Name ______________________________ Date ____________________

Math 10260 Final Exam Review Activity

1. The demand function D and supply function S of a model of jeans for the clothing company Lucky Clover's Wear are given, in term of quantity q, by $p = e^{4-2q}$; $p = e^{q+1}$ $(p \geq 0)$

   (a) Give a sketch of the demand and supply functions. Label clearly which is the demand curve and which is the supply curve.

   (b) Find the consumer surplus and the producer surplus. (Ans: $q_e = 1$, $p_e = e^2$, $CS = 16.22$, $PS = 2.72$)

2. If $f(2) = \ln 2$, $f'(2) = -2$, $f''(2) = 3$. Find the 2nd degree Taylor polynomial of $g(x) = e^{f(x)}$ about 2. (Ans: $P_2(x) = 2 - 4(x-2) + 7(x-2)^2$)

3. Find the equation of the tangent plane to the surface $f(x, y) = xe^{y+2} + y/x^2$ at the point $(1, -2)$.

   (Ans: $z + 1 = 5(x - 1) + 2(y + 2)$)

4. Assume that the weekly profit, in dollars, of a Seattle's Best coffee franchise is normally distributed with the mean $\mu = 4,500$ and the standard deviation $\sigma = 500$. What percentage of the weeks will the profit be between \$4,000 and \$5,500? (Ans: 0.8185)

5. When Jeans company uses x units of labor and y units of capital then it produces $f(x, y) = 2x^{1/2}y^{1/2}$ pairs of pants. Given that each unit of labor cost \$80 and each unit of capital cost \$20, and that the company's budget is \$16,000, what input (x, y) of labor and capital will maximize the company's product output? Draw several level curves of $f(x, y)$ and the constraint curve showing the graphical property that gives maximum output. (Ans: $x = 100, y = 400$)

6. Let $y(t)$ be the solution to the initial value problem $y' = t + 2\ln y$, $y(-1) = 1$.

   (a) Find $P_3(t)$, the 3rd-degree Taylor polynomial of $y(t)$ about -1.

   (Ans: $P_3(t) = 1 - (t + 1) - \frac{1}{2}(t + 1)^2 - \frac{2}{3}(t + 1)^3$)

   (b) Use (a) to approximate $y(-0.9)$. Would you use $P_3(t)$ to estimate $y(1)$?

7. Assume that in a sunflower field the number of the seeds in a sunflower head is normally distributed with expected value $\mu = 500$ and standard deviation $\sigma = 40$. Find the probability that a randomly selected sunflower head has more than 600 seeds. (Ans: 0.0062)

8. Assume that the lifetime in months of a certain electric component is a random variable X with probability density function $f(x) = \frac{1}{10}e^{-x/10}, 0 \leq x \leq \infty$. Find (a) the cumulative distribution function for X, (b) the expected lifetime of this component, and (c) the probability that the component's life is at least 15 months. (Ans: (a) $F(x) = 1 - e^{-x/10}$, (b) $E(X) = 10$, (c) $P(x \geq 15) = e^{-1.5}$)

9. Harry and Sally are supposed to meet at noon on the steps of Metropolitan Museum of Art. Sally is always exactly on time, but Harry is known to arrive anywhere between 10 minutes early and 20 minutes late with a uniform distribution.

   (a) What is the probability that Harry will arrive before Sally?

   (b) What is the probability that Sally has to wait more than 15 minutes?

   (c) On average, how long should Sally expect to wait? (Ans: (a) 1/3, (b) 1/6, (c) 5 minutes)

10. Let $f(x, y) = 10ye^{xy} + x^2y + x\ln(3y + 5)$, $\dfrac{\partial^2 f}{\partial x \partial y} \stackrel{?}{=}$ (Ans: $20ye^{xy} + 10xy^2e^{xy} + 2x + \frac{3}{3y+5}$)

11. Find the limit $\displaystyle\lim_{h \to 0} \frac{(y + h)e^{x(y+h)} - ye^{xy}}{h}$. (Ans: $xye^{xy} + e^{xy}$)

12. Isaac, recovering from an illness, determines that his daily average weight changes according to the differential equation

$$\frac{dw}{dt} = -5\ln\frac{w}{180}$$

where t is in days and w is in pounds. At the beginning, Isaac was 165 pounds.

(a) Is there an average daily weight that he tends not to deviate from once he achieves it? (Ans: 180)

(b) Sketch the graph of his weight $w(t)$ and explain its shape.

(c) Use Euler's method with three steps to estimate his weight after 15 days later. (Ans: 170.6)

13. An urn contains two red and three white chips. Two are chosen, without replacement.

(a) What is the probability that both are red? (Apply Multiplicative rule for (a) and (c))

(b) What is the probability that at least one is white?

(c) What is the probability both are white, given that at least one is white? (Ans: (a) 1/10, (b) 9/10, (c) 1/3)

14. Consider the Taylor series for $\ln(x-2)$ about 3: $\ln(x-2) = \sum_{k=1}^{\infty}(-1)^{k-1}\frac{(x-3)^k}{k}$ for $2 < x \le 4$

(a) Estimate $\ln(1.3)$ with the 4th-degree Taylor Polynomial of $\ln(x-2)$ about 3.

