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HP3 Math 121

Name: SAMPLE ANSWER

Due: Thursday February 09, 2006

1. Using the limit criteria for continuity, find the value of k so that $f(x)$ is a continuous function on any interval. (Do not use graphs to support your answer).

$$f(x) = \begin{cases} kx, & 0 < x < 2 \\ 3x^2, & x \geq 2 \end{cases}$$

$f(x)$ is continuous for all $x \in [0, +\infty)$ except possibly at 2. For $f(x)$ to be continuous at $x = 2$, we need

$$\lim_{x \rightarrow 2} f(x) = f(2) \leftarrow \dots + 12$$

$$\Rightarrow \lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x) = f(2) \leftarrow \dots + 12$$

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} kx = 2k \leftarrow \dots + 12$$

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} 3x^2 = 3(2)^2 = 12 = f(2) \leftarrow \dots + 12$$

Therefore, $2k = 12$ for continuity at $x = 2$

$$\Rightarrow k = 6 \leftarrow \dots + 12$$

2. Using the limit criteria for continuity, find the value of k so that $g(x)$ is a continuous function on any interval. (Do not use graphs to support your answer).

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$$g(x) = \begin{cases} \frac{5x^3 - 10x^2}{x-2}, & x \neq 2 \\ k, & x = 2 \end{cases} = \begin{cases} \frac{5x^2(x-2)}{x-2}, & x \neq 2 \\ k, & x = 2 \end{cases}$$

$$= \begin{cases} 5x^2, & x \neq 2 \\ k, & x = 2 \end{cases} + D$$

$g(x)$ is continuous for all x except possibly $x=2$

For continuity at $x=2$, $\lim_{x \rightarrow 2} g(x) = g(2)$

$$g(2) = k = \lim_{x \rightarrow 2} g(x) = \lim_{x \rightarrow 2} 5x^2$$

$$k = 5(2)^2 = 20$$

OR:

$$g(2) = k = \lim_{x \rightarrow 2} g(x) = \lim_{x \rightarrow 2} \frac{5x^3 - 10x^2}{x-2}$$

$$= \lim_{x \rightarrow 2} \frac{5x^2(x-2)}{(x-2)} = \lim_{x \rightarrow 2} 5x^2$$

$$= 5(2)^2 = 20$$

$$\Rightarrow k = 20$$

