Math 10260 Projects

Why Projects? The goal of these projects is to give you the opportunity to make your own connections between mathematics and modern society by considering a wide variety of problems ranging from economic and environmental issues to social and political situations that can be modeled and solved by mathematical means. They will help you establish connections between Math 10260 and your other courses, and they will allow you to make contributions in areas of your interest and expertise. In addition, they will provide you with an opportunity to interact and collaborate with your classmates.

What Project? Choose a topic from the list below that you are comfortable with, work with your classmates and have fun!

(a) Projects listed in Chapters 5 to 11 of your textbook.

(b) A project listed under "Sample Project Topics."

(c) Other courses you are able to establish a connection with math 10260.

(d) Anything that you find interesting and is approved by your teacher.

The **Rules** are:

(a) You can work in groups of size 1-4 students drawn from any section of Math 10260.

(b) Each group submits two copies of their paper. One must be submitted online via email to the course instructor, while another must be printed and submitted in person.

(c) Each member of the group receives the same score out of 15, which will directly count toward your 20 participation points. This is not all, however, for 10 bonus points will be awarded for great projects. If you and your team create an *exceptional* project, then you may be awarded *extra credit* accordingly.

(d) Your project must clearly display on a cover page the *project title*, the *names* of your team members and the *class sections* each member is from.

(e) The *first draft* is due by the **end of March**.

(f) Your *completed project* is **due by the Last Class Day**.

Sample Project Topics

(1) **What Drives You?** In Walter Isaacson's book "Steve Jobs" (page 570) there is the following statement by Steve Jobs:

"What drove me? I think most creative people want to express appreciation for being able to take advantage of the work that's been done by others before us. I did not invent the language of mathematics I use it. I make little of my own food, none of my own clothes. Everything I do depends on other members of our species and the shoulders that we stand on. And a lot of us want to contribute something back to our species and to add something to the flow. It's about trying to express something in the only way that most of us know how–because we can't write Bob Dylan songs or Tom Stoppard plays. We try to use the talents we do have to express our feelings, to show our appreciation of all the contributions that came before us, and to add something to that flow. That's what has driven me."

Can mathematics help you add something to "the flow" that Steve Jobs is talking about?

(2) **The Social Security.** Some experts project that the Social Security shortfall over the next 75 years will be about four trillion dollars. Is that true? How do they know? Make your contribution in the national debate about saving Social Security using ideas and techniques you learned in Math 10260 (for example, income streams).

(3) **The Deficit.** Visit the Webpage of the Congressional Budget Office (CBO) and try to make sense of the numbers you will find in "Current Budget Projections." How are these estimates made? Are

these figures indicative of a healthy budget? Why or why not? What can be done to make things better? Note that income streams are useful in making projections.

(4) **Sub-prime Loans.** What are sub-prime loans and what do they have to do with the 2008 housing and banking crisis?

(5) **Ponzi Scheme.** In 2008 the Securities and Exchange Commission (SEC) investigated an approximately $50 billion fraud, a Ponzi scheme, perpetrated by Bernard Madoff and the asset management company that he ran. Explain what a Ponzi scheme is and why in Madoff's case it went unnoticed by the SEC for so many years that it became incredibly massive.

(6) **Arctic National Wildlife Refuge: To drill or not to drill?** A question for public debate these days is whether the Arctic National Wildlife Refuge (ANWR) contains enough oil to make its extraction worth both the economic cost and the environmental risk. Weigh scientific predictions about the possible economic cost and environmental risk of expanded oil drilling in these areas versus the benefits that they might bring. Do the math: how do you weigh the cost/benefit analysis?

(7) **Energy Conservation.** Some claim that there are ways for saving about 20% of the energy we consume today. Examine this claim by doing some quantitative analysis on energy consumption and energy wasted.

(8) **Oil Price.** Is the current oil price the result of world demand & supply and/or market manipulation? Draw your own conclusions by collecting data from reliable sources and analyzing them using the mathematics you learned in Math 10260.

(9) **Wind Energy.** Collect data about wind energy production in the U.S. since 2000 and draw a curve that fits these data. Also, draw the oil-price curve using data from reliable sources. Furthermore, compare the shape of these curves and make sense of the current projections of wind energy production for the next 10-20 years. Finally, find out for which country in the world the percentage of the energy it uses from wind is maximum.

(10) **Solar Energy**: Many think that the potential of solar energy is so great, that it will be the future solution to our energy problems. Solar 'farms' in the southwest of the United States *could* provide the energy needs for the entire country. What is stopping us? Some major problems lie in price and efficiency. Collect data about the change in price of solar energy versus that of carbon-based energy in the U.S. since 2000 and model functions to fit the data. What needs to happen before we can switch to solar energy?

