Math 10350 Review for Exam 1

1. Find the $\dfrac{dy}{dx}$ if (a) $y = \dfrac{3x^3 - \sqrt{x} + \pi x^2}{x^2}$, (b) $y = \dfrac{4^t + 1}{2^t}$.
Hint: Divide first.

2. Consider the function $f(x) = \sqrt{x+3}$.

2a. Find the average rate of change of the function $f(x)$ over the interval $[1, 6]$.

2b. Find the average rate of change of the function $f(x)$ over the interval $[1, 1+h]$. Assuming that $h \neq 0$, completely simplify your answer.

2b. Find the derivative of the function $f(x) = \sqrt{x+3}$ at $x = 1$ using the limit definition of derivative. What other names does this value have?

2c. Find the equation of the tangent line to the graph of $y = \sqrt{x+3}$ at $x = 1$.

3. A ball is thrown into the air and its height in feet (measured from the ground) after t seconds is given by $s = -16t^2 + 32t + 48$ until it hits the ground.
(a) What is the initial height of the ball? (Ans: 48 ft)
(b) What is the maximum height of the ball? (Ans: 64 ft)
(c) What is its velocity at moment when the ball hits the ground? (Ans: −64 ft/sec)

4. Solve the x in the following equations: (a) $\dfrac{e^t + 3}{e^t - 2} = 5$; (b) $\log_2(3t^2) - \log_2(t^2 - 1) = 2$

5. An exponential function $y = a \cdot b^x$ passes through the points $(2, 9)$ and $(4, 1)$. (i) Find the values of a and b. Is this an exponential decay or growth function? (ii) For what value of x is $y = 27$?

6. Find c so that $f(x)$ given below is is continuous at $x = 1/3$.

$$f(x) = \begin{cases} \dfrac{9x^2 - 1}{3x^2 - 7x + 2} & x \neq 1/3, -2 \\ \\ c & x = 1/3 \end{cases}$$

7. The position $s(t)$ (in feet) of a particle moving on a straight line at time t minutes is given by the graph below. The line is the tangent line to the graph of $s(t)$ at $t = 2$.
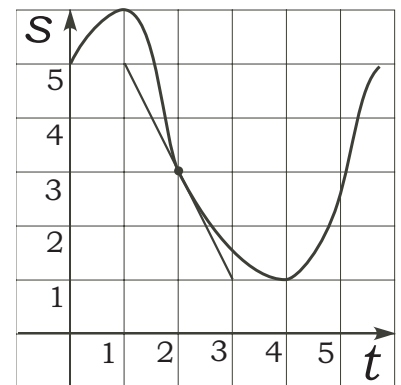
a. Find the average velocity of the particle over the time duration $0 \leq t \leq 3$.

b. Find the instantaneous velocity of the particle at $t = 2$.

c. What is the equation of the tangent line at $t = 3$?

d. Compare the velocities of the particle at $t = 1, 2$ and 5 minutes.

e. Are there any time for which the velocity of the particle is momentarily zero?

8. If $\log_2 3 = x$ and $\log_2 5 = y$ express the following in terms of x and y: (a) $\log_2(45)$; (b) $\log_2(60)$; (c) $\log_2(9/10)$.

9. Consider the curve given by $f(x) = x + \dfrac{1}{x}$.

a. Find the points on the curve $y = f(x)$ at which the tangent lines are horizontal.

b. Find the equations of the tangent lines to the curve $y = f(x)$ which are parallel to the line $3x - 4y = 5$.

10. Consider the function

$$f(x) = \begin{cases} 3 + x & -\infty < x < 5 \\ x^2 & 5 \le x < 11 \\ 5 - 3x & 11 \le x < +\infty \end{cases}$$

Compute the following limits

a. $\displaystyle\lim_{x\to 5^+} 2f(x) - 5$

b. $\displaystyle\lim_{x\to 11^-} f(x) + \dfrac{2}{f(x)}$

c. $\displaystyle\lim_{x\to 7}(f(x))^2$

d. If $f(x)$ continuous? Explain with limits.

11. The graph of the function $f(x)$ is given in Figure 1 below. Find exactly or state that it does not exist each of the following quantity. If it does not exist explain why.

