Name ______________________________ Date ________________

Math 10260 Exam 1 Review Activity

Q 1: Solve the initial value problem $\frac{dy}{dt} = e^{-0.5t} + 3t^2,\ y(0) = 1.$ (Ans: $y = -2e^{-0.5t} + t^3 + 3$)

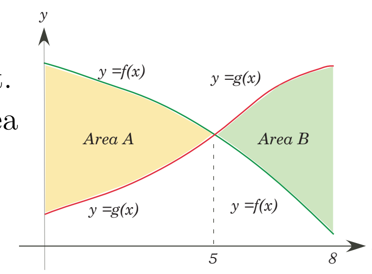
Q 2: Evaluate the following integrals without your calculator:

a. $\int xe^{3x}\,dx$ **b.** $\int \frac{(\ln x)^2}{x}\,dx$ **c.** $\int_1^3 x^2 \ln x\,dx$

d. $\int_0^2 (x-1)^{25}\,dx$ **e.** $\int \frac{x}{x+2}dx$ **f.** $\int \frac{1}{6-x-x^2}\,dx$

Q 3: Let $f(x)$ and $g(x)$ be the functions shown in the graph on the right. If $\int_0^8 [f(x) - g(x)]\ dx = 7$ and $\int_5^8 [f(x) - g(x)]\ dx = -17$, what is the area of A.

(Ans: 24).

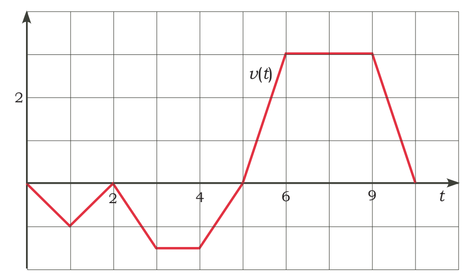


Q 4: The graph of the velocity $v(t)$ of a particle moving on a horizontal straight line is given below. Let $s(t)$ meters be the position of the particle after time t minutes. Assume that $s(0) = 1$. Find the exact value of the following quantities and express each of them with definite integral.

a. The position of the particle after 2 minutes.

b. Total change in position for the duration $1 \le t \le 4$.

(Ans: (a) 0 ft, (b) $-11/4$ ft, (c) 13 ft)



Q 5: Find the intersection points of $f(x) = 5 - x^2$ and $g(x) = x - 1$. Then find the area over the interval $0 \le x \le 3$. (Ans: 10.167)

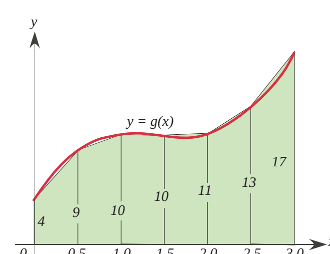
Q 6: Suppose you deposit \$15,000 in an account paying 3% annual interest compounded continuously, and do not make any further deposit or withdrawals. Find the average amount of money in the account during the first 5 years. (Ans: 16183.4)

Q 7: Let $g(x)$ be the function whose graph is shown below.

a. Use Riemann sum with right hand end-points to estimate $\int_0^3 g(x)\,dx$.

b. Use the Trapezoidal rule to estimate the average of $g(x)$ for $0 < x < 3$.

(Ans: (a) 35, (b) $10.58\bar{3}$)



Q 8: Find the equilibrium quantity and price, consumer surplus and producer surplus for the demand curve $D(q) = (q - 10)^2 + 1$ and the supply curve $S(q) = q^2 + 1$ for $0 \le q \le 10$.

(Ans: $q_e = 5, p_e = 26, CS = 500/3, PS = 250/3$)

Q 9: A retired person has \$2 million in an IRA paying an annual interest rate of 5%, compounded continuously. Over the next 20 years she plans to withdraw money continuously at the constant rate of S dollars per year. Find the value of S so that there is \$1 million left in her account at the end of 20 years. (Ans: $S = \frac{0.05(2-e^{-1})}{1-e^{-1}}$ or $\frac{0.1e - 0.05}{e-1}$ million)

Q 10: Which of the following savings accounts yields the most after **ten** years:
(a) For the next 10 years, money is deposited continuously into Account A which pays 6% interest compounded continuously, at a rate of $2000 + 200t$ dollars per year. (Ans: FV_a=$39,743.89)

(b) An Account B with principal $25,000 paying interest at 5% compounded **monthly.** (Ans: FV_b=$41,175.24)