(11) **How much renewable energy do we use?** Visit the Energy Information Administration to find U.S. energy statistics (e.g. see http://tonto.eia.doe.gov/energy_in_brief/renewable_energy.cfm) and provide an answer to this question. Use the quantitative skills you acquired in Math 10260 to make your answer clear and informative.

(12) **Sustainable Development.** How would you define the concept of sustainable development? Is the current socioeconomic system consistent with such a concept? If not, then provide some explanation.

(13) **The nearly $800 billion stimulus package.** The following is from a speech given by President-elect Barack Obama on January 11, 2009, at George Mason University, to defend his stimulus package (with text from New York Times.): "I don't believe it's too late to change course, but it will be if we don't take dramatic action as soon as possible. If nothing is done, this recession could linger for years. The unemployment rate could reach double digits. Our economy could fall $1 trillion short of its full capacity, which translates into more than $12,000 in lost income for a family of four. We could lose a generation of potential and promise, as more young Americans are forced to forgo dreams of college or the chance to train for the jobs of the future. And our nation

could lose the competitive edge that has served as a foundation for our strength and standing in the world. . . . "

Now, using the materials learned in Math 10260 and your business knowledge make a quantitative analysis of the economic stimulus package. You may wish to explore its impact to the Deficit problem the country faces.

(14) **Mountains Beyond Mountains.** In this inspiring book Tracy Kidder describes the quest of Dr. Paul Farmer, a man who would cure the world. Curing infectious diseases and bringing the lifesaving tools of modern medicine to those who need them most is his lifes calling. Read this book and use the mathematics you have learned in Math 10260 to try to understand, analyze and propose possible solutions to the global health problem.

(15) **Universal Health Care.** What are the benefits and problems of a universal health care system? Examine and compare the health care system of the U.S. and one or two from other developed countries like the U.K., Germany, France, Japan, etc.

(16) **The End of Poverty.** In the preface of this book its author Dr. Jeffrey Sachs (Quetelet Professor of Sustainable Development at Columbia University, Director of the Earth Institute, and Director of the United Nations Millennium Project) writes: "When the end of poverty arrives, as it can and should in our generation, it will be citizens in a million communities in rich and poor countries alike, rather than a handful of political leaders, who will have turned the tide. The fight for the end of poverty is a fight that all of us must join in our own way." Read this very interesting book and use the mathematics you have learned in Math 10260 to try to understand (quantify, analyze) poverty as a world problem, and propose possible solutions that our generation can realize. For example, Chapter 13 on "Making the investments needed to end poverty" provides a good application for the Solow growth model.

(17) **Top Ten.** What are the top 10 major challenges for your generation? Provide some numbers to justify your choices.

(18) **The Paradox of Choice.** In this book, Barry Schwartz, among many other things, claims that freedom of choice can turn into a tyranny of choice. He even uses some math to make his point. For example, in pages 67–73 he uses familiar curves to give a general explanation of how we go about evaluating options and making decisions. Write a report on this very interesting book and try to relate it to ideas you learned in Math 10260.

(19) **Demand and Supply.** Read carefully section 6.1 on consumer and producer surplus, compare it with writings in economic literature, and explain how are demand and supply curves determined.

(20) **Flatland.** Imagine that you live in a plane (a 2D-space) and that you are not able to see 3D shapes. Then, think of ways for visualizing such shapes. A good source of ideas is the book "Flatland" by Edwin Abbott. Read this book and extend its ideas to describe how inhabitants of 3D-space (i.e., humans) could visualize 4D shapes.

(21) **Asian Carp Management.** The population of Asian carp in the Illinois river is rapidly increasing threatening to invade the Great Lakes. Develop a mathematical model to predict its future size and propose a control plan for the protection of the Great Lakes. For more information look at: http://AsianCarp.org/

(22) **A. Income Distribution and Lorentz Curves.** The way that income is distributed throughout a given society is an important object of study for economists. The U.S. Census Bureau collects and analyzes income data, which it makes available at its website, www.census.gov. In 2001, for instance, the poorest 20% of the U.S. population received 3.5% of the money income, while the richest 20% received 50.1%. The *cumulative* proportions of population and income are shown in the following table:

proportion of population	proportion of income
0	0
0.20	0.035
0.40	0.123
0.60	0.268
0.80	0.499
1.00	1.00

For instance, the table shows that the lowest 40% of the population received 12.3% of the total income. We can think of the data in this table as being given by a functional equation $y = f(x)$, where x is the cumulative proportion of the population and y is the cumulative proportion of income. For instance, $f(0.60) = 0.268$ and $f(0.80) = 0.499$. Such a function (or, more properly speaking, its graph) is called a **Lorentz curve**.