(a) $\displaystyle\lim_{x\to 2} f(x)$
(b) $\displaystyle\lim_{x\to 1^-} f(x)$
(c) $\displaystyle\lim_{x\to 1} f(x)$
(d) $\displaystyle\lim_{x\to -1} f(x)$
(e) $\displaystyle\lim_{h\to 0} \dfrac{f(h) - f(0)}{h}$
(f) $\displaystyle\lim_{h\to 0} \dfrac{f(2+h) + 2}{h}$
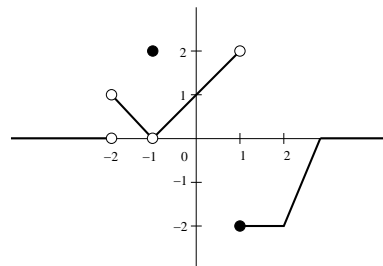
Figure 1

12. If $f'(a) = \displaystyle\lim_{h\to 0} \dfrac{\dfrac{2}{3+h} - \dfrac{2}{3}}{h}$, then $f(x) \overset{?}{=}$ _____ and the value of $a \overset{?}{=}$ ____.

13. If $g'(a) = \displaystyle\lim_{h\to 0} \dfrac{\cos(\pi + h) + 1}{h}$, then $g(x) \overset{?}{=}$ _____ and the value of $a \overset{?}{=}$ ____.

14. In 20 years, the balance of a bank account grew from 1000 to 3000.

14a. If interest is compounded continuously, find the annual interest rate r. Write down a formula for the balance of the account t years after the account is set up. How long would it take for the account to double its balance.

Answer: $r = \ln(3)/20$; double time $= 20\ln(2)/\ln(3)$ years.

14b. If interest is compounded weekly, find the annual interest rate r. Write down a formula for the balance of the account t years after the account is set up. How long would it take for the account to double its balance.

Answer: $r = 52(3^{1/1040} - 1)$; double time $= 20\ln(2)/\ln(3)$ years.

Math 10350: Calculus A
Exam I Sample
September 16, 2038

Name: _____

Class Time: _____

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for one hour and 15 minutes.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 10 pages of the test.

Honor pledge. "As a member of the Notre Dame community, I will not participate in or tolerate academic dishonesty.":

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1. a   b   c   d   e

2. a   b   c   d   e

3. a   b   c   d   e

4. a   b   c   d   e

5. a   b   c   d   e

6. a   b   c   d   e

7. a   b   c   d   e

8. a   b   c   d   e
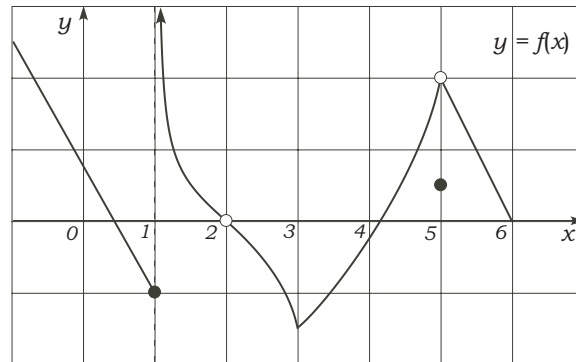
9. a   b   c   d   e

10. a   b   c   d   e

Please do NOT write in this box.

Multiple Choice _____

11. _____

12. _____

13. _____

14. _____

Total _____

Multiple Choice

1.(5 pts.)

The graph of $y = f(x)$ is given above. Which one of the following statements is **FALSE**?

(a) $\lim\limits_{x \to 2} f(x)$ exists.

(b) $\lim\limits_{x \to 1^+} f(x)$ is infinite.

(c) The slope of $f(x)$ at $x = 4$ is positive.

(d) $f(x)$ has a removable discontinuity at $x = 5$.

(e) $\lim\limits_{h \to 0} \dfrac{f(3 + h) - f(3)}{h}$ exists.

2.(5 pts.) Find the average rate of change of the function
$$f(x) = \sqrt{x + 2}$$
over the interval $[-1, 2]$.

(a) 1/6

(b) 1/2

(c) 1/4

(d) 1/3

(e) 1

3.(5 pts.) Find the value of the limit:
$$\lim_{h \to 0} \frac{(2+h)^5 - 2^5}{h}$$
Hint: Think Derivative.

