Name ______________________________ Date ___________________

Math 10260 Exam 2 Review Activity

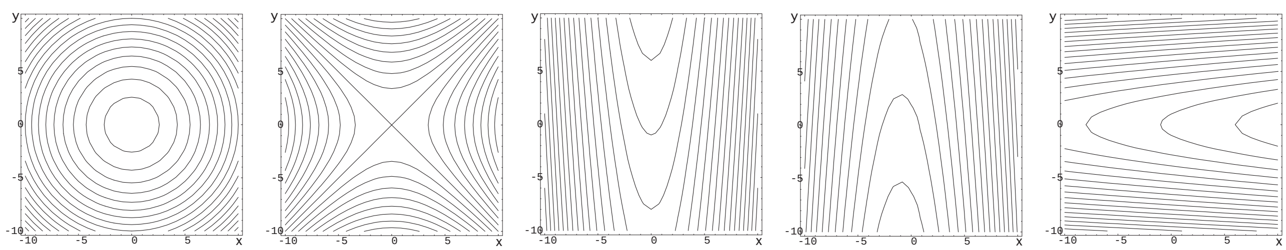
1. Do the points $(1,2,0), (5,6,1)$ and $(9,3,2)$ form a right triangle? Ans. No

2. Let $f(x,y) = x^2 + y^2$.
   - Draw level curves of $f(x,y)$
   - Draw the graph of x-section of $f(x,y)$ where $x = 1$.
   - Draw the graph of y-section of $f(x,y)$ where $y = 1$.

3. Let $f(x,y) = x^3y + \ln(y-x)$. Find the following limits:

   (a) $\lim\limits_{h\to 0} \dfrac{f(1+h,2) - f(1,2)}{h}$ (b) $\lim\limits_{h\to 0} \dfrac{f(1,2+h) - f(1,2)}{h}$ Ans. (a) 5; (b) 2.

4. Which of the following pictures shows level curves of the function $f(x,y) = x^2 - y^2$ Ans. b



a **b** **c** **d** **e**

5. Let $f(x,y)$ be a function, with $f(1,2) = 5$, $\frac{\partial f}{\partial x}(1,2) = 2$, $\frac{\partial f}{\partial y}(1,2) = -3$. Find the equation of the tangent plane to the graph of $f(x,y)$ at the point where $x = 1$ and $y = 2$. Ans. $z = 2x - 3y + 9$

6. Let $f(x,y) = \dfrac{x}{1+y} + e^{xy}$. Find all first and second partial derivatives of f.

   $\frac{\partial f}{\partial x} = \frac{1}{y+1} + ye^{xy}$; $\frac{\partial f}{\partial y} = -x(1+y)^{-2} + xe^{xy}$; $\frac{\partial^2 f}{\partial x^2} = y^2e^{xy}$; $\frac{\partial^2 f}{\partial y^2} = 2x(1+y)^{-3} + x^2e^{xy}$; $\frac{\partial^2 f}{\partial x \partial y} = -(1+y)^{-2} + e^{xy} + xye^{xy} = \frac{\partial^2 f}{\partial y \partial x}$

7. If you wanted to find the least-squares line of the form $y = ax + b$ for the given data $(1,0)$, $(-1,2)$, $(3,1)$, what is the error function $E(a,b)$ you need to minimize in order to determine the values of a and b? Find also a and b by minimizing $E(a.b)$.

   (Ans. $E(a,b) = (a+b)^2 + (-a+b-2)^2 + (3a+b-1)^2$; $a = -0.25$; $b = 1.25$)

8. Find the equation of the plane through the following three points $(1,1,2)$, $(1,3,3)$, $(5,5,0)$.

   Ans. $z - 2 = -(x-1) + \frac{1}{2}(y-1)$

9. Let $f(x,y) = x^2 - y^2 + xy + 3x - 4y + 5$. Find the critical points of f and then use the second derivative test, if possible, to determine the nature of each critical point. Ans. saddle point at $(-2/5, -11/5)$

10. Let $f(x,y) = y^3 - x^3 - 3xy + 5$. Find the critical points of f and then use the second derivative test, if possible, to determine the nature of each critical point. Ans. saddle point at $(0,0)$ and max at (1,-1)

11. Let $f(x,y)$ be a function with a critical point at $(1,2)$ and the second order partial derivatives of f at this point given by:

$$\frac{\partial^2 f}{\partial x^2}(1,2) = -2, \quad \frac{\partial^2 f}{\partial y^2}(1,2) = -4, \quad \frac{\partial^2 f}{\partial x \partial y}(1,2) = 1.$$

Use the second derivative test to determine whether the critical point $(1,2)$ is a place of local maximum, minimum or saddle point, or the test is inconclusive. Ans. max at $(1,2)$

12. Let $f(x,y) = ye^x + 2x - 2y + 5$. Find the linear approximation of $f(x,y)$ at the point $(0,0)$. Then use it to estimate $f(-0.05, 0.01)$ Ans. $f(-0.05, 0.01) \approx 4.89$.