(Hint: Check your work against the calculator)

(b) Write the error (remainder term) for the estimate in (a) as an infinite sum. Give at least the first three terms of the infinite sum.

15. Show that the function $f(x) = \ln x$ is a probability density function for a continuous random variable X with range $[1, e]$. Find also the expected value and variance of X.

(Ans: $E(X) = \frac{e^2+1}{4}$; $Var(X) = \frac{2e^3+1}{9} - \frac{(e^2+1)^2}{16}$)

16. Let $g(y)$ be the function whose graph is as shown below. Consider the autonomous differential equation $\frac{dy}{dt} = g(y)$

(a) What are the equilibrium solutions of the differential equation?

(Ans: $y=-2$, $y=0$, $y=2$)

(b) Determine the nature of each equilibrium solutions.

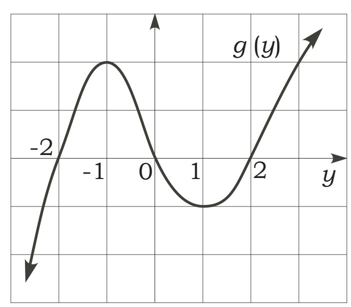
(Ans: $y=-2$ (unstable), $y=0$ (asymptotically stable), $y=2$ (unstable))

(c) State the values of y where the solutions change their concavity.

(Ans: $y=-1$, $y=1$)

(d) If $y(0) = -1$, find the limit $\lim_{t\to\infty} y(t)$.

(Ans: 0)



17. Suppose that statistical evidence indicates that the probability of a new born being boy is 0.51. For a randomly selected family of five children, what is the probability that two of them are boys? the mean number of boys in such a family? standard deviation? (Ans:$10(0.51)^2(0.49)^3$, $5(0.51)$, $\sqrt{5(0.51)(0.49)}$)

18. Suppose that 2% of the computer chips manufactured by a certain company are defective. 98% of those defective are rejected by quality control, but 1% of those good are incorrectly rejected by quality control. Find (a) the probability of a chip being rejected by quality control, (b) the probability of finding a defective given it is rejected by quality control. (Ans: (a) 0.0294; (b) 2/3)

19. A Pharmaceutical company estimates that the demand function for one of its drugs in US is $p_1 = 25 - 0.1x$, while the demand function for the same drug in China market is $p_2 = 20 - 0.05y$, where x and y are in thousands of units and p_1 and p_2 are in dollars per unit. If the cost function is $C(x, y) = 10 + 2(x + y)$, in thousands of dollars, then find the quantities x and y and the prices that maximize the company profit. (Ans: (115,180))

20. A recent weather report indicates that there is a 69% chance that is it would rain, a 21% chance that it would hail, and a 71% chance that it would rain **or** hail. Let R be the event that it rains and H be the event that it would hail. Compute $P(R|H)$ and $P(H|R)$. Are R and H independent?

(Ans: 19/21, 19/69, $P(R \cap H) \stackrel{?}{=} P(R) \cdot P(H)$)

21. Let X be a continuous random variable with probability function $f(x) = c(4 - x^2)$ on the interval $[0, 2]$, where c is some constant. Find (i) c, (ii) the expected value, (iii) variance and (iv) standard deviation. (Ans: (i) 3/16, (ii)3/4, (iii) 19/80, (iv) $\sqrt{19/80}$)

22. A patient is injected once a day with 10 units of a certain drug. Suppose this drug is eliminated by the body exponentially with any single injection leaving an amount of $10e^{-0.6t}$ remaining after t days. Use an infinite geometric series to approximate the number of units of the drug remaining in the patient's system after a very long time. We assume that the measurement is done before each injection. (Ans: 12.1637)

23. A philanthropist invests $1,000,000 in January to set up a perpetual fund for an annual scholarship. The first withdrawal is $5000 one year later. Withdrawals continue annually, with the amount increasing by rate r every year. If the money is invested in an account that pays 8% annual interest compounded **four times a year**, what is the present value of the **third** withdrawal? Find the largest r that ensures perpetual funding of the scholarship. Round your answer to 4 decimal places.