(a)   80

(b)   Does not exist.

(c)   24

(d)   10

(e)   0

4.(5 pts.) Find the equation of the tangent line to the graph of the function
$$f(x) = \sqrt[5]{x} + 4$$
at $x = -1$.

(a) $y - 3 = \dfrac{x^{-4/5}}{5}(x+1)$

(b) $y + 3 = \dfrac{x^{-4/5}}{5}(x-1)$

(c) $y + 1 = \dfrac{1}{5}(x-3)$

(d) $y - 3 = \dfrac{1}{5}(x+1)$

(e) $y + 3 = \dfrac{1}{5}(x-1)$

5.(5 pts.) Consider the function

$$f(x) = \begin{cases} \dfrac{x-1}{k(\sqrt{x}-1)} & \text{if } x < 1 \\[2mm] 2x+3 & \text{if } x \geq 1 \end{cases}$$

Find the value of k, if it exists, so that $f(x)$ is continuous at $x = 1$.

(a)   3

(b)   2

(c) $\dfrac{2}{5}$

(d)   Does not exist.

(e) $\dfrac{1}{2}$

6.(5 pts.) Find the derivative of the function

$$y = \frac{3x^5 - 4x^3 + x^2}{x^3}.$$

(a) $\dfrac{15x^6 - 12x^4 + 2x^3}{3x^4}$

(b) $x^3 - 4x - x^{-2}$

(c) $6x - x^{-2}$

(d) $\dfrac{15x^4 - 12x^2 + 2x}{3x^2}$

(e) $6x^3 - 1$

4

7.(5 pts.) Find the value of the **left-hand** limit: $\displaystyle \lim_{x\to 2^-} \frac{|x-2|}{4-x^2}$

(a) $-\dfrac{1}{4}$

(b) $+\infty$

(c) Does not exist.

(d) -1

(e) $\dfrac{1}{4}$

8.(5 pts.) Find the constant c so that the function

$$f(x) = \begin{cases} x^3 + 3x - 3c, & x > 1 \\ (x-1)^2 + c, & x \le 1. \end{cases}$$

is continuous.

(a) 1

(b) 2

(c) -2

(d) -1

(e) 0

9.(5 pts.) Find the value of the limit:
$$\lim_{x \to 2} \frac{\sqrt{x+2} - \sqrt{6-x}}{x-2}$$

(a) 1/3

(b) 1

(c) $+\infty$

(d) 1/2

(e) Does not exist.

10.(5 pts.) The temperature F (in Fahrenheit) of a hot liquid at time t hours is given by the equation
$$F(t) = 100(0.5)^t + 70.$$
Find the time at which the temperature is 95 °F.

(a) $\ln(2)$ hours

(b) 2 hours

(c) 1 hour

(d) 1/2 hour

(e) $[\ln(0.25) - \ln(0.5)]$ hours

Partial Credit
You must show your work on the partial credit problems to receive credit!

11.(12 pts.)

11a. Find ALL equations of the vertical asymptotes of the curve $y = \dfrac{x^2 - 9}{x^2 - x - 6}$

(Remark: Your answers should be in the form: $x = c$.)

11b. Find ALL values of x for which the graph of $g(x) = 16x + \dfrac{1}{x^2}$ has a horizontal tangent line.

12.(12 pts.) **(Part A)** Solve for x that satisfies the equation below.

$2 \cdot e^{2x} - 5 \cdot e^{5x} = 0$

(Part B) [Not related to Part A]. Using completing the square, rewrite $-5x^2 + 30x + 1$ in the form $a(x+b)^2 + c$ where a, b and c are numbers to be determined.

13.(12 pts.) The position function of a ball thrown upward , measured from ground level, is given by the function
$$s(t) = -5t^2 + 4t + 1.$$
13a. Find the time at which the ball hits the ground.

13b. Find the instantaneous **velocity** at time t.

13c. Find the instantaneous rate of change of the **velocity** at time t.

13d. Find the velocity at the moment when the ball hits the ground.

