty$, $C(6) = 324$, $\lim_{x \to \infty} 3x^2 + \frac{1296}{x} = \infty$

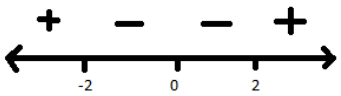
VI. Answer: $x = 6m$, $y = \frac{9}{2}m$, $Cost = \$324$

5.

(a) $f'(x) = 1 - \frac{4}{x^2} = \frac{x^2 - 4}{x^2}$

(b) CP: $f'(x) = 0 \Rightarrow x^2 - 4 = 0 \Rightarrow x = \pm 2$

$f'(x)$ DNE $\Rightarrow x^2 = 0 \Rightarrow x = 0$



(c) By First Derivative Test we have:

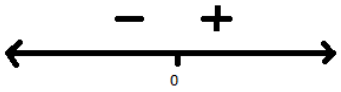
(i) $x = -2$, Coordinates: $(-2, -4)$

(ii) $x = 2$, Coordinates: $(2, 4)$

(iii) $f(x)$ increasing: $(\infty, -2), (2, \infty)$

(iv) $f(x)$ decreasing: $(-2, 0), (0, 2)$

(d) $f''(x) = \frac{8}{x^3}$

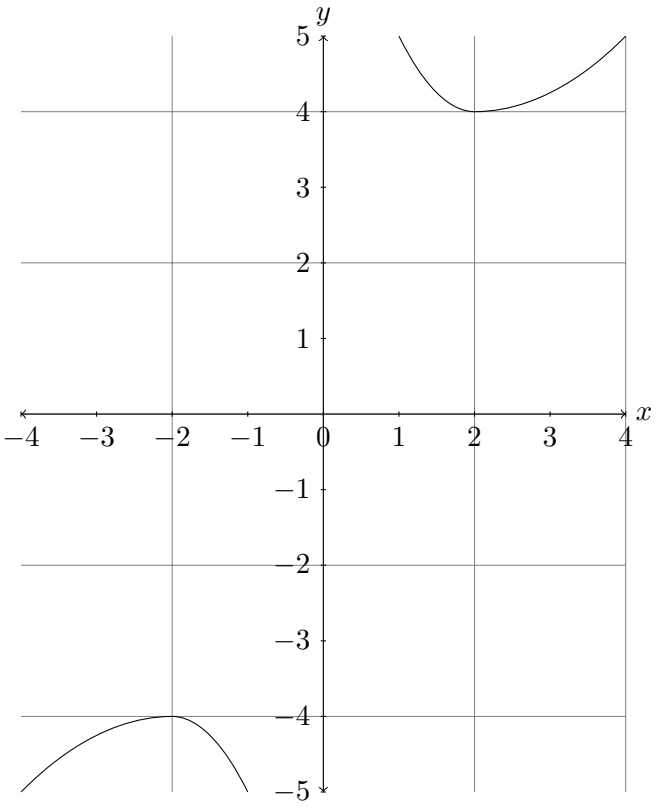


$f(x)$ concave up: $(0, \infty)$

$f(x)$ concave down: $(-\infty, 0)$

IP: None since $f(x)$ is not defined at 0.

(e) Graph of $f(x)$



6. Since the interval is closed and bounded, the absolute max. or min. cann occur ar CP or endpoints. First we find CP: $f'(x) = 6x^2 - 6x - 12 = 6(x-2)(x+1)$

If $f'(x) = 0 \Rightarrow 6(x-2)(x+1) = 0 \Rightarrow x = 2$ or $x = -1$. Also since $f'(x)$ is a polynomial then there is no value of x for which $f'(x)$ DNE.

The CP is only $x = -1$ since 2 is not in the interval we are considering.

Finally, $f(-2) = -4$, $f(-1) = 7$, $f(1) = -13$. So the Abs. Max. is 7 and Abs. Min. is -13.

7.

(a) We use the fact that $a = e^{\ln(a)}$

$$\lim_{x\to 0^+}\left(1+\frac{1}{x^2}\right)^x = \lim_{x\to 0^+} e^{\ln\left(1+\frac{1}{x^2}\right)^x} = e^{\lim\limits_{x\to 0^+} x\ln\left(1+\frac{1}{x^2}\right)}$$

But,

$$\lim_{x\to 0^+} x\ln\left(1+\frac{1}{x^2}\right) = \lim_{x\to 0^+}\frac{\ln\left(1+\frac{1}{x^2}\right)}{\frac{1}{x}} =_{LH} \lim_{x\to 0^+}\frac{\frac{\frac{-2}{x^3}}{1+\frac{1}{x^2}}}{\frac{-1}{x^2}} = \lim_{x\to 0^+}\frac{-2}{x^3}\cdot\frac{x^2}{x^2+1}\cdot\frac{-x^2}{1} = 0$$