(Ans: $5000(1 + r)^2(1.02)^{-12}$; 0.0774)

24. If you wanted to find the least-squares line of the form $y = ax + b$ for the given data $(1, -1)$, $(-1, 4)$, $(2, 0)$, what would be the error function $E(a, b)$ you need to minimize in order to determine the values of a and b? Find a and b. (Ans: $(a + b + 1)^2 + (-a + b - 4)^2 + (2a + b)^2$; $a = -1.5$; $b = 2$)

25. The rate of change of world population $p(t)$, in billions, is given below. If the world population is 4.45 billion in the year of 2000, use Euler's Method with $\Delta t = 5$ to estimate the population in the year of 2015. (Ans: 5.615 billion)

t	0	5	10	15
$p'(t)$	0.076	0.078	0.079	0.080

26. The size p(t) of a certain population is model by $\dfrac{dp}{dt} = 0.2p(p - 2)\left(1 - \dfrac{p}{10}\right)$. Find the threshold level of this population. Sketch the graphs of the population if (a) $p(0) = 1$; (b) $p(0) = 3$ and (c) $p(0) = 13$. (Ans: 2)

27. Which of the following savings accounts yields the most after **ten** years:

(a) For the next 10 years, money is deposited continuously into Account A which pays 6% interest compounded continuously, at a rate of $600t$ dollars per year. (Ans: FV_a=37,019.80$)

(b) An Account B with principal $25,000 paying interest at 5% compounded **monthly.**

(Ans: FV_b=$41,175.24)

(c) On the first of January this year and for the next four years (no more after), $6000 is deposited into Account C earning 4% annual interest compounded continuously. (Ans: FV_c=$41,379.97)

28. Find the present value of each account in the question above. (Ans: PV_a=$20,316.90; PV_b=$25,000; PV_c=$27,737.82)

29. A woman retires on her 65th birthday and begins to withdraw \$3000 a **month** from her savings account, which earns 7%, compounded continuously. At that rate, she will run out of money on her 100th birthday. Let $M(t)$ be the amount in the account t years after her 65th birthday. Find an initial value problem satisfied by $M(t)$. You may assume that money is withdrawn continuously from the account. (Ans: $M'(t) = -36000 + 0.07M(t)$, $M(35) = 0$)

30. Find the present value of a perpetual income stream flowing continuously at a rate of \$80,000 per year, with interest compounded continuous at 10%. Also write down but do not solve an initial value problem that models the amount of money in the account t years after it has been set up.

(Ans: PV=\$800,000; $M'(t) = 80000 + 0.1M(t)$, $M(0) = 0$)

31. Assuming that the pattern continues, which of the following series are geometric series? Find the sum of the geometric series that converges.

(a) $8 + 8(1.03) + 8(1.03)^2 + 8(1.03)^3 + 8(1.03)^4 + \cdots$ (Ans: Divergent geometric series)

(b) $5 - 5(0.03) + 5(0.03)^2 - 5(0.03)^3 + 5(0.03)^4 - \cdots$ (Ans: $\frac{5}{1.03}$)

(c) $5 + 6(0.03) + 7(0.03)^2 + 8(0.03)^3 + 9(0.03)^4 + \cdots$ (Ans: Not geometric)

(d) $2 + \frac{4}{e} + \frac{8}{e^2} + \frac{16}{e^3} + \frac{32}{e^4} + \cdots$ (Ans: $\frac{2}{1 - 2/e}$)

32. The 4th-degree polynomial of the function $f(x)$ about -2 is given by

$$P_4(x) = 5 - 2(x+2) + 3(x+2)^2 - (x+2)^4.$$

(a) What is the slope of $f(x)$ at $x = -2$? Find the equation of the tangent line to the graph of $f(x)$ at $x = -2$. Give your answer in the form $y = mx + b$. (Ans: $y = -2x + 1$)

(b) Write down the values of $f(-2)$, $f''(-2)$, $f'''(-2)$, and $f^{(4)}(-2)$.

(Ans: $f(-2) = 5$, $f''(-2) = 6$, $f'''(-2) = 0$, and $f^{(4)}(-2) = -24$)

33. Evaluate the following indefinite integrals:

a. $\int e^{2x} - 2e^{-x} + e + x^{3/2}\,dx$

d. $\int \frac{(\ln x)^2}{x}\,dx$

g. $\int xe^{3x}\,dx$

b. $\int x(x^2+1)^{3/2}\,dx$

e. $\int_0^2 (x-1)^{25}\,dx$

h. $\int_1^3 x^2 \ln x\,dx$

c. $\int \frac{x^2}{\sqrt{1-x}}\,dx$

f. $\int_1^4 \frac{1}{x^2}\sqrt{1 + \frac{1}{x}}\,dx$

i. $\int \frac{1}{6 - x - x^2}\,dx$

34. The marginal cost of a product is given by $MC(x) = 4x^3 - \frac{1}{x^2}$ where x is the number of items produced in units of millions. Find the total change in the cost (in dollars) if production level changes from 2 million to 4 million. (Ans: \$239.75)

35. A manufacturer of a certain product determines that its production function is given by $Q(L, K) = \sqrt{LK}$, where L is the number of units of labor and K is the number of units of capital. Find the marginal productivity of labor and marginal productivity of capital for $L = 25$ and $K = 64$

(Ans: $MPK = 5/16$, $MPL = 4/5$)

36. Consider the three points $A(1, 1, 2)$, $B(-1, 2, 0)$, and $C(2, 3, 1)$. Compute the distance AB and BC. Is ABC a right angle triangle? Find also the equation of the plane containing ABC.

(Ans: 3, $\sqrt{11}$, $z = \frac{3}{5}(x+1) - \frac{4}{5}(y-2)$)