14.(12 pts.) Consider the function
$$f(x) = \frac{1}{x}.$$
14a. Write down the average rate of change of $f(x)$ over the interval $2 \le x \le 2+h$. You may assume that $h \ne 0$.

14b. Using Part (a) above and limits (only), find the **slope** of the curve $y = \frac{1}{x}$ at $x = 2$.

Math 10350: Calculus A
Exam I Sample
September 16, 2038

Name: _____

Class Time: _ANSWERS_ _____

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- The exam lasts for one hour and 15 minutes.
- Be sure that your name is on every page in case pages become detached.
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Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1. | a | b | c | d | ● |
2. | a | b | c | ● | e |
3. | ● | b | c | d | e |
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7. | a | b | c | d | ● |
8. | ● | b | c | d | e |
9. | a | b | c | ● | e |
10. | a | ● | c | d | e |

Please do NOT write in this box.

Multiple Choice _____

11. _____

12. _____

13. _____

14. _____

Total _____

Math 10350: Calculus A
Exam I
September 22, 2039

Name: _____

Class Time: _____

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- No calculators.
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1. a   b   c   d   e

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3. a   b   c   d   e

4. a   b   c   d   e

5. a   b   c   d   e

6. a   b   c   d   e

7. a   b   c   d   e

8. a   b   c   d   e

Please do NOT write in this box.

Multiple Choice _____

9. _____

10. _____

11. _____

12. _____

13. _____

Total _____

Partial Credit
You must show your work on the partial credit problems to receive credit!

11.(12 pts.)

11a. Find ALL equations of the vertical asymptotes of the curve $y = \dfrac{x^2 - 9}{x^2 - x - 6}$

(Remark: Your answers should be in the form: $x = c$.)

$$\frac{x^2 - 9}{x^2 - x - 6} = \frac{(x - 3)(x + 3)}{(x - 3)(x + 2)} = \frac{x + 3}{x + 2} \quad \text{when } x \neq 3.$$

$x = -2$ is the only vertical asymptotes

$\left(\text{at } x = 3 \text{, the function is undefined} \right)$

11b. Find ALL values of x for which the graph of $\quad g(x) = 16x + \dfrac{1}{x^2} \quad$ has a horizontal tangent line.

$g(x)$ has horizontal tangent line when $g'(x) = 0$.

$$16 - 2x^{-3} = 0 \implies 16 - \frac{2}{x^3} = 0$$

$$\implies \frac{2}{x^3} = 16 \implies x^3 = \frac{2}{16} = \frac{1}{8}$$

$$\implies x = \sqrt[3]{\frac{1}{8}} = \frac{1}{2}$$

12.(12 pts.) (**Part A**) Solve for x that satisfies the equation below.

$2 \cdot e^{2x} - 5 \cdot e^{5x} = 0 \implies 2e^{2x} = 5e^{5x}$

$\implies \dfrac{2e^{2x}}{e^{5x}} = 5 \implies 2e^{-3x} = 5 \implies e^{-3x} = \dfrac{5}{2}$

$\implies \ln\left(e^{-3x}\right) = \ln\left(\dfrac{5}{2}\right) \implies -3x = \ln\dfrac{5}{2}$

$\implies x = -\dfrac{1}{3}\ln\left(\dfrac{5}{2}\right)$

(**Part B**) [**Not related to Part A**]. Using completing the square, rewrite $-5x^2+30x+1$ in the form $a(x+b)^2+c$ where a, b and c are numbers to be determined.

$-5x^2+30x+1 = -5(x^2-6x)+1$

$= -5\left(x^2-6x+\left(\dfrac{-6}{2}\right)^2 - \left(\dfrac{-6}{2}\right)^2\right)+1$

$\underbrace{\hspace{1cm}}_{(-3)^2} \quad \underbrace{\hspace{1cm}}_{(-3)^2}$

$= -5\left((x-3)^2 - 9\right) + 1$

$= -5(x-3)^2 + 45 + 1$

$= -5(x-3)^2 + 46$

13.(12 pts.) The position function of a ball thrown upward , measured from ground level, is given by the function

$$s(t) = -5t^2 + 4t + 1.$$

$s = 0$

13a. Find the time at which the ball hits the ground.