So, $\lim\limits_{x\to 0^+}\left(1+\frac{1}{x^2}\right)^x = e^{\lim\limits_{x\to 0^+} x\ln\left(1+\frac{1}{x^2}\right)} = e^0 = 1$

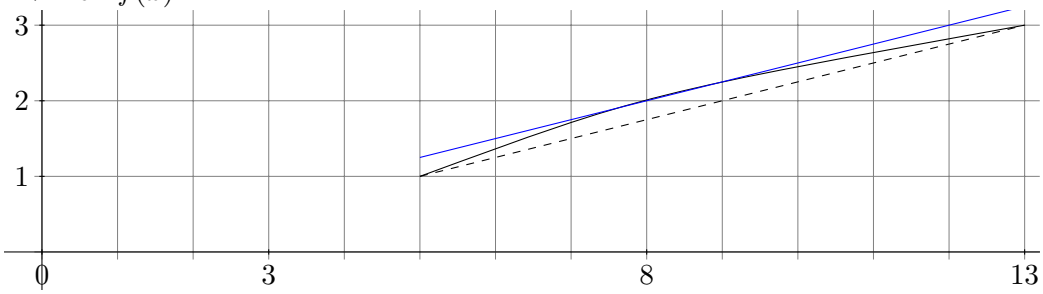
(b) $\lim\limits_{x\to\infty}\dfrac{e^x-1}{\sin(x)}$ DNE. As $x\to\infty$, $e^x\to\infty$, but $\sin(x)$ fluctuates between -1 and 1, causing $\dfrac{e^x-1}{\sin(x)}$ to fluctuate between $-\infty$ and ∞.

(c) $\lim\limits_{x\to\frac{\pi}{2}}\tan(x)-\sec(x) = \lim\limits_{x\to\frac{\pi}{2}}\dfrac{\sin(x)}{\cos(x)} - \dfrac{1}{\cos(x)} = \lim\limits_{x\to\frac{\pi}{2}}\dfrac{\sin(x)-1}{\cos(x)} =_{LH} \lim\limits_{x\to\frac{\pi}{2}}\dfrac{\cos(x)}{-\sin(x)} = 0$

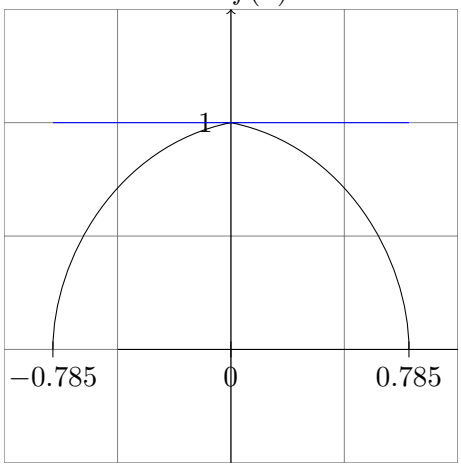
8. Since $f(x)$ is differentiable in $[1, 2]$ we can apply the MVT.

$$\begin{aligned} f'(c) = \frac{f(2) - f(1)}{2 - 1} &\Rightarrow 1 + \ln(c) = \frac{2\ln(2) - \ln(1)}{1} \\ &\Rightarrow \ln(c) = \ln(2^2) - 1 \\ &\Rightarrow c = e^{\ln(4)-1} \\ &\Rightarrow c = 4e^{-1} \end{aligned}$$

9. MVT on $f(x)$:



10. Rolle's Thm. on $f(x)$



11. We follow the usual steps:

I. Control equation: $40 = x + y$

II. Equation to optimize (in terms of x): $P(x) = 100\sqrt{x} + 150\sqrt{40 - x}$

III. Domain: $0 \leq x \leq 40$

IV. Find CP:

$$\begin{aligned} P'(x) = 0 \Rightarrow \frac{50}{\sqrt{x} - \frac{75}{\sqrt{40-x}}} &= 0 \\ \Rightarrow \frac{50\sqrt{40-x} - 75\sqrt{x}}{\sqrt{x(40-x)}} &= 0 \\ \Rightarrow 50\sqrt{40-x} - 75\sqrt{x} &= 0 \\ \Rightarrow 2500(40-x) &= 5625x \\ \Rightarrow x &= \frac{160}{13} \end{aligned}$$

V. Check: $P(0) = 300\sqrt{10}$, $P\left(\frac{160}{13}\right) = 100\sqrt{130}$, $P(40) = 200\sqrt{10}$

VI. Answer: $x = \$12.31$ per acre, $y = \$27.69$ per acre

12.