13. When Gap uses K units of capital and L units of labor then it produces $P(K,L) = 20K^{1/3}L^{2/3}$ units of clothing. Given that each unit of capital costs \$10 and each unit of labor costs \$20, and that the company's budget is \$60,000, what input (K,L) of capital and labor will maximize the company's output? Draw a picture containing level curves and the constraint curve that depicts Lagrange's method. Ans. max production at (2,000,2,000)

14. Use Lagrange multipliers to find the maximum and minimum values of the function $f(x,y) = xy$ on the curve $x^2 + 2y^2 = 1$. Ans. max $= \sqrt{2}/4$ at $(\sqrt{2}/2, 1/2)$ & $(-\sqrt{2}/2, -1/2)$; min $= -\sqrt{2}/4$ at $(-\sqrt{2}/2, 1/2)$ & $(\sqrt{2}/2, -1/2)$.

15. The function $f(x,y,z) = x^2 + y^2 + z^2$ has a minimum subject to the constraint $2x - 2y + z = 9$. Find it by the method of Lagrange multipliers. Ans. min $= 9$ at $(2, -2, 1)$

16. Figure 1 shows four level curves of a Cobb-Douglas production function $Q(K,L) = AK^{\alpha}L^{1-\alpha}$. Find the maximum output $Q(K,L)$ subject to the constraint

$$4K + 7L = 280,000.$$

Ans: 800

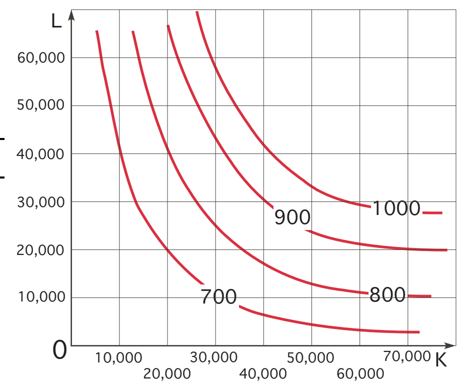


Figure 1

17. You are hiking across the twin peaks of a mountain following the path ABCDEFGH shown on a topographical map as in Figure 2.
(a) Find the height of the mountain at the points A, B, C, D, E, F, G, and H.
(b) Where is the height along this path maximum?
(c) Where is the tangent line to the path the same with the tangent line to one of the level curves shown in Figure 2?
(d) Draw a sketch of this mountain in 3D.

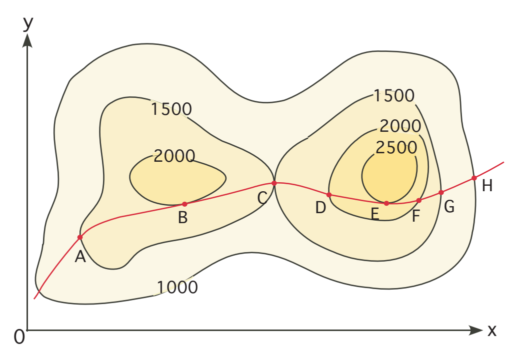


Figure 2

18. A small company produces two items, X and Y. Its **fixed** cost per week is \$500. If the company produces 100 items of X and 50 items of Y per week then its cost is \$750; if it produces 50 items of X and 100 items of Y per week then its cost is \$700. Write down a formula for the cost function $C(x,y)$, where x is the number of items of X and y is the number of items of Y produced per week, assuming that $C(x,y)$ is **linear** in x and y. Ans. $C(x,y) = 2x + y + 500$

19. A Pharmaceutical company estimates that the demand function for one of its drugs in the U.S. is $p_1 = 20 - 0.2x$, while the demand function for the same drug in foreign market is $p_2 = 10 - 0.05y$, where x and y are in thousands of units and p_1 and p_2 are in dollars per unit. Its cost function is $C(x,y) = 55 + 0.2(2x + y)$, in thousands of dollars. Find the quantities x, y and the prices that maximize the company profit. Ans. max profit at $(49, 98)$