$$S(t) = 0 \implies -5t^2 + 4t + 1 = 0$$

$$\implies (-5t + 1)(t - 1) = 0$$

$$\implies t = -1/5 \text{ (rejected)} , 1.$$

$$\text{so } \boxed{t = 1}$$

13b. Find the instantaneous **velocity** at time t.

$$V = S'(t) = -10t + 4$$

13c. Find the instantaneous rate of change of the **velocity** at time t.

$$V'(t) = -10$$

13d. Find the velocity at the moment when the ball hits the ground.

$$V(1) = -10 + 4 = -6$$

14.(12 pts.) Consider the function

$$f(x) = \frac{1}{x}.$$

14a. Write down the average rate of change of $f(x)$ over the interval $2 \leq x \leq 2+h$. You may assume that $h \neq 0$.

$$\frac{f(2+h) - f(2)}{h} = \frac{\frac{1}{2+h} - \frac{1}{2}}{h} = \frac{\frac{2 - (2+h)}{2(2+h)}}{h}$$

$$= \frac{\frac{2 - 2 - h}{2(2+h)}}{} \times \frac{1}{h} = \frac{-h}{2(2+h)} \times \frac{1}{h}$$

$$= \frac{-1}{2(2+h)}$$

14b. Using Part (a) above and limits (only), find the **slope** of the curve $y = \frac{1}{x}$ at $x = 2$.

The slope of $\frac{1}{x}$ at $x=2$ is

$$\lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \to 0} \frac{-1}{2(2+h)}$$

$$f'(2)$$

$$= \frac{-1}{2(2)} = \boxed{-\frac{1}{4}}$$

Multiple Choice

1.(5 pts.)
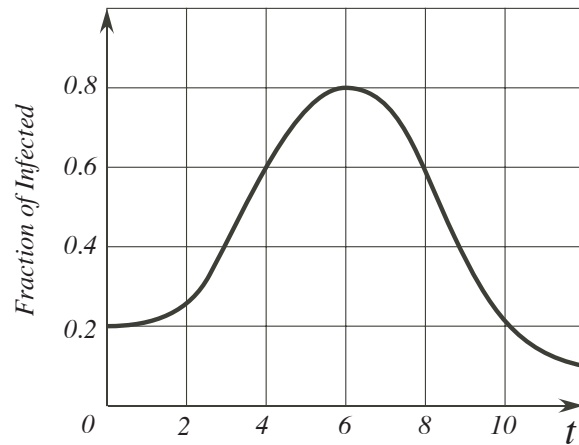
The graph above shows the fraction of the population of a city infected by a flu virus over the a 12-month duration. What is average rate of change of the fraction of infected people over the duration of the first six months?

(a)   0.6

(b)   0.1

(c) $\dfrac{5}{3}$

(d)   10

(e) -0.6

2.(5 pts.) Referring to the graph above, let $v(t)$ be the **instantaneous** rate of change of the fraction of infected people in the city at time t where $0 < t < 12$. Listing the values of $v(2)$, $v(6)$, and $v(9)$ from the smallest to the biggest, we have:

(a) $v(6) < v(2) < v(9)$

(b) $v(2) < v(6) < v(9)$

(c) $v(6) < v(9) < v(2)$

(d) $v(2) < v(9) < v(6)$

(e) $v(9) < v(6) < v(2)$

3.(5 pts.) Find the equation of the tangent line to the graph of the function
$$f(x) = \frac{1}{x^4} + 3$$
at $x = -1$.

(a) $y - 4 = 4(x + 1)$

(b) $y - 4 = -4x^{-5}(x + 1)$

(c) $y + 4 = 4(x - 1)$

(d) $y + 1 = 4(x - 4)$

(e) $y + 4 = -4x^{-5}(x - 1)$

4.(5 pts.) If $\log_3 x = -1$ and $\log_3 y = 4$, find the exact value of
$$\log_3 \left(81 x \sqrt{y}\right)$$

(a)   7

(b) -8

(c) -5

(d)   5

(e) -7

5.(5 pts.) Consider the function
$$f(x) = \begin{cases} \dfrac{x^2 + 3x - 4}{x^2 - 3x + 2} & \text{if } x < 1 \\[2mm] kx + 1 & \text{if } x \geq 1 \end{cases}$$

Find the value of k, if it exists, so that $f(x)$ is continuous at $x = 1$.