I. Monotonicity

$f'(x) = 0 \Rightarrow 15x^2(x-2)(x+2) = 0$

$f(x)$ Inc.: $(-\infty, -2)$, $(2, \infty)$

$f(x)$ Dec.: $(-2, 0)$, $(0, 2)$

II. Concavity

$f''(x) = 0 \Rightarrow 60x(x - \sqrt{2})(x + \sqrt{2}) = 0$

$f(x)$ C. Up: $(-\sqrt{2}, 0)$, $(\sqrt{2}, \infty)$

$f(x)$ C. Down: $(-\infty, \sqrt{2})$, $(0, \sqrt{2})$

Solutions to Partial Credit Questions

Exam 03 Nov. 18, 2018

13

(a) Since the cylinder is closed, the surface area of the lids must be taken into account.

$54\pi = 2\pi rh + 2\pi r^2 \Rightarrow h = \frac{27}{r} - r$

Now that we have an expression of h in terms of r, we can express the volume only in terms of r.

$V(r) = \pi r^2 h \Rightarrow V(r) = \pi r^2 \left(\frac{27}{r} - r\right) \Rightarrow \boxed{V(r) = 27\pi r - \pi r^3}$

(b) Domain: $0 < r < \sqrt{27}$

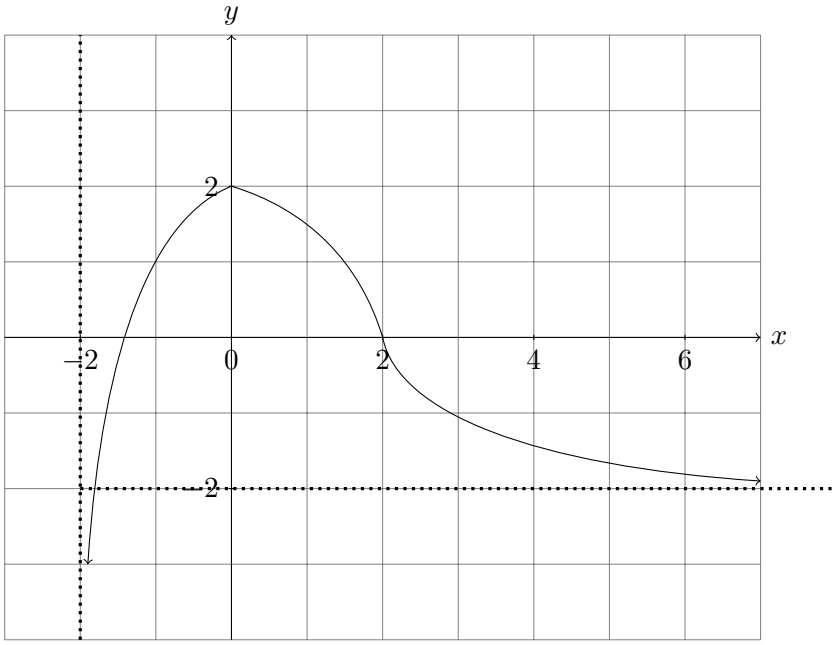
(c) Find first the CP: $V'(r) = 0 \Rightarrow 27\pi - 3\pi r^2 = 0 \Rightarrow r = 3$

Then compare CP with Boundary Values:

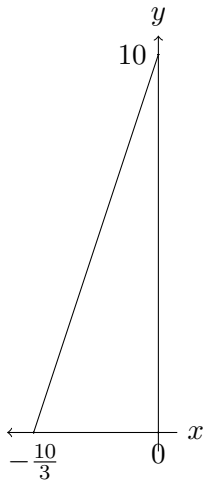
$\lim_{x \to 0^+} 27\pi r - \pi r^3 = 0$, $V(3) = 54$, $\lim_{x \to \sqrt{27}^-} 27\pi r - \pi r^3 = 0$

$\boxed{\text{The Largest Volume is } 54m^3}$

14. Graph of $f(x)$



15. First we draw the problem:



- $D(x) = \sqrt{x^2 + y^2} \Rightarrow D(x) = \sqrt{x^2 + (3x+10)^2} \Rightarrow \boxed{D(x) = \sqrt{10x^2} + 60x + 100}$
- Domain: $-\frac{10}{3} \leq x \leq 0$
- Find CP: $D^{'}(x) = \dfrac{1}{2} \cdot \dfrac{20x + 60}{\sqrt{10x^2 + 60x + 100}} = \dfrac{10(x+3)}{\sqrt{10x^2 + 60x + 100}}$
  $D^{'}(x) = 0 \Rightarrow 20x + 60 = 0 \Rightarrow x = -3$
  $D^{'}(x)$ DNE $\Rightarrow 10x^2 + 60x + 100 = 0 \Rightarrow$ Never
- Check CP and Endpoints: $f(-\frac{10}{3}) = \frac{10}{3}$, $f(-3) = \sqrt{10}$, $f(0) = 10$
- Answer: $\boxed{\text{Closest Point is } (\sqrt{10}, 3\sqrt{10}, 10)}$

16.