(c) On the first of January this year and for the next four years (no more after), $6000 is deposited into Account C earning 4% annual interest compounded continuously. (Ans: FV_c=$41,379.97)
Q 11: Find the present value of each account in Q10. (Ans: PV_a=$21,811.91; PV_b=$25,000; PV_c=$27,737.82)

Q 12: If a continuous income stream flows into a saving account at a constant rate of $10,000 per year and earns 7% interest, compounded continuously, find the time required for the balance to become $1,000,000. (Ans: $\frac{\ln 8}{0.07}$)

Q 13: Find all solutions for the differential equation $\dfrac{dy}{dx} = 2x - xy.$ (Ans: $y = 2 - Ke^{-x^2/2}$)

Q 14: Solve the initial value problem $\dfrac{dy}{dt} = y^2(4t^3 + 1), \quad y(0) = -1.$ (Ans: $y = \frac{-1}{t^4+t+1}$)

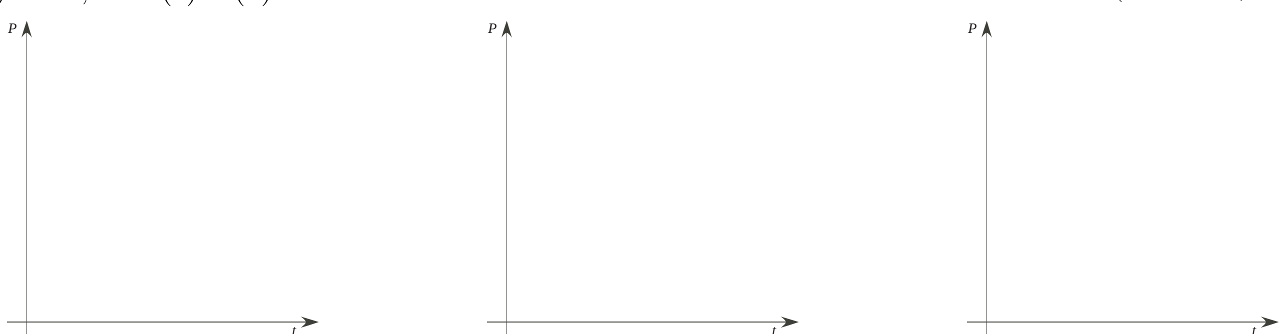
Q 15: A home buyer can afford to spend no more than $900 per month on mortgage payments. Suppose that the annual interest rate is 8%, compounded continuously, that the term of the mortgage is 30 years, and that payments are also made continuously.
(a) Determine the maximum amount that this buyer can afford to borrow. (Ans: $122,753)
(b) Determine the total interest paid during the term of the mortgage. (Ans: $201,247)

Q 16: If for the US population we assume that the intrinsic growth rate is 0.015 and the carrying capacity is 1 billion, then write the logistic equation that models it. Then, solve it using the information that its size now is about 0.3 billion. (Ans: $\frac{dP}{dt} = 0.015P(1-P); P(0) = 0.3 \Rightarrow P(t) = \frac{3e^{0.015t}}{7 + 3e^{0.015t}}$)

Q 17: Find the intrinsic growth rate and the carrying capacity of the logistic equation

$$\frac{dp}{dt} = 0.2p - 0.004p^2$$

Without solving for $P(t)$, sketch the graph of the solution of the logistic equation if (a) $P(0) = 60$, (b) $P(0) = 50$, and (c) $P(0) = 20$ (Ans : $r = 0.2$, $K = 50$) .



(a) $P(0) = 60$ (b) $P(0) = 50$ (c) $P(0) = 20$

Q 18: A company has determined that when it produces at least 100 units of its product, its marginal cost is given by $MC(x) = 0.25x + 1500$ and its marginal revenue $MR(x) = -6x^2 + 500x + 20000$. If the company is currently operating at a production level of 100 units per day, is it profitable for the company to increase production to 110 units per day? Justify your answer (Ans : Total Change in profit = 47,737.50)

Q 19: Your apartment in LA produces a perpetual income stream flowing continuously at a rate of $70,000 per year. This income can be invested at the annual rate of 7 %, compounded continuously. Find the present value of this stream. What is the maximum market value of your appartment? Show your work clearly using limits. (Ans: 1 million)

Q 20: Using limits compute, $\displaystyle\int_0^1 \frac{1}{(1-x)^{1/2}}\, dx$ (Ans: 2)