(a) -5 　　　　　　　 (b) -6 　　　　　　　 (c) 6

(d)   Does not exist. 　　　 (e) 5

6.(5 pts.) On her 20th birthday, Joan decided to set up a certificate of deposit (CD) which will mature when she is 30 years old. If her initial deposit was \$2000 and interests is compounded **monthly** at a rate of 3% per year, how much will she receive when the certificate reaches maturity?

(a)   None of these.

(b) $2000 \left(1 + \dfrac{0.03}{12}\right)^{30}$

(c) $2000 \left(1 + 0.03\right)^{10}$

(d) $2000 \left(1 + \dfrac{0.03}{12}\right)^{10}$

(e) $2000 \left(1 + \dfrac{0.03}{12}\right)^{120}$

7.(5 pts.) Find $\dfrac{dy}{dx}$ if $y = \dfrac{2x^6 + 9x^2 - 1}{x^2}$.

(a) $\dfrac{2}{5}x^5 + 9x + x^{-1}$

(b) $\dfrac{12x^6 + 18x^3 - x}{2x^3}$

(c) $8x^4 + 2x^{-1}$

(d) $8x^3 + 2x^{-3}$

(e) $\dfrac{12x^5 + 18x}{2x}$

8.(5 pts.) Let $f(x)$ be the function whose graph is shown below. Which of the following statements is **FALSE**?
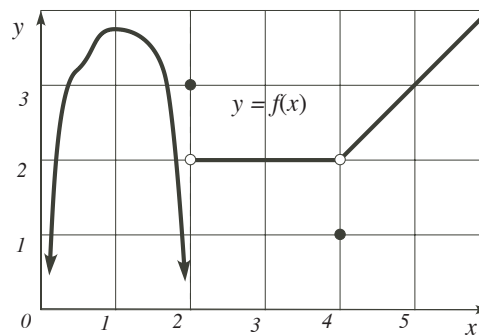
(a) $\displaystyle\lim_{x \to 4} f(x) = 1$.

(b) $f(2) = 3$.

(c) $f(x)$ is not continuous at $x = 4$.

(d) $f(x)$ is continuous at $x = 3$.

(e) $\displaystyle\lim_{x \to 2^-} f(x) = -\infty$.

Partial Credit
You must show your work on the partial credit problems to receive credit!

9.(12 pts.) Completely simplify the following expression giving your answer in the form $\dfrac{Ae^{4x} + Be^{2x} + C}{(e^{2x} + 3)^n}$ where A, B, C and n are constants to be determined.

$$\frac{(e^{2x} + 3)^4 \cdot 2e^{2x} - 4(e^{2x} + 3)^3 \cdot 2e^{2x}(e^{2x} + 1)}{(e^{2x} + 3)^8}$$

10.(12 pts.) The monthly profit function for the sales of a limited edition vase of a members only

$$P(x) = 450 - \frac{1}{2}x^2 + 10x \quad \text{thousands of dollars}$$

where x is the number of vases sold in a month.

Part A. Using completing the square, find the number of vases that must be sold so that the profit is maximum. What is the maximum monthly profit?

Part B. Find the number of vases sold to make a profit of 50 thousand dollar.

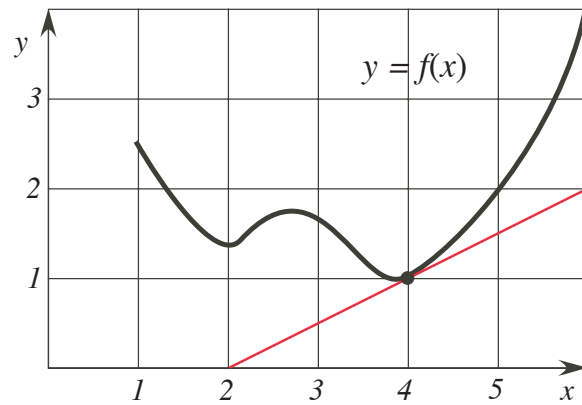**11.**(12 pts.) Solve for x that satisfies each of the following equations:

Part A. $\log_{10}(3x + 2) - \log_{10}(x - 1) = 1$

Part B. (Not related to above.) $\dfrac{e^x + 1}{2e^x - 1} = 3$

12.(12 pts.)