(a) If the area is $200ft^2$, then $xy = 200 \Rightarrow y = \frac{200}{y}$.
Then, $C(x) = (1)(2x) + (1)y + 3(y) = 2x + 4\left(\frac{200}{x}\right) \Rightarrow \boxed{C(x) = 2x + \dfrac{800}{x}}$

(b) Domain: $0 < x < \infty$

(c) $C^{'}(x) = 2 - \frac{800}{x^2} = \frac{2x^2 - 800}{x^2}$
Find CP: $C^{'}(x) = 0 \Rightarrow 2x^2 - 800 = 0 \Rightarrow x = 20$
$C^{'}(x)$ DNE $\Rightarrow x = 0$

Check: $\lim_{x\to 0^+} 2x + \frac{800}{x} = \infty$, $C(20) = 80$, $\lim_{x\to\infty} 2x\frac{800}{x} = \infty$

$\boxed{\text{The Minimal Cost is \$80.00}}$.

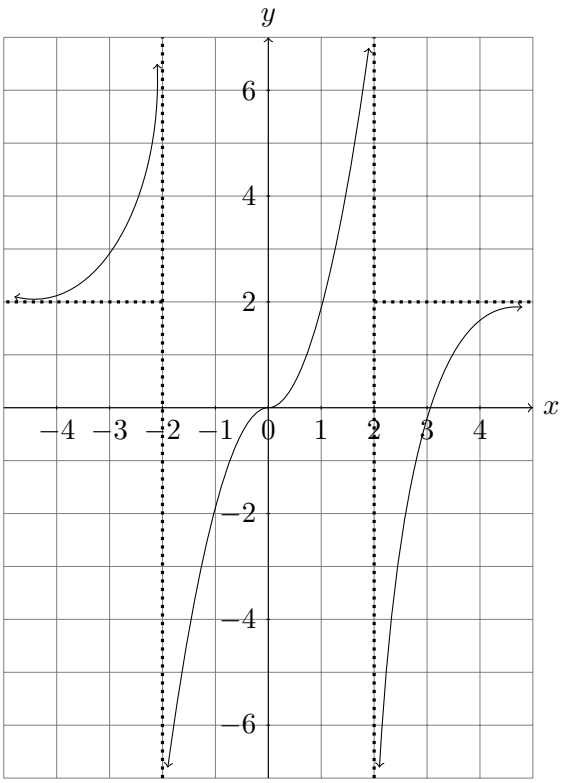
Solutions to Partial Credit Questions

Exam 03 Nov. 17, 2019

13

- $f'(x) = (-2x+2)e^{-x^2+2x} = 2(1-x)e^{-x^2+2x}$
- Find CP: $f'(x) = 0 \Rightarrow 1 - x = 0 \Rightarrow x = 1$
  $f'(x)$ DNE $\Rightarrow e^{x^2} = 0 \Rightarrow$ Never
- Check CP and Endpoints: $f(-1) = e^{-3}$, $f(1) = e$, $f(2) = 0$
- $\boxed{\text{Abs. Max. } (1, e)}$ $\boxed{\text{Abs. Min. } (-1, e^{-3})}$

13. Part A.

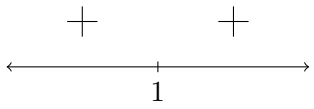


Part B

First we find the possible IP: $g''(x) = \dfrac{1}{(x-1)^2}$

$g''(x) = 0 \Rightarrow$ Never

$g''(x)$ DNE $\Rightarrow x = 1$



Answer: Concave up: $(-\infty, 1), (1, \infty)$, Concave down: Never

14.

(a) Since the Volume is $8m^3$, then $x^2y = 8 \Rightarrow y = \frac{8}{x^2}$

Then, $A(x) = 2x^2 + 4xy = 2x^2 + 4x \cdot \frac{8}{x^2} \Rightarrow \boxed{A(x) = 2x^2 + \dfrac{32}{x}}$

(b) Domain: $0 < x < \infty$

(c) $A'(x) = 4x - \frac{32}{x^2} = \frac{4(x^3-8)}{x^2}$

Find CP: $A'(x) = 0 \Rightarrow x^3 - 8 = 0 \Rightarrow x = 2$

Check CP and Boundary Points: $\lim\limits_{x\to 0^+} 2x^2 + \dfrac{32}{x} = \infty$, $f(2) = 32$, $\lim\limits_{x\to\infty} 2x^2 + \dfrac{32}{x} = \infty$

$\boxed{\text{Value of } x \text{ that minimizes surface area is } 2}$.

15. Graph of $f(x)$