(i) Show that $f(x) = 0.1x + 0.9x^2$ is a possible Lorentz curve. Also, compute the income received by the lowest 0 %, 50%, and 100% of the population.

(ii) Show that $f(x) = 0.3x + 0.9x^2$ is **not** a Lorentz curve.

(iii) For the Lorentz curve in (i) show the following properties:

(a) $f(0) = 0, f(1) = 1,$ and $0 \leq f(x) \leq 1$ for all $0 \leq x \leq 1$,

(b) $f(x)$ is an increasing function,

(c) $f(x) \leq x$ for all x, $0 \leq x \leq 1$.

(iv) Explain why properties (a)–(c) hold for every Lorentz curve.

(v) Write many other different formulas for Lorentz curves.

(vi) Using real data produce Lorentz curves for the U.S. in 2006.

(vii) Sketch the graph of a Lorentz curve and compare it with the line $y = x$.

B. Coefficient of Inequality. If the Lorentz curve of a country is given by $f(x) = x$ then its total income is distributed equally. Otherwise there are inequalities present in the distribution of income, which are measured by the following number:

$$\textbf{coefficient of inequality} = 2\int_0^1 [x - f(x)]dx,$$

which is also called the **Gini Index**.

(i) Compute the coefficient of inequality when $f(x) = 0.1x + 0.9x^2$.

(ii) Show that the Gini Index is the **ratio** of the area of the region between $y = f(x)$ and $y = x$ to the area of the region under $y = x$, and provide an economic inerpretation of this ratio.

(iii) Using real data estimate the Gini Index of the U.S. in 2006.

(23) **A.** (**The Cobb-Douglas Production Function**) Show that the production function $Q(K, L)$ having the properties:

(i) (Marginal Product of Capital) $\cdot$ (Capital) $= \alpha \cdot$ (Output),

(ii) (Marginal Product of Labor) $\cdot$ (Labor) $= (1 - \alpha) \cdot$ (Output),

for some constant α, $0 < \alpha < 1$, **must be** of the form $Q(K, L) = AK^{\alpha}L^{1-\alpha}$, for some constant A.

B. Read and understand the Solow Growth Model (Section 9.3) and do exercise 1, or 2, or 3 on page 607.

(24) **A.** You are 35 years old and your company offers you the following three retirement plans:

(Plan 1) At the beginning it deposits \$50,000 into an account A and nothing more during the next 30 years.

(Plan 2) For the next 30 years it deposits money continuously into an account B at a rate of $10,000$ dollars per year.

(Plan 3) At the age of 65 you will receive \$1,200,000 and nothing more during the next 30 years you will be working there.

If the accounts A and B yields 8% interest, compounded continuously, which option will you choose? Explain your answer.

B. Do part A again with interest rate at 10% compounded monthly. For Plan 2, assume that money will be deposited monthly into account B. To complete this part, you will have to set up a geometric series that gives the value of your retirement account. Go to your notes for continuous compounding and modify the set up for discrete compounding. Explain what each of the terms in the geometric series means. You should state clearly the first term, common ratio, and the formula you use to obtain the value of your retirement account from the geometric series. How would this change your decision in part A?

(25) **A.** A homeowner takes out a 20-year mortgage with an interest rate of 5% compounded continuously. The homeowner plans to make payments totalling \$1,500 per month. Let $M(t)$ be the amount owed after t years. Write an initial value problem modelling this situation. Then find the maximum amount of mortgage that the homeowner can afford.

B. Do part A again with interest rate at 5% compounded **quarterly**. To complete this part, you will have to set up a geometric series that gives the value of the mortgage. Go to your notes for continuous compounding and modify the set up for discrete compounding. Explain what each of the terms in the geometric series means. You should state clearly the first term, common ratio, and the formula you use to obtain the mortgage value from the geometric series. (Hint: You should prorate the interest because you are paying monthly.)

(26) Read carefully section 6.4 on population models and then do exercises 27 and 28 on page 445.

(27) **What does calculus have to do with change?** The two central concepts in calculus are the derivative (instantaneous rate of change) and the integral (total change). Both are based on the fundamental calculus idea of *"using elementary concepts (like slope of a line and area of a rectangle) to approximate advanced concepts (like slope of a curve and area enclosed by a curve)."* Write in your own words the way you understand these concepts. Give examples from mathematics and its applications to demonstrate them.