12a. The figure above describes the graph of $y = f(x)$ and its tangent line at $x = 4$. Answer the problems below:

i. $f(4) \overset{?}{=}$ _____ and $f'(4) \overset{?}{=}$ _____

ii. Find the equation of the tangent line at $x = 4$. Give your answer in slope-intercept form.

12b. (Not related to above.)

Find the equations of the tangent lines to the graph of $f(x) = 4x^3$ such that they are parallel to the line $y - 12x = 8$.

13.(12 pts.) Consider the function $f(x) = x^2 + 2x$.

13a. Compute the average rate of change of $f(x)$ over the interval $2 \leq x \leq 2 + h$. You may assume that $h \neq 0$ and simplify your answer.

13b. Using Part (a) above and limits (only), find the **slope** of the curve $y = x^2 + 2x$ at $x = 2$.

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1. a   ●   c   d   e

2. a   b   c   d   ●

3. ●   b   c   d   e

4. a   b   c   ●   e

5. a   ●   c   d   e

6. a   b   c   d   ●

7. a   b   c   ●   e

8. ●   b   c   d   e

Please do NOT write in this box.

Multiple Choice _____

9. _____

10. _____

11. _____

12. _____

13. _____

Total _____

Partial Credit
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9.(12 pts.) Completely simplify the following expression giving your answer in the form $\frac{Ae^{4x} + Be^{2x} + C}{(e^{2x} + 3)^n}$ where A, B, C and n are constants to be determined.

$$\frac{(e^{2x} + 3)^4 \cdot 2e^{2x} - 4(e^{2x} + 3)^3 \cdot 2e^{2x}(e^{2x} + 1)}{(e^{2x} + 3)^8}$$

$$= \frac{2e^{2x}(e^{2x} + 3)^3\left[(e^{2x} + 3) - 4(e^{2x} + 1)\right]}{(e^{2x} + 3)^8}$$

$$= \frac{2e^{2x}\left(e^{2x} + 3 - 4e^{2x} - 4\right)}{(e^{2x} + 3)^5}$$

$$= \frac{2e^{2x}\left(-3e^{2x} - 1\right)}{(e^{2x} + 3)^5}$$

$$= \frac{-6e^{4x} - 2e^{2x}}{(e^{2x} + 3)^5}.$$

$A = -6$

$B = -2$

$C = 0$

$n = 5$

Apply $a^{\log_a y} = y$

11. (12 pts.) Solve for x that satisfies each of the following equations:

Part A. $\log_{10}(3x + 2) - \log_{10}(x - 1) = 1$

$$\log_{10}\left(\frac{3x+2}{x-1}\right) = 1 \implies \frac{3x+2}{x-1} = 10^1$$

$$3x + 2 = 10(x-1)$$

$$3x + 2 = 10x - 10$$

$$3x - 10x = -10 - 2$$

$$-7x = -12 \implies \boxed{x = \frac{12}{7}}$$

Part B. (Not related to above.) $\frac{e^x + 1}{2e^x - 1} = 3 \implies e^x + 1 = 3(2e^x - 1)$

$$\implies e^x + 1 = 6e^x - 3$$

$$\implies e^x - 6e^x = -3 - 1$$

$$\implies -5e^x = -4 \implies e^x = \frac{4}{5}$$

$$\implies \ln e^x = \ln\frac{4}{5} \implies \boxed{x = \ln\frac{4}{5}}$$

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10.(12 pts.) The monthly profit function for the sales of a limited edition vase of a members only

$$P(x) = 450 - \frac{1}{2}x^2 + 10x \quad \text{thousands of dollars}$$

where x is the number of vases sold in a month.

Part A. Using completing the square, find the number of vases that must be sold so that the profit is maximum. What is the maximum monthly profit?

$$P(x) = -\frac{1}{2}x^2 + 10x + 450 = -\frac{1}{2}(x^2 - 20x) + 450$$

$$= -\frac{1}{2}\left(x^2 - 20x + \left(\frac{-20}{2}\right)^2 - \left(\frac{-20}{2}\right)^2\right) + 450$$

$$\underbrace{\left(\frac{-20}{2}\right)^2}_{(-10)^2} \quad \underbrace{\left(\frac{-20}{2}\right)^2}_{(-10)^2}$$

$$= -\frac{1}{2}\left((x-10)^2 - 100\right) + 450$$

$$= -\frac{1}{2}(x-10)^2 + 50 + 450 = 500 - \frac{1}{2}(x-10)^2$$

Max profit = 500 thousand dollars occuring when $x = 10$.

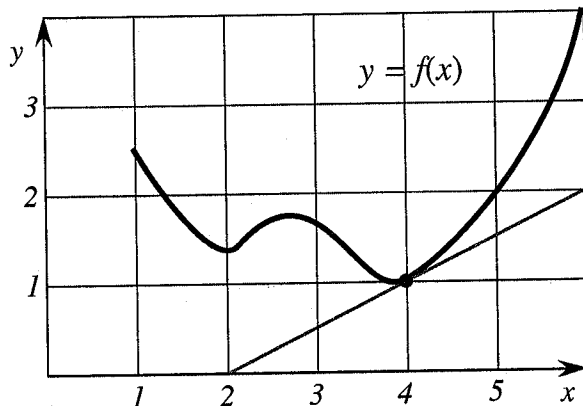**Part B.** Find the number of vases sold to make a profit of 50 thousand dollar.

$$P(x) = 50 = 450 - \frac{1}{2}x^2 + 10x$$

$$0 = 400 - \frac{1}{2}x^2 + 10x \implies \frac{1}{2}x^2 - 10x - 400 = 0$$

$$\implies x^2 - 20x - 800 = 0 \implies (x+20)(x-40) = 0$$

$$\implies x = 40, \quad -20 \text{ (rejected)}.$$

14.(12 pts.)

14a. The figure above describes the graph of $y = f(x)$ and its tangent line at $x = 4$. Answer the problems below:

i. $f(4) \stackrel{?}{=} \underline{\qquad 1 \qquad}$ and $f'(4) \stackrel{?}{=} \underline{\qquad \frac{2}{4} = \frac{1}{2} \qquad}$

ii. Find the equation of the tangent line at $x = 4$. Give your answer in slope-intercept form.

$$y - 1 = \tfrac{1}{2}(x - 4) \Rightarrow y - 1 = \tfrac{1}{2}x - 2$$
$$\Rightarrow y = \tfrac{1}{2}x - 1$$

14b. (Not related to above.)

Find the equations of the tangent lines to the graph of $f(x) = 4x^3$ such that they are parallel to the line $y - 12x = 8$.

$$f'(x) = 12x^2 \; ; \qquad y = 12x + 8$$
$$\text{Set } f'(x) = 12 \Rightarrow 12x^2 = 12 \Rightarrow x^2 = 1$$
$$\Rightarrow x = \pm 1$$

$x = 1 \Rightarrow y = f(1) = 4.$ One tangent line
is $y - 4 = 12(x - 1)$ is. $y = 12x - 8.$

$x = -1 \Rightarrow y = f(-1) = -4.$ The other tangent line is
$y - (-4) = 12(x - (-1)) \Rightarrow y = 12x + 8.$

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15.(12 pts.) Consider the function $f(x) = x^2 + 2x$.

15a. Compute the average rate of change of $f(x)$ over the interval $2 \leq x \leq 2 + h$. You may assume that $h \neq 0$ and simplify your answer.

$$\frac{f(2+h) - f(2)}{h} = \frac{(2+h)^2 + 2(2+h) - (2^2 + 2(2))}{h}$$

$$= \frac{4 + 4h + h^2 + 4 + 2h - 8}{h} = \frac{6h + h^2}{h}$$

$$= \frac{h(6+h)}{h}$$

$$= 6 + h.$$

15b. Using Part (a) above and limits (only), find the **slope** of the curve $y = x^2 + 2x$ at $x = 2$.

The slope of $y = x^2 + 2x$ at $x = 2$ is

$$\frac{dy}{dx}\bigg|_{x=2} = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h \to 0} (6+h) = 6$$